## A LEVEL

## Specification

## MATHEMATICS B (MEI)

## H640

For first assessment in 2018

# MEI <br> Innovators in ${ }^{\circledR}$ <br> Mathematics <br> Education 

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Learners must complete components 01,02 and 03 to be awarded OCR A Level in Mathematics B (MEI). Contents is in these three areas:
1 Pure mathematics
2 Mechanics
3 Statistics.

## Content Overview

| Component 01 assesses content from |
| :---: |
| areas 1 and 2 |


| Component 02 assesses content from |
| :---: |
| areas 1 and 3 |


| Pure Mathematics and |
| :---: |
| Mechanics |
| $(01)$ |
| 100 marks |
| 2 hour written paper |

$36.4 \%$
of total
A Level

| Pure Mathematics and |
| :---: |
| Statistics |
| (02) |
| 100 marks |
| 2 hour written paper |

Component 03 assesses content from area 1 (areas 2 and 3 are assumed knowledge)

| Pure Mathematics and |
| :---: |
| Comprehension |
| $(03)$ |
| 75 marks |
| 2 hour written paper |

$36.4 \%$
of total
A Level

## 27.3\% <br> of total <br> A Level

Percentages in the table above are rounded to 1 decimal place, exact component proportions are:
$36 \frac{4}{11}, 36 \frac{4}{11}, 27 \frac{3}{11}$.

All A level qualifications offered by OCR are accredited by Ofqual, the Regulator for qualifications offered in England. The accreditation number for OCR's A Level in Mathematics B (MEI) is QN: 603/1002/9.

Learners will be given formulae in each assessment on pages 2 and 3 of the question paper. See section $5 d$ for a list of these formulae.

## 1a. Why choose an OCR A Level in Mathematics B (MEI)?

OCR A Level Mathematics B (MEI) has been developed in partnership with Mathematics in Education and Industry (MEI).

OCR A Level in Mathematics B (MEI) provides a framework within which a large number of young people continue the subject beyond GCSE (9-1). It supports their mathematical needs across a broad range of subjects at this level and provides a basis for subsequent quantitative work in a very wide range of higher education courses and in employment. It also supports the study of AS and A Level Further Mathematics.

OCR A Level in Mathematics B (MEI) builds from GCSE (9-1) level mathematics and introduces calculus and its applications. It emphasises how mathematical ideas are interconnected and how mathematics can be applied to model situations using algebra and other representations, to help make sense of data, to understand the physical world and to solve problems in a variety of contexts, including social sciences and business. It prepares students for further study and employment in a wide range of disciplines involving the use of mathematics.

AS Level Mathematics B (MEI), which can be co-taught with the A Level as a separate qualification, consolidates and develops GCSE level mathematics and supports transition to higher education or employment in any of the many disciplines that make use of quantitative analysis, including those involving calculus.

This qualification is part of a wide range of OCR mathematics qualifications, which allows progression from Entry Level Certificate and Functional Skills, through GCSE (9-1) and Additional Maths to Core Maths, AS and A level.

All of our mathematics specifications have been developed by OCR and MEI with subject and teaching experts. We have worked in close consultation with teachers from learned societies, industry and representatives from Higher Education (HE).

We recognise that teachers want to be able to choose qualifications that suit their learners so we offer two suites of qualifications in mathematics and further mathematics.

Mathematics A builds on our existing popular course. We've based the redevelopment of our current suite around an understanding of what works well in centres and have updated areas of content and assessment where stakeholders have identified that improvements could be made.

Mathematics $B$ (MEI) is based on the existing suite of qualifications assessed by OCR. MEI is a long established, independent curriculum development body. MEI provides advice and CPD relating to all the curriculum and teaching aspects of the course. It also provides teaching resources, which for this specification can be found on the website (www.mei.org.uk).

## 1b. What are the key features of this specification?

Exemplar content.
Clear command words and guidance on calculator use.
Separate Question Papers and Answer Booklets so that students can always see the whole of a question at one time and to allow for diagrams and tables for them to work on.

Easy to follow mark schemes with complete solutions and clear guidance.
Stretch and challenge questions designed to allow the most able learners the opportunity to demonstrate the full extent of their knowledge and skills, and to support the awarding of A* grade at A Level.

Applied content (statistics and mechanics) assessed on separate papers so that the content domains assessed on any given paper don't cover both at once.

| Mathematics A H240 | Mathematics B (MEI) H640 |
| :--- | :--- |
| Single pre-release data set designed to last the life <br> of the qualification. <br> Components 02 and 03 are in two sections: <br> section A on the Pure Mathematics content; <br> section B on either Statistics or Mechanics. | Three data sets available at all times, so that you can <br> use all three for teaching, but for each cohort of <br> students just one will be the context for some of the <br> questions in the exam. Each data set will be clearly <br> labelled as to when it is used. <br> Components 01 and 02 are in two sections: <br> section A consists of shorter questions with minimal <br> reading and interpretation; <br> section B includes longer questions and problem <br> solving. |

## 1c. Aims and learning outcomes

OCR A Level in Mathematics B (MEI) encourages learners to:

- understand mathematics and mathematical processes in a way that promotes confidence, fosters enjoyment and provides a strong foundation for progress to further study
- extend their range of mathematical skills and techniques
- understand coherence and progression in mathematics and how different areas of mathematics are connected
- apply mathematics in other fields of study and be aware of the relevance of mathematics to the world of work and to situations in society in general
- use their mathematical knowledge to make logical and reasoned decisions in solving problems both within pure mathematics and in a variety of contexts, and communicate the mathematical rationale for these decisions clearly
- reason logically and recognise incorrect reasoning
- generalise mathematically
- construct mathematical proofs
- use their mathematical skills and techniques to solve challenging problems which require them to decide on the solution strategy
- recognise when mathematics can be used to analyse and solve a problem in context
- represent situations mathematically and understand the relationship between problems in context and mathematical models that may be applied to solve them
- draw diagrams and sketch graphs to help explore mathematical situations and interpret solutions
- make deductions and inferences and draw conclusions by using mathematical reasoning
- interpret solutions and communicate their interpretation effectively in the context of the problem
- read and comprehend mathematical arguments, including justifications of methods and formulae, and communicate their understanding
- read and comprehend articles concerning applications of mathematics and communicate their understanding
- use technology such as calculators and computers effectively and recognise when such use may be inappropriate
- take increasing responsibility for their own learning and the evaluation of their own mathematical development.


## 1d. How do I find out more information?

If you are already using OCR specifications you can contact us at: www.ocr.org.uk.

If you are not already a registered OCR centre then you can find out more information on the benefits of becoming one at: www.ocr.org.uk.

Get in touch with one of OCR's Subject Advisors:

Email: maths@ocr.org.uk

Twitter: @OCR Maths

Customer Contact Centre: 01223553998

Teacher resources, blogs and support: available from: www.ocr.org.uk

Sign up for our monthly Maths newsletter, Total Maths

Access CPD, training, events and support through OCR's CPD Hub

Access our online past papers service that enables you to build your own test papers from past OCR exam questions through OCR's ExamBuilder

Access our free results analysis service to help you review the performance of individual learners or whole schools through Active Results

## 2 The specification overview

## 2a. Content of A Level in Mathematics B (H640)

This A level qualification builds on the skills, knowledge and understanding set out in the whole GCSE (9-1) subject content for mathematics for first teaching from 2015.

A Level Mathematics $B(M E I)$ is a linear qualification, with no options. The content is listed below, under three areas:

1. Pure mathematics includes proof, algebra, graphs, sequences, trigonometry, logarithms, calculus and vectors
2. Mechanics includes kinematics, motion under gravity, working with forces including friction, Newton's laws and simple moments
3. Statistics includes working with data from a sample to make inferences about a population, probability calculations, using binomial and Normal distributions as models and statistical hypothesis testing.

There will be three examination papers, at the end of the course, to assess all the content:

- Pure Mathematics and Mechanics (01), a 2 hour paper assessing pure mathematics and mechanics
- Pure Mathematics and Statistics (02), a 2 hour paper assessing pure mathematics and statistics
- Pure Mathematics and Comprehension (03), a 2 hour paper assessing pure mathematics.

Although the content is listed under three separate areas, links should also be made between pure mathematics and each of mechanics and statistics; some of the links that can be made are indicated in the notes in the detailed content.

The overarching themes should be applied, along with associated mathematical thinking and understanding, across the whole of the detailed content in pure mathematics, statistics and mechanics.

## 2b. Pre-release material

Pre-release material will be made available in advance of the examinations. It will be relevant to some (but not all) of the questions in component 02. The pre-release material will be a large data set (LDS) that can be used as teaching material throughout the course. It is comparable to a set text for a literature course. It will be published in advance of the course. In the examination it will be assumed that learners are familiar with the contexts covered by this data set and that they have used a spreadsheet or other statistical software when working with the data. Questions based on these assumptions will be set in Pure Mathematics and Statistics (02).

The intention is that these questions should give a material advantage to learners who have studied, and are familiar with, the prescribed large data set. They might include questions requiring learners to interpret data in ways which would be too demanding in an unfamiliar context.

Learners will NOT have a printout of the pre-release data set available to them in the examination but selected data or summary statistics from the data set may be provided, within the examination paper.

Different LDS will be issued for different years; three large data sets will be available at any time. Only one of these data sets will be used in a given series of examinations. Each data set will be clearly labelled with the year of the examination series in which it will be used. The expectation is that teachers will use all three data sets but they do have the choice of concentrating more on the examination data set (or of using just that one if they wish). To support progression and co-teachability, students taking AS Level Mathematics $B$ (MEI) will use the same data set if they take A Level Mathematics B (MEI) in the following year.

The intention of the large data set is that it and associated contexts are explored in the classroom using technology, and that learners become familiar with the context and main features of the data.

To support the teaching and learning of statistics with the large data set, we suggest that the following activities are carried out throughout the course:

1. Exploratory data analysis: Learners should explore the LDS with both quantitative and visual techniques to develop insight into underlying patterns and structures, suggest hypotheses to test and to provide a reason for further data collection. This will include the use of the following techniques.

- Creating diagrams: Learners should use spreadsheets or statistical graphing tools to create diagrams from data.
- Calculations: Learners should use appropriate technology to perform statistical calculations.
- Investigating correlation: Learners should use appropriate technology to explore correlation between variables in the LDS.

2. Modelling: Learners should use the LDS to provide estimates of probabilities for modelling and to explore possible relationships between variables.
3. Repeated sampling: Learners should use the LDS as a model for the population to perform repeated sampling experiments to investigate variability and the effect of sample size. They should compare the results from different samples with each other and with the results from the whole LDS.
4. Hypothesis testing: Learners should use the LDS as the population against which to test hypotheses based on their own sampling.

## 2c. Use of technology

It is assumed that learners will have access to appropriate technology when studying this course such as mathematical and statistical graphing tools and spreadsheets. When embedded in the mathematics classroom, the use of technology can facilitate the visualisation of abstract concepts and deepen learners' overall understanding. Learners are not expected to be familiar with any particular software, but they are expected to be able to use their calculator for any function it can perform, when appropriate. Examination questions may include printouts from software which learners will need to complete or interpret. They should be familiar with language used to describe spreadsheets such as row, column and cell.

To support the teaching and learning of mathematics using technology, we suggest that the following activities are carried out through the course:

1. Graphing tools: Learners should use graphing software to investigate the relationships between graphical and algebraic representations, e.g. understanding the effect of changing the parameter $k$ in the graphs of $y=\frac{1}{x}+k$ or $y=x^{2}-k x$; e.g. investigating tangents to curves.
2. Spreadsheets: Learners should use spreadsheets to generate tables of values for functions, to investigate functions numerically and as an example of applying algebraic notation. Learners should also use spreadsheet software to investigate numerical methods for solving equations and for modelling in statistics and mechanics.
3. Statistics: Learners should use spreadsheets or statistical software to explore data sets and statistical models including generating tables and diagrams, and performing standard statistical calculations.
4. Mechanics: Learners should use graphing and/or spreadsheet software for modelling, including kinematics and projectiles.
5. Computer Algebra System (CAS): Learners could use CAS software to investigate algebraic relationships, including derivatives and integrals, and as an investigative problem solving tool. This is best done in conjunction with other software such as graphing tools and spreadsheets.

## Use of calculators

Calculators must comply with the published Instructions for conducting examinations, which can be found at http://www.jcq.org.uk/

It is expected that calculators available in the examinations will include the following features:

- An iterative function such as an ANS key.
- The ability to compute summary statistics and access probabilities from the binomial and Normal distributions

When using calculators, candidates should bear in mind the following:

1. Candidates are advised to write down explicitly any expressions, including integrals, that they use the calculator to evaluate.
2. Candidates are advised to write down the values of any parameters and variables that they input into the calculator. Candidates are not expected to write down data transferred from question paper to calculator.
3. Correct mathematical notation (rather than "calculator notation") should be used; incorrect notation may result in loss of marks.

## 2d. Command words

It is expected that learners will simplify algebraic and numerical expressions when giving their final answers, even if the examination question does not explicitly ask them to do so.

Example 1:
$80 \frac{\sqrt{3}}{2}$ should be written as $40 \sqrt{3}$.

Example 2:
$\frac{1}{2}(1+2 x)^{-\frac{1}{2}} \times 2$ should be written as either $(1+2 x)^{-\frac{1}{2}}$ or $\frac{1}{\sqrt{1+2 x}}$.

Example 3:
$\ln 2+\ln 3-\ln 1$ should be written as $\ln 6$.

## Example 4:

The equation of a straight line should be given in the form $y=m x+c$ or $a x+b y=c$ unless otherwise stated.

The meanings of some instructions and words used in this specification are detailed below.

Other command words, for example "explain" or "calculate", will have their ordinary English meaning.

## Exact

An exact answer is one where numbers are not given in rounded form. The answer will often contain an irrational number such as $\sqrt{3}, \mathrm{e}$ or $\pi$ and these numbers should be given in that form when an exact answer is required.

The use of the word 'exact' also tells learners that rigorous (exact) working is expected in the answer to the question.

Example 1:

Find the exact solution of $\ln x=2$.
The correct answer is $\mathrm{e}^{2}$ and not 7.389056 .

## Example 2:

Find the exact solution of $3 x=2$
The correct answer is $x=\frac{2}{3}$ or $x=0.6$, not $x=0.67$ or similar.

## Prove

Learners are given a statement and must provide a formal mathematical argument which demonstrates its validity.

A formal proof requires a high level of mathematical detail, with candidates clearly defining variables, correct algebraic manipulation and a concise conclusion.

## Example Question

Prove that the sum of the squares of any three consecutive positive integers cannot be divided by 3 .

## Example Response

Let the three consecutive positive integers be $n-1, n$ and $n+1$
$(n-1)^{2}+n^{2}+(n+1)^{2}=3 n^{2}+2$
This always leaves a remainder of 2 and so cannot be divided by 3 .

## Show that

Learners are given a result and have to show that it is true. Because they are given the result, the explanation has to be sufficiently detailed to cover every step of their working.

## Example Question

Show that the curve $y=x \ln x$ has a stationary point $\left(\frac{1}{\mathrm{e}},-\frac{1}{\mathrm{e}}\right)$.

## Example Response

$\frac{\mathrm{d} y}{\mathrm{~d} x}=1 . \ln x+x \cdot \frac{1}{x}=\ln x+1$
$\frac{\mathrm{d} y}{\mathrm{~d} x}=0$ for stationary point
When $x=\frac{1}{\mathrm{e}} \Rightarrow \frac{\mathrm{d} y}{\mathrm{~d} x}=\ln \frac{1}{\mathrm{e}}+1=0$ so stationary
When $x=\frac{1}{\mathrm{e}}, y=\frac{1}{\mathrm{e}} \ln \frac{1}{\mathrm{e}} \Rightarrow y=-\frac{1}{\mathrm{e}}$ so $\left(\frac{1}{\mathrm{e}},-\frac{1}{\mathrm{e}}\right)$ is a stationary point on the curve.

## Verify

A clear substitution of the given value to justify the statement is required.

## Example Question

Verify that the curve $y=x \ln x$ has a stationary point at $x=\frac{1}{\mathrm{e}}$.
Example Response

```
\(\frac{\mathrm{d} y}{\mathrm{~d} x}=\ln x+1\)
At \(x=\frac{1}{\mathrm{e}}, \frac{\mathrm{d} y}{\mathrm{~d} x}=\ln \frac{1}{\mathrm{e}}+1=-1+1=0\) therefore it is a stationary point.
```


## Find, Solve, Calculate

These command words indicate that, while working may be necessary to answer the question, no justification is required. A solution could be obtained from the efficient use of a calculator, either graphically or using a numerical method.

## Example Question

Find the coordinates of the stationary point of the curve $y=x \ln x$.
Example Response
( $0.368,-0.368$ )

## Determine

This command word indicates that justification should be given for any results found, including working where appropriate.

## Example Question

Determine the coordinates of the stationary point of the curve $y=x \ln x$.

## Example Response

$\frac{\mathrm{d} y}{\mathrm{~d} x}=1 \cdot \ln x+x \cdot \frac{1}{x}=\ln x+1$
$\ln x+1=0 \Rightarrow x=0.368 \ldots$
When $x=0.368 \ldots, y=0.368 \ldots \times \ln \frac{1}{0.368 \ldots}=-0.368 \ldots$
So $(0.368,-0.368)$

## Give, State, Write down

These command words indicate that neither working nor justification is required.

## Example Question

State the coordinates of the stationary point of the curve $y=(x-3)^{2}+4$.
Example Response
$(3,4)$

## In this question you must show detailed reasoning.

When a question includes this instruction learners must give a solution which leads to a conclusion showing a detailed and complete analytical method. Their solution should contain sufficient detail to allow the line of their argument to be followed. This is not a restriction on a learner's use of a calculator when tackling the question, e.g. for checking an answer or evaluating a function at a given point, but it is a restriction on what will be accepted as evidence of a complete method.

In these examples variations in the structure of the answers are possible, for example using a different base for the logarithms in example 1, and different intermediate steps may be given.

Example 1:
Use logarithms to solve the equation $3^{2 x+1}=4^{100}$, giving your answer correct to 3 significant figures.
The answer is $x=62.6$, but the learner must include the steps $\log 3^{2 x+1}=\log 4^{100},(2 x+1) \log 3=\log 4^{100}$ and an intermediate evaluation step, for example $2 x+1=126.18 \ldots$... Using the solve function on a calculator to skip one of these steps would not result in a complete analytical method.

Example 2:
Evaluate $\int_{0}^{1} x^{3}+4 x^{2}-1 \mathrm{~d} x$
The answer is $\frac{7}{12}$, but the learner must include at least $\left[\frac{1}{4} x^{4}+\frac{4}{3} x^{3}-x\right]_{0}^{1}$ and the substitution $\frac{1}{4}+\frac{4}{3}-1$. Just writing down the answer using the definite integral function on a calculator would therefore not be awarded any marks.

Example 3:
Solve the equation $3 \sin 2 x=\cos x$ for $0^{\circ} \leq x \leq 180^{\circ}$.
The answer is $x=9.59^{\circ}, 90^{\circ}$ or $170^{\circ}$ (to 3 sf ), but the learner must include $\ldots 6 \sin x \cos x-\cos x=0$, $\cos x(6 \sin x-1)=0, \cos x=0$ or $\sin x=\frac{1}{6}$.
A graphical method which investigated the intersections of the curves $y=3 \sin 2 x$ and $y=\cos x$ would be acceptable to find the solution at $90^{\circ}$ if carefully verified, but the other two solutions must be found analytically, not numerically.

## Hence

When a question uses the word 'hence', it is an indication that the next step should be based on what has gone before. The intention is that learners should start from the indicated statement.

You are given that $\mathrm{f}(x)=2 x^{3}-x^{2}-7 x+6$. Show that $(x-1)$ is a factor of $\mathrm{f}(x)$.

Hence find the three factors of $\mathrm{f}(x)$.

## Hence or otherwise

This is used when there are multiple ways of answering a given question. Learners starting from the indicated statement may well gain some information about the solution from doing so, and may already be some way towards the answer. The command phrase is used to direct learners towards using a particular piece of information to start from or to a particular method. It also indicates to learners that valid alternate methods exist which will be given full credit, but that they may be more time-consuming or complex.

Example:
Show that $(\cos x+\sin x)^{2}=1+\sin 2 x$ for all $x$.

Hence or otherwise, find the derivative of $(\cos x+\sin x)^{2}$.

## You may use the result

When this phrase is used it indicates a given result that learners would not normally be expected to know, but which may be useful in answering the question.

The phrase should be taken as permissive; use of the given result is not required.

## Plot

Learners should mark points accurately on the graph in their printed answer booklet. They will either have been given the points or have had to calculate them. They may also need to join them with a curve or a straight line, or draw a line of best fit through them.

Example:
Plot this additional point on the scatter diagram.

## Sketch

Learners should draw a diagram, not necessarily to scale, showing the main features of a curve. These are likely to include at least some of the following.

- Turning points
- Asymptotes
- Intersection with the $y$-axis
- Intersection with the $x$-axis
- Behaviour for large $x$ (+ or - )

Any other important features should also be shown.

Example:
Sketch the curve with equation $y=\frac{1}{(x-1)}$

## Draw

Learners should draw to an accuracy appropriate to the problem. They are being asked to make a sensible judgement about this.

Example 1:
Draw a diagram showing the forces acting on the particle.

Example 2:
Draw a line of best fit for the data.

## 2e. Overarching Themes

These Overarching Themes should be applied, along with associated mathematical thinking and understanding, across the whole of the detailed content in this specification. These statements are intended to direct the teaching and learning of A Level Mathematics, and they will be reflected in assessment tasks.

OT1 Mathematical argument, language and proof

|  | Knowledge/Skill |
| :--- | :--- |
| OT1.1 | Construct and present mathematical arguments through appropriate use of diagrams; sketching <br> graphs; logical deduction; precise statements involving correct use of symbols and connecting <br> language, including: constant, coefficient, expression, equation, function, identity, index, term, <br> variable |
| OT1.2 | Understand and use mathematical language and syntax as set out in the content |
| OT1.3 | Understand and use language and symbols associated with set theory, as set out in the content <br> Apply to solutions of inequalities and probability |
| OT1.4 | Understand and use the definition of a function; domain and range of functions |
| OT1.5 | Comprehend and critique mathematical arguments, proofs and justifications of methods and <br> formulae, including those relating to applications of mathematics |

## OT2 Mathematical problem solving

|  | Knowledge/Skill |
| :--- | :--- |
| OT2.1 | Recognise the underlying mathematical structure in a situation and simplify and abstract <br> appropriately to enable problems to be solved |
| OT2.2 | Construct extended arguments to solve problems presented in an unstructured form, including <br> problems in context |
| OT2.3 | Interpret and communicate solutions in the context of the original problem |
| OT2.4 | Understand that many mathematical problems cannot be solved analytically, but numerical <br> methods permit solution to a required level of accuracy |
| OT2.5 | Evaluate, including by making reasoned estimates, the accuracy or limitations of solutions, <br> including those obtained using numerical methods |
| OT2.6 | Understand the concept of a mathematical problem solving cycle, including specifying the <br> problem, collecting information, processing and representing information and interpreting <br> results, which may identify the need to repeat the cycle |
| OT2.7 | Understand, interpret and extract information from diagrams and construct mathematical <br> diagrams to solve problems, including in mechanics |

## OT3 Mathematical modelling

|  | Knowledge/Skill |
| :--- | :--- |
| OT3.1 | Translate a situation in context into a mathematical model, making simplifying assumptions |
| OT3.2 | Use a mathematical model with suitable inputs to engage with and explore situations (for a given <br> model or a model constructed or selected by the student) |
| OT3.3 | Interpret the outputs of a mathematical model in the context of the original situation (for a given <br> model or a model constructed or selected by the student) |
| OT3.4 | Understand that a mathematical model can be refined by considering its outputs and simplifying <br> assumptions; evaluate whether the model is appropriate |
| OT3.5 | Understand and use modelling assumptions |

## Mathematical Problem Solving Cycle

Mathematical problem solving is a core part of mathematics. The problem solving cycle gives a general strategy for dealing with problems which can be solved using mathematical methods; it can be used for problems within mathematical contexts and for problems in real-world contexts.


| Process | Description |
| :--- | :--- |
| Problem <br> specification <br> and analysis | The problem to be addressed needs to be formulated in a way which allows mathematical <br> methods to be used. It then needs to be analysed so that a plan can be made as to how <br> to go about it. The plan will almost always involve the collection of information in some <br> form. The information may already be available (e.g. online) or it may be necessary to <br> carry out some form of experimental or investigational work to gather it. <br> In some cases the plan will involve considering simple cases with a view to generalising from <br> them. In others, physical experiments may be needed. In statistics, decisions need to be <br> made at this early stage about what data will be relevant and how they will be collected. <br> The analysis may involve considering whether there is an appropriate standard model to <br> use (e.g. the Normal distribution or the particle model) or whether the problem is similar <br> to one which has been solved before. <br> At the completion of the problem solving cycle, there needs to be consideration of <br> whether the original problem has been solved in a satisfactory way or whether it is <br> necessary to repeat the problem solving cycle in order to gain a better solution. <br> For example, the solution might not be accurate enough or only apply in some cases. |
| Information <br> collection | This stage involves getting the necessary inputs for the mathematical processing that will <br> take place at the next stage. This may involve deciding which are the important variables, <br> finding key measurements or collecting data. |
| Processing and | This stage involves using suitable mathematical techniques, such as calculations, graphs <br> or diagrams, in order to make sense of the information collected in the previous stage. <br> This stage ends with a provisional solution to the problem. |
| representation |  |
| Interpretation | This stage of the process involves reporting the solution to the problem in a way which <br> relates to the original situation. Communication should be in clear plain English which <br> can be understood by someone who has an interest in the original problem but is not <br> an expert in mathematics. This should lead into reflection on the solution to consider <br> whether it is satisfactory or if further work is needed. |

## The Modelling Cycle

The examinations will assume that learners have used the full modelling cycle during the course.

Mathematics can be applied to a wide variety of problems arising from real situations but real life is complicated, and can be unpredictable, so some assumptions need to be made to simplify the situation and allow mathematics to be used. Once answers have been obtained, we need to compare with experience to make sure that the answers are useful. For example, the government might want to
know how many primary school children there will be in the future so that they can make sure that there are enough teachers and school places. To find a reasonable estimate, they might assume that the birth rate over the next five years will be similar to that for the last five years and those children will go to school in the area they were born in. They would evaluate these assumptions by checking whether they fit in with new data and review the estimate to see whether it is still reasonable.


## Learning outcomes

Learning outcomes are designed to help users by clarifying the requirements, but the following points need to be noted:

- Content that is covered by a learning outcome with a reference code may be tested in an examination question without further guidance being given.
- Learning outcomes marked with an asterisk * are assumed knowledge and will not form the focus of any examination questions. These statements are included for clarity and completeness.
- Many examination questions will require learners to use two or more learning outcomes at the same time without further guidance being given. Learners are expected to be able to make links between different areas of mathematics.
- Learners are expected to be able to use their knowledge and understanding to reason mathematically and solve problems both within mathematics and in context. Content that is covered by any learning outcome may be required in problem solving, modelling and reasoning tasks even if that is not explicitly stated in the learning outcome.
- Learning outcomes have an implied prefix: 'A learner should...'.
- Each reference code for a learning outcome is unique. For example, in the code Mc1, M refers to Mathematics, c refers to 'calculus' (see below) and 1 means that it is the first such learning outcome in the list.
- The letters used in assigning reference codes to learning outcomes are common to all qualifications in spec B (H630, H640, H635 and H645). Only those used in A level Mathematics $B$ (MEI) are shown below.

| a | algebra | A |
| :---: | :---: | :---: |
| b |  | B |
| c | calculus | C |
| d |  | D |
| e | equations | E |
| f | functions | F |
| g | geometry, graphs | G |
| h |  | H |
| , |  | I |
|  |  | J |
| k | kinematics | K |
| , |  | L |
| m |  | M |
| n | Newton's laws | N |
| o |  | 0 |
| p | mathematical processes (modelling, proof, etc) | P |
| q |  | Q |
| r |  | R |
| s | sequences and seriestrigonometry | S |
| t |  | T |
| u | probability (uncertainty) | U |
| v | vectors | V |
| w |  | W |
| x |  | X |
| y | projectiles | Y |
|  |  | Z |

calculus
C
D Data presentation \& interpretation
Exponentials and logarithms
Forces
Hypothesis testing

## Notes, notation and exclusions

The notes, notation and exclusions columns in the specification are intended to assist teachers and learners.

- The notes column provides examples and further detail for some learning outcomes. All exemplars contained in the specification are for illustration only and do not constitute an exhaustive list
- The notation column shows notation and terminology that learners are expected to know, understand and be able to use
- The exclusions column lists content which will not be tested, for the avoidance of doubt when interpreting learning outcomes.

2f. Content of A Level Mathematics B (MEI)

| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| PURE MATHEMATICS: PROOF (1) |  |  |  |  |  |
| Proof | Mp1 | Understand and be able to use the structure of mathematical proof. <br> Use methods of proof, including proof by deduction and proof by exhaustion. | Proceeding from given assumptions through a series of logical steps to a conclusion. |  |  |
|  | p2 | Be able to disprove a conjecture by the use of a counter example. |  |  |  |
| PURE MATHEMATICS: PROOF (2) |  |  |  |  |  |
| Proof | p3 | Understand and be able to use proof by contradiction. | Including proof of the irrationality of $\sqrt{2}$ and the infinity of primes, and application to unfamiliar proofs. |  |  |


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| PURE MATHEMATICS: ALGEBRA (1) |  |  |  |  |  |
| Algebraic language | Ma1 | Know and be able to use vocabulary and notation appropriate to the subject at this level. | Vocabulary includes constant, coefficient, expression, equation, function, identity, index, term, variable, unknown. | $\mathrm{f}(x)$ |  |
| Solution of equations | * | Be able to solve linear equations in one unknown. | Including those containing brackets, fractions and the unknown on both sides of the equation. |  |  |
|  | * | Be able to change the subject of a formula. | Including cases where the new subject appears on both sides of the original formula, and cases involving squares, square roots and reciprocals. |  |  |
|  | Ma2 | Be able to solve quadratic equations. | By factorising, completing the square, using the formula and graphically. Includes quadratic equations in a function of the unknown. |  |  |
|  | a3 | Be able to find the discriminant of a quadratic function and understand its significance. | The condition for distinct real roots of $a x^{2}+b x+c=0$ is: <br> Discriminant $>0$. <br> The condition for repeated roots is: <br> Discriminant $=0$. <br> The condition for no real roots is: Discriminant < 0 . | For $a x^{2}+b x+c=0$ the discriminant is $b^{2}-4 a c$. | Complex roots. |
|  | a4 | Be able to solve linear simultaneous equations in two unknowns. | By elimination and by substitution. |  |  |
|  | a5 | Be able to solve simultaneous equations in two unknowns with one equation linear and one quadratic. | By elimination and by substitution. |  |  |
|  | a6 | Know the significance of points of intersection of two graphs with relation to the solution of equations. | Including simultaneous equations. |  |  |


| Specification | Ref. | Learning outcomes | Notes | Notation |
| :---: | :---: | :---: | :---: | :---: |

PURE MATHEMATICS: ALGEBRA (1)

| Inequalities | Ma7 | Be able to solve linear inequalities in one variable. Be able to represent and interpret linear inequalities graphically e.g. $y>x+1$. | Including those containing brackets and fractions. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | a8 | Be able to solve quadratic inequalities in one variable. Be able to represent and interpret quadratic inequalities graphically e.g. $y>a x^{2}+b x+c$. | Algebraic and graphical treatment of solution of quadratic inequalities. <br> For regions defined by inequalities learners must state clearly which regions are included and whether the boundaries are included. No particular shading convention is expected. |  | Complex roots |
|  | a9 | Be able to express solutions of inequalities through correct use of 'and' and 'or', or by using set notation. | Learners will be expected to express solutions to quadratic inequalities in an appropriate version of one of the following ways. <br> - $x \leq 1$ or $x \geq 4$ <br> - $\{x: x \leq 1\} \cup\{x: a x \geq 4\}$ <br> - $2<x<5$ <br> - $\quad x<5$ and $x>2$ <br> - $\{x: x<5\} \cap\{x: x>2\}$ | $\{x: x>4\}$ |  |
| Surds Indices | a10 | Be able to use and manipulate surds. |  |  |  |
|  | a11 | Be able to rationalise the denominator of a surd. | e.g. $\frac{1}{5+\sqrt{3}}=\frac{5-\sqrt{3}}{22}$ |  |  |
|  | a12 | Understand and be able to use the laws of indices for all rational exponents. | $x^{a} \times x^{b}=x^{a+b}, x^{a} \div x^{b}=x^{a-b},\left(x^{a}\right)^{n}=x^{a n}$ |  |  |
|  | a13 | Understand and be able to use negative, fractional and zero indices. | $x^{-a}=\frac{1}{x^{a}}, x^{0}=1(x \neq 0), x^{\frac{1}{a}}=\sqrt[a]{x}$ |  |  |


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| PURE MATHEMATICS: ALGEBRA (1) |  |  |  |  |  |
| Proportion | a14 | Understand and use proportional relationships and their graphs. | For one variable directly or inversely proportional to a power or root of another. |  |  |
| PURE MATHEMATICS: ALGEBRA (2) |  |  |  |  |  |
| Partial fractions | a15 | Be able to express algebraic fractions as partial fractions. | Fractions with constant or linear numerators and denominators up to three linear terms. Includes squared linear terms in denominator. |  | Fractions with a quadratic or cubic which cannot be factorised in the denominator. |
| Rational expressions | a16 | Be able to simplify rational expressions. | Including factorising, cancelling and simple algebraic division. Any correct method of algebraic division may be used. |  | Division by non-linear expressions. |


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| PURE MATHEMATICS: FUNCTIONS (1) |  |  |  |  |  |
| Polynomials | Mf1 | Be able to add, subtract, multiply and divide polynomials. | Expanding brackets and collecting like terms. |  | Division by non-linear expressions. |
|  | f2 | Understand the factor theorem and be able to use it to factorise a polynomial or to determine its zeros. | $\mathrm{f}(a)=0 \Leftrightarrow(x-a)$ is a factor of $\mathrm{f}(x)$. Including when solving a polynomial equation. |  | Equations of degree $>4$ |

PURE MATHEMATICS: FUNCTIONS (2)

| The language of functions | f3 | Understand the definition of a function, and be able to use the associated language. | A function is a mapping from the domain to the range such that for each $x$ in the domain, there is a unique $y$ in the range with $\mathrm{f}(x)=y$. The range is the set of all possible values of $\mathrm{f}(x)$. | Many-to-one, one-to-one, domain, range. $\mathrm{f}: x \rightarrow y$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | f4 | Understand and use composite functions. | Includes finding the correct domain of gf given the domains of $f$ and $g$. | $\mathrm{gf}(x)$ |  |
|  | f5 | Understand and be able to use inverse functions and their graphs. Know the conditions necessary for the inverse of a function to exist and how to find it. | Includes using reflection in the line $y=x$ and finding domain and range of an inverse function. <br> e.g. $\ln x(x>0)$ is the inverse of $\mathrm{e}^{x}$. | $\mathrm{f}^{-1}(x)$ |  |
| The modulus function | f6 | Understand and be able to use the modulus function. | Graphs of the modulus of linear functions involving a single modulus sign. |  |  |
|  | f7 | Be able to solve simple inequalities containing a modulus sign. | Including the use of inequalities of the form $\|x-a\| \leq b$ to express upper and lower bounds, $a \pm b$, for the value of $x$. |  | Inequalities involving more than one modulus sign or modulus of non-linear functions. |
| Modelling | f8 | Be able to use functions in modelling. | Including consideration of limitations and refinements of the models. |  |  |


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :--- | :--- | :--- | :--- | :--- | :--- |

PURE MATHEMATICS: GRAPHS (1)

| Graphs | MC1 | Understand and use graphs of functions. |  |  |
| :--- | ---: | :--- | :--- | :--- | :--- |
| Sketching <br> curves | C2 | Understand how to find intersection points <br> of a curve with coordinate axes. | Including relating this to the solution of an <br> equation. |  |
|  | C3 | Understand and be able to use the method <br> of completing the square to find the line of <br> symmetry and turning point of the graph of a <br> quadratic function and to sketch a quadratic <br> curve (parabola). | The curve $y=a(x+p)^{2}+q$ has <br> a minimum at $(-p, q)$ for $a>0$ or <br> a maximum at $(-p, q)$ for $a<0$ <br> a line of symmetry $x=-p$. |  |
|  | C4 | Be able to sketch and interpret the graphs of <br> simple functions including polynomials. | Including cases of repeated roots for <br> polynomials. |  |
|  | C5 | Be able to use stationary points when curve <br> sketching. | Including distinguishing between maximum <br> and minimum turning points. |  |


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| PURE MATHEMATICS: GRAPHS (1) |  |  |  |  |  |
| Transformations | MC7 | Be able to sketch curves of the forms $y=a \mathrm{f}(x), y=\mathrm{f}(x)+a, y=\mathrm{f}(x+a)$ and $y=\mathrm{f}(a x)$, given the curve of $y=\mathrm{f}(x)$ and describe the associated transformations. Be able to form the equation of a graph following a single transformation. | Including working with sketches of graphs where functions are not defined algebraically. | Map(s) onto. <br> Translation, stretch, reflection. |  |
| PURE MATHEMATICS: GRAPHS (2) |  |  |  |  |  |
| Transformations | C8 | Understand the effect of combined transformations on a graph and be able to form the equation of the new graph and to sketch it. Be able to recognise the transformations that have been applied to a graph from the graph or its equation. |  | Vector notation may be used for a translation. $\binom{a}{b}, a \mathbf{i}+b \mathbf{j}$ |  |
| Sketching curves | C9 | Be able to use stationary points of inflection when curve sketching. |  |  |  |


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| PURE MATHEMATICS: COORDINATE GEOMETRY (1) |  |  |  |  |  |
| The coordinate geometry of straight lines | * | Understand and use the equation $y=m x+c$. |  |  |  |
|  | Mg1 | Know and be able to use the relationship between the gradients of parallel lines and perpendicular lines. | For parallel lines $m_{1}=m_{2}$. <br> For perpendicular lines $m_{1} m_{2}=-1$. |  |  |
|  | g2 | Be able to calculate the distance between two points. |  |  |  |
|  | g3 | Be able to find the coordinates of the midpoint of a line segment joining two points. |  |  |  |
|  | g4 | Be able to form the equation of a straight line. | Including $y-y_{1}=m\left(x-x_{1}\right)$ and $a x+b y+c=0$ |  |  |
|  | g5 | Be able to draw a line given its equation. | By using gradient and intercept or intercepts with axes as well as by plotting points. |  |  |
|  | g6 | Be able to find the point of intersection of two lines. | By solution of simultaneous equations. |  |  |
|  | g7 | Be able to use straight line models. | In a variety of contexts; includes considering the assumptions that lead to a straight line model. |  |  |
| Equations of straight lines |  |  |  |  |  |
| Many learners taking A Level Mathematics will be familiar with the equation of a straight line in the form $y=m x+c$. Their understanding at A Level should extend to different forms of the equation of a straight line including $y-y_{1}=m\left(x-x_{1}\right), a x+b y+c=0$ and $\frac{y-y_{1}}{y_{2}-y_{1}}=\frac{x-x_{1}}{x_{2}-x_{1}}$. |  |  |  |  |  |


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| PURE MATHEMATICS: COORDINATE GEOMETRY (1) |  |  |  |  |  |
| The coordinate geometry of curves | Mg8 | Be able to find the point(s) of intersection of a line and a curve or of two curves. |  |  |  |
|  | g9 | Be able to find the point(s) of intersection of a line and a circle. |  |  |  |
|  | g10 | Understand and use the equation of a circle in the form $(x-a)^{2}+(y-b)^{2}=r^{2}$. | Includes completing the square to find the centre and radius. |  |  |
|  | g11 | Know and be able to use the following properties: <br> - the angle in a semicircle is a right angle; <br> - the perpendicular from the centre of a circle to a chord bisects the chord; <br> - the radius of a circle at a given point on its circumference is perpendicular to the tangent to the circle at that point. | These results may be used in the context of coordinate geometry. |  |  |

Parametric
equations

| g12 | Understand the meaning of the terms parameter and parametric equations. |  |  |
| :---: | :---: | :---: | :---: |
| g13 | Be able to convert between cartesian and parametric forms of equations. | When converting from cartesian to parametric form, guidance will be given as to the choice of parameter. |  |
| g14 | Understand and use the equation of a circle written in parametric form. |  |  |
| g15 | Be able to find the gradient of a curve defined in terms of a parameter by differentiation. | $\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{\left(\frac{\mathrm{d} y}{\mathrm{~d} t}\right)}{\left(\frac{\mathrm{d} x}{\mathrm{~d} t}\right)}$ | Second and higher derivatives. |
| g16 | Be able to use parametric equations in modelling. | Contexts include kinematics and projectiles in mechanics. Including modelling with a parameter with a restricted domain. |  |


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| PURE MATHEMATICS: SEQUENCES AND SERIES (1) |  |  |  |  |  |
| Binomial expansions | Ms1 | Understand and use the binomial expansion of $(a+b x)^{n}$ where $n$ is a positive integer. |  |  |  |
|  | s2 | Know the notations $n$ ! and ${ }_{n} C_{r}$ and that ${ }_{n} C_{r}$ is the number of ways of selecting $r$ distinct objects from $n$. | The meaning of the term factorial. $n$ a positive integer. <br> Link to binomial probabilities. | $\begin{aligned} & { }_{n} \mathrm{C}_{r}=\frac{n!}{r!(n-r)!} \\ & n!=1.2 .3 \ldots n \\ & { }_{n} C_{0}={ }_{n} C_{n}=1 \\ & 0!=1 \\ & { }^{n} C_{r},\binom{n}{r} \end{aligned}$ | ${ }_{n} C_{r}$ will only be used in the context of binomial expansions and binomial probabilities. |

## PURE MATHEMATICS: SEQUENCES AND SERIES (2)

## Binomial

expansions

| s3 | Use the binomial expansion of $(1+x)^{n}$ where $n$ is <br> any rational number. | For $\|x\|<1$ when $n$ is not a positive integer. | General term. |
| ---: | :--- | :--- | :--- | :--- |
| s4 | Be able to write $(a+b x)^{n}$ in the form $a^{n}\left(1+\frac{b x}{a}\right)^{n}$ <br> and hence expand $(a+b x)^{n}$. | $\left\|\frac{b x}{a}\right\|<1$ when $n$ is not a positive integer. | Proof of <br> convergence. |
| s5 | Be able to use binomial expansions with $n$ rational <br> to find polynomials which approximate $(a+b x)^{n}$. | Includes finding approximations to <br> rational powers of numbers. |  |


| Ms6 | Know what a sequence of numbers is and the meaning of finite and infinite with reference to sequences. |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| s7 | Be able to generate a sequence using a formula for the $k^{\text {th }}$ term, or a recurrence relation of the form $a_{k+1}=\mathrm{f}\left(a_{k}\right)$. | $\begin{aligned} & \text { e.g. } a_{k}=2+3 k \\ & a_{k+1}=a_{k}+3 \text { with } a_{1}=5 . \end{aligned}$ | $k^{\text {th }}$ term: $a_{k}$ |  |
| s8 | Know that a series is the sum of consecutive terms of a sequence. | Starting from the first term. |  |  |
| s9 | Understand and use sigma notation. |  | $\sum_{r=1}^{n} r=1+2+\ldots+n$ |  |
| s10 | Be able to recognise increasing, decreasing and periodic sequences. |  |  |  |
| s11 | Know the difference between convergent and divergent sequences. | Including when using a sequence as a model or when using numerical methods. | Limit to denote the value to which a sequence converges. | Formal tests for convergence. |
| s12 | Understand and use arithmetic sequences and series. | The term arithmetic progression (AP) may also be used for an arithmetic sequence. | First term, $a$ <br> Last term, $l$ <br> Common difference, $d$. |  |
| s13 | Be able to use the standard formulae associated with arithmetic sequences and series. | The $n$th term, the sum to $n$ terms. Including the sum of the first $n$ natural numbers. | $S_{n}$ |  |
| s14 | Understand and use geometric sequences and series. | The term geometric progression (GP) may also be used for a geometric sequence. | First term, $a$ Common ratio, $r$. |  |
| s15 | Be able to use the standard formulae associated with geometric sequences and series. | The $n$th term, the sum to $n$ terms. | $S_{n}$ |  |
| s16 | Know the condition for a geometric series to be convergent and be able to find its sum to infinity. |  | $S_{\infty}=\frac{a}{1-r},\|r\|<1$ |  |
| s17 | Be able to use sequences and series in modelling. |  |  |  |


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| PURE MATHEMATICS: TRIGONOMETRY (1) |  |  |  |  |  |
| Basic trigonometry | * | Know how to solve right-angled triangles using trigonometry. |  |  |  |
| Trig. functions | Mt1 | Be able to use the definitions of $\sin \theta, \cos \theta$ and $\tan \theta$ for any angle. | By reference to the unit circle, $\sin \theta=y, \cos \theta=x, \tan \theta=\frac{y}{x}$. |  |  |
|  | t2 | Know and use the graphs of $\sin \theta, \cos \theta$ and $\tan \theta$ for all values of $\theta$, their symmetries and periodicities. | Stretches, translations and reflections of these graphs. <br> Combinations of these transformations. | Period. |  |
|  | * | Know and be able to use the exact values of $\sin \theta$ and $\cos \theta$ for $\theta=0^{\circ}, 30^{\circ}, 45^{\circ}, 60^{\circ}$ and $90^{\circ}$ and the exact values of $\tan \theta$ for $\theta=0^{\circ}, 30^{\circ}, 45^{\circ}$ and $60^{\circ}$. |  |  |  |
| Area of triangle; sine and cosine rules | t3 | Know and be able to use the fact that the area of a triangle is given by $1 / 2 a b \sin C$. |  |  |  |
|  | t4 | Know and be able to use the sine and cosine rules. | Use of bearings may be required. |  |  |
| Identities | t5 | Understand and be able to use $\tan \theta=\frac{\sin \theta}{\cos \theta}$. | e.g. solve $\sin \theta=3 \cos \theta$ for $0^{\circ} \leq \theta \leq 360^{\circ}$. |  |  |
|  | t6 | Understand and be able to use the identity $\sin ^{2} \theta+\cos ^{2} \theta=1$. | e.g. solve $\sin ^{2} \theta=\cos \theta$ for $0^{\circ} \leq \theta \leq 360^{\circ}$. |  |  |
| Equations | t7 | Be able to solve simple trigonometric equations in given intervals and know the principal values from the inverse trigonometric functions. | $\begin{aligned} & \text { e.g. } \sin \theta=0.5 \text {, in }\left[0^{\circ}, 360^{\circ}\right] \\ & \Leftrightarrow \theta=30^{\circ}, 150^{\circ} \end{aligned}$ <br> Includes equations involving multiples of the unknown angle e.g. $\sin 2 \theta=3 \cos 2 \theta$ <br> Includes quadratic equations. | $\begin{array}{ll} \hline \arcsin x & \sin ^{-1} x \\ \arccos x & \cos ^{-1} x \\ \arctan x & \tan ^{-1} x \end{array}$ | General solutions. |


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :--- | :--- | :--- | :--- | :--- | :--- |

PURE MATHEMATICS: TRIGONOMETRY (2)


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| PURE MATHEMATICS: TRIGONOMETRY (2) |  |  |  |  |  |
| Compound angle formulae | Mt16 | Understand and use the identities for $\sin (\theta \pm \phi)$, $\cos (\theta \pm \phi), \tan (\theta \pm \phi)$. | Includes understanding geometric proofs. The starting point for the proof will be given. |  | Proofs using de Moivre's theorem will not be accepted. |
|  | t17 | Know and use identities for $\sin 2 \theta, \cos 2 \theta, \tan 2 \theta$. | Includes understanding derivations from $\sin (\theta+\phi), \cos (\theta+\phi)$, $\tan (\theta+\phi)$. $\begin{aligned} & \cos 2 \theta=\cos ^{2} \theta-\sin ^{2} \theta \\ & \cos 2 \theta=2 \cos ^{2} \theta-1 \\ & \cos 2 \theta=1-2 \sin ^{2} \theta \end{aligned}$ |  |  |
|  | t18 | Understand and use expressions for $a \cos \theta \pm b \sin \theta$ in the equivalent forms $R \sin (\theta \pm \alpha)$ and $R \cos (\theta \pm \alpha)$. | Includes sketching the graph of the function, finding its maximum and minimum values and solving equations. |  |  |
| Equations | t19 | Use trigonometric identities, relationships and definitions in solving equations. |  |  |  |
| Proofs and problems | t20 | Construct proofs involving trigonometric functions and identities. |  |  |  |
|  | t21 | Use trigonometric functions to solve problems in context, including problems involving vectors, kinematics and forces. | The argument of the trigonometric functions is not restricted to angles. |  |  |


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| PURE MATHEMATICS: EXPONENTIALS AND LOGARITHMS (1) |  |  |  |  |  |
| Exponentials and Logarithms | ME1 | Know and use the function $y=a^{x}$ and its graph. | For $a>0$. |  |  |
|  | E2 | Be able to convert from an index to a logarithmic form and vice versa. | $x=a^{y} \Leftrightarrow y=\log _{a} x$ for $a>0$ and $x>0$. |  |  |
|  | E3 | Understand a logarithm as the inverse of the appropriate exponential function and be able to sketch the graphs of exponential and logarithmic functions. | $y=\log _{a} x \Leftrightarrow a^{y}=x$ for $a>0$ and $x>0$. Includes finding and interpreting asymptotes. |  |  |
|  | E4 | Understand the laws of logarithms and be able to apply them, including to taking logarithms of both sides of an equation. | $\begin{aligned} & \log _{a}(x y)=\log _{a} x+\log _{a} y \\ & \log _{a}\left(\frac{x}{y}\right)=\log _{a} x-\log _{a} y \\ & \log _{a}\left(x^{k}\right)=k \log _{a} x \end{aligned}$ <br> Including, for example $k=-1$ and $k=-\frac{1}{2}$ |  | Change of base of logarithms. |
|  | E5 | Know and use the values of $\log _{a} a$ and $\log _{a} 1$. | $\log _{a} a=1, \log _{a} 1=0$ |  |  |
|  | E6 | Be able to solve an equation of the form $a^{x}=b$. | Includes solving related inequalities. |  |  |
|  | E7 | Know how to reduce the equations $y=a x^{n}$ and $y=a b^{x}$ to linear form and, using experimental data, to use a graph to estimate values of the parameters. | By taking logarithms of both sides and comparing with the equation $y=m x+c$ <br> Learners may be given graphs and asked to select an appropriate model. |  |  |


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| PURE MATHEMATICS: EXPONENTIALS AND LOGARITHMS (1) |  |  |  |  |  |
| Exponentials and natural logarithms | ME8 | Know and be able to use the function $y=\mathrm{e}^{x}$ and its graph. |  |  |  |
|  | E9 | Know that the gradient of $\mathrm{e}^{k x}$ is $k \mathrm{e}^{k x}$ and hence understand why the exponential model is suitable in many applications. |  |  |  |
|  | E10 | Know and be able to use the function $y=\ln x$ and its graph. Know the relationship between $\ln x$ and $\mathrm{e}^{x}$. | $\ln x$ is the inverse function of $\mathrm{e}^{x}$. | $\log _{\mathrm{e}} x=\ln x$ |  |
| Exponential growth and decay | E11 | Be able to solve problems involving exponential growth and decay; be able to consider limitations and refinements of exponential growth and decay models. | Understand and use exponential growth and decay: use in modelling (examples may include the use of e in continuous compound interest, radioactive decay, drug concentration decay, exponential growth as a model for population growth); consideration of limitations and refinements of exponential models. Finding long term values. |  |  |

## Graphs with gradient proportional to one of the coordinates

## $\frac{\mathrm{d} y}{\mathrm{~d} x} \propto x$ results in a quadratic graph.


$\frac{\mathrm{d} y}{\mathrm{~d} x} \propto y$ results in an exponential graph.


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| PURE MATHEMATICS: CALCULUS (1) |  |  |  |  |  |
| Basic differentiation | Mc1 | Know and use that the gradient of a curve at a point is given by the gradient of the tangent at the point. |  |  |  |
|  | c2 | Know and use that the gradient of the tangent at a point A on a curve is given by the limit of the gradient of chord AP as P approaches A along the curve. |  |  | The modulus function. |
|  | c3 | Understand and use the derivative of $\mathrm{f}(x)$ as the gradient of the tangent to the graph of $y=\mathrm{f}(x)$ at a general point ( $x, y$ ). Know that the gradient function d $y$ $\frac{\mathrm{d} y}{\mathrm{~d} x}$ gives the gradient of the curve and measures the rate of change of $y$ with respect to $x$. | Be able to deduce the units of rate of change for graphs modelling real situations. The term derivative of a function. | $\begin{aligned} & \frac{\mathrm{d} y}{\mathrm{~d} x}=\operatorname{Lim}_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} \\ & \mathrm{f}^{\prime}(x)=\operatorname{Lim}_{h \rightarrow 0}\left(\frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}\right) \end{aligned}$ |  |
|  | c4 | Be able to sketch the gradient function for a given curve. |  |  |  |
| Differentiation of functions | c5 | Be able to differentiate $y=k x^{n}$ where $k$ is a constant and $n$ is rational, including related sums and differences. | Differentiation from first principles for small positive integer powers. |  |  |
| Applications of differentiation to functions and graphs | c6 | Understand and use the second derivative as the rate of change of gradient. |  | $\mathrm{f}^{\prime \prime}(x)=\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}$ |  |
|  | c7 | Be able to use differentiation to find stationary points on a curve: maxima and minima. | Distinguish between maximum and minimum turning points. |  |  |
|  | c8 | Understand the terms increasing function and decreasing function and be able to find where the function is increasing or decreasing. | In relation to the sign of $\frac{\mathrm{d} y}{\mathrm{~d} x}$. |  |  |
|  | c9 | Be able to find the equation of the tangent and normal at a point on a curve. |  |  |  |


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| PURE MATHEMATICS: CALCULUS (2) |  |  |  |  |  |
| Differentiation of functions | Mc10 | Be able to differentiate $\mathrm{e}^{k x}, a^{k x}$ and $\ln x$. | Including related sums, differences and constant multiples. |  |  |
|  | c11 | Be able to differentiate the trigonometrical functions: $\sin k x ; \cos k x ; \tan k x$ for $x$ in radians. | Including their constant multiples, sums and differences. Differentiation from first principles for $\sin x$ and $\cos x$. |  |  |
| Product, quotient and chain rules | c12 | Be able to differentiate the product of two functions. | The product rule: $y=u v$, $\begin{aligned} & \frac{\mathrm{d} y}{\mathrm{~d} x}=u \frac{\mathrm{~d} v}{\mathrm{~d} x}+v \frac{\mathrm{~d} u}{\mathrm{~d} x} \\ & \text { Or }[\mathrm{f}(x) \mathrm{g}(x)]^{\prime}=\mathrm{f}(x) \mathrm{g}^{\prime}(x)+\mathrm{f}^{\prime}(x) \mathrm{g}(x) \end{aligned}$ |  |  |
|  | c13 | Be able to differentiate the quotient of two functions. | $\begin{aligned} & y=\frac{u}{v}, \frac{\mathrm{~d} y}{\mathrm{~d} x}=\frac{v \frac{\mathrm{~d} u}{\mathrm{~d} x}-u \frac{\mathrm{~d} v}{\mathrm{~d} x}}{v^{2}} \\ & \operatorname{Or}\left[\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right]^{\prime}=\frac{\mathrm{g}(x) \mathrm{f}^{\prime}(x)-\mathrm{f}(x) \mathrm{g}^{\prime}(x)}{[\mathrm{g}(x)]^{2}} \end{aligned}$ |  |  |
|  | c14 | Be able to differentiate composite functions using the chain rule. | $\begin{aligned} & y=\mathrm{f}(u), u=\mathrm{g}(x), \frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{\mathrm{d} y}{\mathrm{~d} u} \times \frac{\mathrm{d} u}{\mathrm{~d} x} \text { or } \\ & \{\mathrm{f}[\mathrm{~g}(x)]\}^{\prime}=\mathrm{f}^{\prime}[\mathrm{g}(x)] \mathrm{g}^{\prime}(x) \end{aligned}$ |  |  |
|  | c15 | Be able to find rates of change using the chain rule, including connected rates of change and differentiation of inverse functions. | $\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{1}{\left(\frac{\mathrm{~d} x}{\mathrm{~d} y}\right)}$ |  |  |
| Implicit differentiation | c16 | Be able to differentiate a function or relation defined implicitly. | e.g. $(x+y)^{2}=2 x$. |  | Second and higher derivatives. |


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| PURE MATHEMATICS: CALCULUS (2) |  |  |  |  |  |
| Applications of differentiation to functions and graphs | c17 | Understand that a section of curve which has increasing gradient (and so positive second derivative) is concave upwards. Understand that a section of curve which has decreasing gradient (and so negative second derivative) is concave downwards. | concave upwards (convex downwards) <br> concave downwards (convex upwards) | The wording "concave upwards" or "concave downwards" will be used in examination questions. |  |
|  | c18 | Understand that a point of inflection on a curve is where the curve changes from concave upwards to concave downwards (or vice versa) and hence that the second derivative at a point of inflection is zero. Be able to use differentiation to find stationary and non-stationary points of inflection. | Learners are expected to be able to find and classify points of inflection as stationary or non-stationary. Distinguish between maxima, minima and stationary points of inflection. |  |  |


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| PURE MATHEMATICS: CALCULUS (1) |  |  |  |  |  |
| Integration as reverse of differentiation | Mc19 | Know that integration is the reverse of differentiation. | Fundamental Theorem of Calculus. |  |  |
|  | c20 | Be able to integrate functions of the form $k x^{n}$ where $k$ is a constant and $n \neq-1$. | Including related sums and differences. |  |  |
|  | c21 | Be able to find a constant of integration given relevant information. | e.g. Find $y$ as a function of $x$ given that $\frac{\mathrm{d} y}{\mathrm{~d} x}=x^{2}+2 \text { and } y=7 \text { when } x=1 .$ |  |  |
| Integration to find area under a curve | c22 | Know what is meant by indefinite and definite integrals. <br> Be able to evaluate definite integrals. | e.g. $\int_{1}^{3}\left(3 x^{2}+5 x-1\right) \mathrm{d} x$. |  |  |
|  | c23 | Be able to use integration to find the area between a graph and the $x$-axis. | Includes areas of regions partly above and partly below the $x$-axis. General understanding that the area under a graph can be found as the limit of a sum of areas of rectangles. |  | Formal understanding of the continuity conditions required for the Fundamental Theorem of Calculus. |

## The Fundamental Theorem of Calculus

One way to define the integral of a function is as follows.
The area under the graph of the function is approximately the sum of the areas of narrow rectangles (as shown). The limit of this sum as the rectangles become narrower (and there are more of them) is the integral. The fundamental theorem of calculus says that this is the same as doing the reverse of differentiation.


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| PURE MATHEMATICS: CALCULUS (2) |  |  |  |  |  |
| Differential equations | c31 | Be able to formulate first order differential equations using information about rates of change. | Contexts may include kinematics, population growth and modelling the relationship between price and demand. |  |  |
|  | c32 | Be able to find general or particular solutions of first order differential equations analytically by separating variables. | Equations may need to be factorised using a common factor before variables can be separated. |  |  |
|  | c33 | Be able to interpret the solution of a differential equation in the context of solving a problem, including identifying limitations of the solution. | Includes links to kinematics. |  |  |


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| PURE MATHEMATICS: NUMERICAL METHODS (2) |  |  |  |  |  |
| Solution of equations | Me1 | Be able to locate the roots of $\mathrm{f}(x)=0$ by considering changes of sign of $\mathrm{f}(x)$ in an interval of $x$ in which $\mathrm{f}(x)$ is sufficiently well-behaved. | Finding an interval in which a root lies. This is often used as a preliminary step to find a starting value for the methods in Me3 and Me4. |  |  |
|  | e2 | Be aware of circumstances under which change of sign methods may fail. | e.g. when the curve of $y=\mathrm{f}(x)$ touches the $x$-axis. e.g. when the curve of $y=\mathrm{f}(x)$ has a vertical asymptote. e.g. there may be several roots in the interval. |  |  |
|  | e3 | Be able to carry out a fixed point iteration after rearranging an equation into the form $x=\mathrm{g}(x)$ and be able to draw associated staircase and cobweb diagrams. | e.g. write $x^{3}-x-4=0$ as $x=\sqrt[3]{x+4}$ and use the iteration $x_{n+1}=\sqrt[3]{x_{n}+4}$ with an appropriate starting value. <br> Includes use of ANS key on calculator. | iteration, iterate |  |
|  | e4 | Be able to use the Newton-Raphson method to find a root of an equation and represent the process on a graph. |  |  |  |
|  | e5 | Understand that not all iterations converge to a particular root of an equation. | Know how Newton-Raphson and fixed point iteration can fail and be able to show this graphically. |  |  |
| Integration | Mc34 | Be able to find an approximate value of a definite integral using the trapezium rule, and decide whether it is an over- or an under-estimate. | In an interval where the curve is either concave upwards or concave downwards. | Number of strips. |  |
|  | c35 | Use the sum of a series of rectangles to find an upper and/or lower bound on the area under a curve. |  |  |  |
| Problem solving | Me6 | Use numerical methods to solve problems in context. |  |  |  |


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :--- | :--- | :--- | :--- | :--- | :--- |

## PURE MATHEMATICS: VECTORS (1)

| General vectors | Mv1 | Understand the language of vectors in two dimensions. | Scalar, vector, modulus, magnitude, direction, position vector, unit vector, cartesian components, equal vectors, parallel vectors, collinear. | Vectors printed in bold. <br> Unit vectors $\mathbf{i}, \mathbf{j}, \hat{\mathbf{r}}$ <br> The magnitude of the vector a is written $\|\mathbf{a}\|$ or $a$. $\mathbf{a}=\binom{a_{1}}{a_{2}}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | v2 | Be able to add and subtract vectors using a diagram or algebraically, multiply a vector by a scalar, and express a vector as a combination of others. | Geometrical interpretation. Includes general vectors not expressed in component form. |  |  |
|  | v3 | Be able to calculate the magnitude and direction of a vector and convert between component form and magnitude-direction form. |  | Magnitude-direction |  |
| Position vectors | v4 | Understand and use position vectors. | Including interpreting components of a position vector as the cartesian coordinates of the point. $\overrightarrow{\mathrm{AB}}=\mathbf{b}-\mathbf{a}$ | $\overrightarrow{\mathrm{OB}}$ or $\mathbf{b}$. $\mathbf{r}=\binom{x}{y}$ |  |
|  | v5 | Be able to calculate the distance between two points represented by position vectors. |  |  |  |
| Using vectors | v6 | Be able to use vectors to solve problems in pure mathematics and in context, including problems involving forces. | Includes interpreting the sum of vectors representing forces as the resultant force. |  |  |
| PURE MATHEMATICS: VECTORS (2) |  |  |  |  |  |
| General vectors | Mv7 | Understand the language of vectors in three dimensions. | Extend the work of Mv2 to Mv 6 to include vectors in three dimensions. | Unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}, \hat{\mathbf{r}}$ $\mathbf{a}=\left(\begin{array}{l} a_{1} \\ a_{2} \\ a_{3} \end{array}\right)$ |  |


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| STATISTICS: SAMPLING (1) |  |  |  |  |  |
| Population and sample | Mp21 | Understand and use the terms population and sample. |  |  |  |
|  | p22 | Be able to use samples to make informal inferences about a population, recognising that different samples might lead to different conclusions. | e.g. using sample mean or variance as an estimate of population mean or variance. |  |  |
| Sampling techniques | p23 | Understand and be able to use the concept of random sampling. | Simple random sampling. Every sample of the required size has the same probability of being selected. |  |  |
|  | p24 | Understand and be able to use a variety of sampling techniques. | Opportunity sampling, systematic sampling, stratified sampling, quota sampling, cluster sampling, selfselected samples. <br> Any other techniques will be explained in the question. |  |  |
|  | p25 | Be able to select or evaluate sampling techniques in the context of solving a statistical problem. | Includes recognising possible sources of bias and being aware of the practicalities of implementation. |  |  |

Population in statistics means all the individuals we are interested in for a particular investigation e.g. all cod in an area of the sea. A population can be infinite e.g. all possible tosses of a particular coin. A probability distribution can be used to model some characteristic of the population which is of interest e.g a Normal distribution could be used to model lengths of cod.
A sample is a set of items chosen from a population. When sampling from an infinite population it does not matter whether the sampling is with or without replacement. When taking a sample of individuals, e.g. for a sample survey, it is usual to sample without replacement to avoid getting data from the same individual more than once

| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| STATISTICS: DATA PRESENTATION AND INTERPRETATION (1) |  |  |  |  |  |
| Data presentation for single variable | MD1 | Be able to recognise and work with categorical, discrete, continuous and ranked data. Be able to interpret standard diagrams for grouped and ungrouped single-variable data. | Includes knowing this vocabulary and deciding what data presentation methods are appropriate: bar chart, dot plot, histogram, vertical line chart, pie chart, stem-and-leaf diagram, box-and-whisker diagram (box plot), frequency chart. <br> Learners may be asked to add to diagrams in examinations in order to interpret data. | A frequency chart resembles a histogram with equal width bars but its vertical axis is frequency. A dot plot is similar to a bar chart but with stacks of dots in lines to represent frequency. | Comparative pie charts with area proportional to frequency. |
|  | D2 | Understand that the area of each bar in a histogram is proportional to frequency. Be able to calculate proportions from a histogram and understand them in terms of estimated probabilities. | Includes use of area scale and calculation of frequency from frequency density. |  |  |
|  | D3 | Be able to interpret a cumulative frequency diagram. |  |  |  |
|  | D4 | Be able to describe frequency distributions. | Symmetrical, unimodal, bimodal, skewed (positively and negatively). |  | Measures of skewness. |


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| STATISTICS: DATA PRESENTATION AND INTERPRETATION (1) |  |  |  |  |  |
| Data presentation | MD5 | Understand that diagrams representing unbiased samples become more representative of theoretical probability distributions with increasing sample size. | e.g. A bar chart representing the proportion of heads and tails when a fair coin is tossed tends to have the proportion of heads increasingly close to $50 \%$ as the sample size increases. |  |  |
|  | D6 | Be able to interpret a scatter diagram for bivariate data, interpret a regression line or other best fit model, including interpolation and extrapolation, understanding that extrapolation might not be justified. | Including the terms association, correlation, regression line. <br> Leaners should be able to interpret other best fit models produced by software (e.g. a curve). <br> Learners may be asked to add to diagrams in examinations in order to interpret data. |  | Calculation of equation of regression line from data or summary statistics. |
|  | D7 | Be able to recognise when a scatter diagram appears to show distinct sections in the population. Be able to recognise and comment on outliers in a scatter diagram. | An outlier is an item which is inconsistent with the rest of the data. <br> Outliers in scatter diagrams should be judged by eye. |  |  |
|  | D8 | Be able to recognise and describe correlation in a scatter diagram and understand that correlation does not imply causation. | Positive correlation, negative correlation, no correlation, weak/strong correlation. |  |  |
|  | D9 | Be able to select or critique data presentation techniques in the context of a statistical problem. | Including graphs for time series. |  |  |

## Bivariate data, association and correlation

Bivariate data consists of two variables for each member of the population or sample. An association between the two variables is some kind of relationship between them. Correlation measures linear relationships. At A Level, learners are expected to judge relationships from scatter diagrams by eye and may be asked to interpret given correlation coefficients - see MAH10.

| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :--- | :--- | :--- | :--- | :--- | :--- |

## STATISTICS: DATA PRESENTATION AND INTERPRETATION (1)

$\left.\begin{array}{|l|c|l|l|l|l|}\hline \begin{array}{l}\text { Summary } \\ \text { measures }\end{array} & \text { MD10 } & \begin{array}{l}\text { Know the standard measures of central } \\ \text { tendency and be able to calculate and interpret } \\ \text { them and to decide when it is most appropriate } \\ \text { to use one of them. }\end{array} & \begin{array}{l}\text { Median, mode, (arithmetic) mean, } \\ \text { midrange. The main focus of questions } \\ \text { will be on interpretation rather than } \\ \text { calculation. }\end{array} & \text { Mean }=\bar{x} \\ \text { Includes understanding when it is } \\ \text { appropriate to use a weighted mean e.g. } \\ \text { when using populations as weights. }\end{array}\right]$.

| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| STATISTICS: DATA PRESENTATION AND INTERPRETATION (1) |  |  |  |  |  |
| Summary measures | MD12 | Know how to calculate and interpret variance and standard deviation for raw data, frequency distributions, grouped frequency distributions. <br> Be able to use the statistical functions of a calculator to find mean and standard deviation. | sample variance: $s^{2}=\frac{S_{x x}}{n-1}$ where $S_{x x}=\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}$ sample standard deviation: $s=\sqrt{\text { variance }}$ | $s^{2}$ <br> $s$ | Corrections for class interval in these calculations. |
|  | D13 | Understand the term outlier and be able to identify outliers. Know that the term outlier can be applied to an item of data which is: <br> - at least 2 standard deviations from the mean; <br> OR <br> - at least $1.5 \times \mathrm{IQR}$ beyond the nearer quartile. | An outlier is an item which is inconsistent with the rest of the data. |  |  |
|  | D14 | Be able to clean data including dealing with missing data, errors and outliers. |  |  |  |
| Notation for sample variance and sample standard deviation |  |  |  |  |  |

The notations $s^{2}$ and $s$ for sample variance and sample standard deviation, respectively, are written into both British Standards (BS3534-1, 2006) and International Standards (ISO 3534). The definitions are those given above in equations ( $\dagger$ ) and ( $\ddagger$ ). The calculations are carried out using divisor ( $n-1$ ).

In this specification, the usage will be consistent with these definitions. Thus the meanings of 'sample variance', denoted by $s^{2}$, and 'sample standard deviation', denoted by $s$, are defined to be calculated with divisor $(n-1)$.

In early work in statistics it is common practice to introduce these concepts with divisor $n$ rather than $(n-1)$. However there is no recognised notation to denote the quantities so derived.

Students should be aware of the variations in notation used by manufacturers on calculators and know what the symbols on their particular models represent.

| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| STATISTICS: PROBABILITY (1) |  |  |  |  |  |
| Probability of events in a finite sample space | * | Be able to calculate the probability of an event. | Using modelling assumptions such as equally likely outcomes. | $\mathrm{P}(A)$ |  |
|  | * | Understand the concept of a complementary event and know that the probability of an event may be found by means of finding that of its complementary event. |  | $A^{\prime}$ is the event "not- $A$ ". |  |
| Probability of two or more events | * | Be able to calculate the expected frequency of an event given its probability. |  | Expected frequency $=n \mathrm{P}(A)$ |  |
|  | * | Be able to use appropriate diagrams to assist in the calculation of probabilities. | E.g. tree diagrams, sample space diagrams, Venn diagrams. |  |  |
|  | Mu1 | Understand and use mutually exclusive events and independent events. |  |  |  |
|  | u2 | Know to add probabilities for mutually exclusive events. | E.g. to find $\mathrm{P}(A$ or $B)$. |  |  |
|  | u3 | Know to multiply probabilities for independent events. | E.g. to find $\mathrm{P}(A$ and $B)$. Including the use of complementary events, e.g. finding the probability of at least one 6 in five throws of a dice. |  |  |


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| STATISTICS: PROBABILITY (2) |  |  |  |  |  |
| Probability of two or more events | u4 | Understand and use mutually exclusive events and independent events and associated notation and definitions. | For mutually exclusive events $\mathrm{P}(A \cap B)=0$ for any pair of events. |  |  |
|  | u5 | Be able to use Venn diagrams to assist in the calculations of probabilities. Know how to calculate probabilities for two events which are not mutually exclusive. | Venn diagrams for up to three events. Learners should understand the relation: $\mathrm{P}(A \cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A \cap B)$ |  | Probability of a general or infinite number of events. Formal proofs. |
| Conditional probability | u6 | Be able to calculate conditional probabilities by formula, from tree diagrams, two-way tables, Venn diagrams or sample space diagrams. | $\mathrm{P}(A \mid B)=\frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ | $\mathrm{P}(A \mid B)$ | Finding reverse conditional probability i.e. calculating $\mathrm{P}(B \mid A)$ given $\mathrm{P}(A \mid B)$ and additional information. |
|  | u7 | Know that $\mathrm{P}(B \mid A)=\mathrm{P}(B) \Leftrightarrow B$ and $A$ are independent. | In this case $\mathrm{P}(A \cap B)=\mathrm{P}(A) \cdot \mathrm{P}(B)$ |  |  |


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| STATISTICS: PROBABILITY DISTRIBUTIONS (1) |  |  |  |  |  |
| Situations leading to a binomial distribution | MR1 | Recognise situations which give rise to a binomial distribution. |  |  |  |
|  | R2 | Be able to identify the probability of success, $p$, for the binomial distribution. | The binomial distribution as a model for observed data. | $\mathrm{B}(n, p), q=1-p$ <br> ~ means 'has the distribution'. |  |
| Calculations relating to binomial distribution | R3 | Be able to calculate probabilities using the binomial distribution. | Including use of calculator functions. |  |  |
| Mean and expected frequencies for binomial distribution | R4 | Understand and use mean $=n p$. |  |  | Derivation of mean $=n p$ |
|  | R5 | Be able to calculate expected frequencies associated with the binomial distribution. |  |  |  |
| Discrete probability distributions | R6 | Be able to use probability functions, given algebraically or in tables. Know the term discrete random variable. | Restricted to simple finite distributions. | $X$ for the random variable. <br> $x$ or $r$ for a value of the random variable. |  |
|  | R7 | Be able to calculate the numerical probabilities for a simple distribution. Understand the term discrete uniform distribution. | Restricted to simple finite distributions. | $\begin{aligned} & \mathrm{P}(X=x) \\ & \mathrm{P}(X \leq x) \end{aligned}$ | Calculation of $\mathrm{E}(X)$ or $\operatorname{Var}(\mathrm{X})$. |

## Situations which give rise to a binomial distribution

- An experiment or trial is conducted a fixed number of times.
- There are exactly 2 outcomes, which can be thought of as "success" or "failure".
- The probability of "success" is the same each time.
- The probability of "success" on any trial is independent of what has happened in previous trials.
- The random variable of interest is "the number of successes".

| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| STATISTICS: PROBABILITY DISTRIBUTIONS (2) |  |  |  |  |  |
| Normal distribution | MR8 | Be able to use the Normal distribution as a model. | Includes recognising when a Normal distribution may not be appropriate. <br> Understand how and why a continuity correction is used when using a Normal distribution as a model for a distribution of discrete data. <br> Recognise from the shape of the distribution when a binomial distribution can be approximated by a Normal distribution. | $X \sim \mathrm{~N}\left(\mu, \sigma^{2}\right)$ | Knowing conditions for Normal approximation to binomial. |
|  | R9 | Know the shape of the Normal curve and understand that histograms from increasingly large samples from a Normal distribution tend to the Normal curve. | Includes understanding that the area under the Normal curve represents probability. |  |  |
|  | R10 | Know that linear transformation of a Normal variable gives another Normal variable and know how the mean and standard deviation are affected. Be able to standardise a Normal variable. | $y_{i}=a+b x_{i} \Rightarrow \bar{y}=a+b \bar{x}, s_{y}^{2}=b^{2} s_{x}^{2}$ | Standard Normal $\begin{gathered} Z \sim \mathrm{~N}(0,1) \\ Z=\frac{X-\mu}{\sigma} \end{gathered}$ | Proof |
|  | R11 | Know that the line of symmetry of the Normal curve is located at the mean and the points of inflection are located one standard deviation away from the mean. |  |  |  |
|  | R12 | Be able to calculate and use probabilities from a Normal distribution. | Including use of calculator functions. |  |  |
| Modelling with probability | R13 | Be able to model with probability and probability distributions, including recognising when the binomial or Normal model may not be appropriate. | Including critiquing assumptions made and the likely effect of more realistic assumptions. |  |  |

## Mean and variance of a Normal distribution

The Normal distribution is a probability model; its mean and variance are calculated using techniques beyond the scope of A Level Mathematics. At this level, students should understand the mean and variance of a Normal distribution as the limiting values from calculating the mean and variance of increasingly large samples from a Normal distribution.

| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| STATISTICS: STATISTICAL HYPOTHESIS TESTING (1) |  |  |  |  |  |
| Hypothesis testing | MH1 | Understand the process of hypothesis testing and the associated language. | Null hypothesis, alternative hypothesis. Significance level, test statistic, 1-tail test, 2-tail test. <br> Critical value, critical region (rejection region), acceptance region, $p$-value. |  |  |
|  | H2 | Understand when to apply 1-tail and 2-tail tests. |  |  |  |
|  | H3 | Understand that a sample is being used to make an inference about the population and appreciate that the significance level is the probability of incorrectly rejecting the null hypothesis. | For a binomial hypothesis test, the probability of the test statistic being in the rejection region will always be less than or equal to the intended significance level of the test, and will usually be less than the significance level of the test. Learners will not be tested on this distinction. If asked to give the probability of incorrectly rejecting the null hypothesis for a particular binomial test, either the intended significance level or the probability of the test statistic being in the rejection region will be acceptable. |  |  |
| Null and alternative hypotheses |  |  |  |  |  |

The null hypothesis for a hypothesis test is the default position which will only be rejected in favour of the alternative hypothesis if the evidence is strong enough. Assuming the null hypothesis is true, as a default position, allows the calculation of values of the test statistic which would be unlikely (have low probability) if the null hypothesis were true; this is the critical region (rejection region).


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| STATISTICS: STATISTICAL HYPOTHESIS TESTING (2) |  |  |  |  |  |
| Informal hypothesis testing for correlation/ association | MH10 | Understand correlation as a measure of how close data points lie to a straight line. <br> Understand that a rank correlation coefficient measures the correlation between the data ranks rather than actual data values. | Learners are not required to know the names of particular correlation coefficients. | $r$ | Calculation of correlation coefficient |
|  | H11 | Be able to use a given correlation coefficient for a sample to make an inference about correlation or association in the population for given $p$-value or critical value. | Association refers to a more general relationship between the variables. <br> The (often implicit) null hypothesis is of the form either that there is no correlation or no association in the population. Questions will use an appropriate correlation coefficient and indicate whether correlation or association is being tested for. | Questions may require understanding of notation from software; sufficient guidance will be given in the question. | Knowledge of bivariate Normal distribution |

## Calculating correlation

Learners are expected to use technology to work with real data, including the pre-release data. Calculators, spreadsheets and other software will calculate correlation coefficients. Learners may be asked to interpret such correlation coefficients in the examination. The following points should be noted:

- A correlation coefficient measures the strength of a linear relationship. A correlation between the ranks of the data values may be used for a more general relationship.
- Correlation coefficients will only be used for data where both variables are random (not, for example, for time series data where one variable occurs at set intervals).
- Outliers or distinct sections of data in the scatter diagram can affect the value of the correlation coefficient.


## Conclusion from a hypothesis test

## Learners are expected to make non-assertive conclusions in context.

E.g. "There is not enough evidence to conclude that the proportion of... has increased."
E.g. "There is enough evidence to indicate that the probability of ..... has changed."
E.g. "There is insufficient evidence to indicate that the true mean of ..... is lower than......"
E.g. "There is sufficient evidence to suggest that there is positive correlation between..... and ....."
E.g. "There is not sufficient evidence to suggest that there is association between ... and ...."

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| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :--- | :--- | :--- | :--- | :--- | :--- |

MECHANICS: MODELS AND QUANTITIES (1)


## MECHANICS: KINEMATICS IN 1 DIMENSION (1)



MECHANICS: KINEMATICS IN 2 DIMENSIONS (2)

Motion in 2
dimensions

| Mk9 | Understand the language of kinematics appropriate to motion in 2 dimensions. Know the difference between, displacement, distance from and distance travelled; velocity and speed, and between acceleration and magnitude of acceleration. | Position vector, relative position. <br> Average speed $=$ distance travelled $\div$ elapsed time <br> Average velocity = overall <br> displacement : elapsed time |  | Relative velocity |
| :---: | :---: | :---: | :---: | :---: |
| k10 | Be able to extend the scope of techniques from motion in 1 dimension to that in 2 dimensions by using vectors. | The use of calculus and the use of constant acceleration formulae. | $\begin{aligned} & \mathbf{a}=\dot{\mathbf{v}}=\frac{\mathrm{d} \mathbf{v}}{\mathrm{~d} t}, \mathbf{v}=\dot{\mathbf{r}}=\frac{\mathrm{d} \mathbf{r}}{\mathrm{~d} t} \\ & \mathbf{r}=\int \mathbf{v} t, \mathbf{v}=\int \mathbf{a d} t \\ & \mathbf{s}=\mathbf{u} t+\frac{1}{2} \mathbf{a} t^{2} \\ & \mathbf{s}=\mathbf{v} t-\frac{1}{2} \mathbf{a} t^{2} \\ & \mathbf{v}=\mathbf{u}+\mathbf{a} t \\ & \mathbf{s}=\frac{1}{2}(\mathbf{u}+\mathbf{v}) t \end{aligned}$ | Vector form of $v^{2}-u^{2}=2 a s$ |
| k11 | Be able to find the cartesian equation of the path of a particle when the components of its position vector are given in terms of time. |  |  |  |
| k12 | Be able to use vectors to solve problems in kinematics. | Includes relative position of one particle from another. <br> Includes knowing that the velocity vector gives the direction of motion and the acceleration vector gives the direction of resultant force. |  |  |


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| MECHANICS: PROJECTILES (2) |  |  |  |  |  |
| Motion under gravity in 2 dimensions | My1 | Be able to model motion under gravity in a vertical plane using vectors. Be able to formulate the equations of motion of a projectile using vectors. | Standard modelling assumptions for projectile motion are as follows. <br> - No air resistance. <br> - The projectile is a particle. <br> - Horizontal distance travelled is small enough to assume that gravity is always in the same direction. <br> - Vertical distance travelled is small enough to assume that gravity is constant. |  | Calculations involving air resistance. |
|  | y2 | Know how to find the position and velocity at any time of a projectile and find range and maximum height. |  |  |  |
|  | y3 | Be able to find the initial velocity of a projectile given sufficient information. |  |  |  |
|  | y4 | Be able to eliminate time from the component equations that give the horizontal and vertical displacement in terms of time to obtain the equation of the trajectory. |  |  |  |
|  | y5 | Be able to solve simple problems involving projectiles. |  |  | Maximum range on inclined plane Bounding parabola. |


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| MECHANICS: FORCES (1) |  |  |  |  |  |
| Identifying and representing forces | MF1 | Understand the language relating to forces. | Weight, tension, thrust or compression, normal reaction (or normal contact force), frictional force, resistance, driving force. <br> Understand that the value of the normal reaction depends on the other forces acting. <br> Understand that there may be frictional force when the surface is not smooth (i.e. is rough). |  |  |
|  | F2 | Know that the acceleration due to gravity is not a universal constant but depends on location in the universe. Know that on earth, the acceleration due to gravity is often modelled to be a constant, $g \mathrm{~m} \mathrm{~s}^{-2}$. | $g \approx 10, g \approx 9.8$ <br> Unless otherwise specified, in examinations the value of $g$ should be taken to be 9.8. | Acceleration <br> due to <br> gravity, <br> $g \mathrm{~m} \mathrm{~s}^{-2}$. | Inverse square law for gravitation. |
|  | F3 | Be able to identify the forces acting on a system and represent them in a force diagram. Understand the difference between external and internal forces and be able to identify the forces acting on part of the system. |  |  |  |
| Vector treatment of forces | F4 | Be able to find the resultant of several concurrent forces when the forces are parallel or in two perpendicular directions or in simple cases of forces given as 2-D vectors in component form. |  |  |  |
|  | F5 | Understand the concept of equilibrium and know that a particle is in equilibrium if and only if the vector sum of the forces acting on it is zero in the cases where the forces are parallel or in two perpendicular directions or in simple cases of forces given as 2-D vectors in component form. |  |  |  |

## Acceleration due to gravity

The acceleration due to gravity ( $\mathrm{g} \mathrm{m} \mathrm{s}^{-2}$ ) varies on earth between 9.76 and 9.83. It depends on latitude and height above sea level. The standard acceleration due to gravity is internationally agreed to be 9.80665 ; this value is stored in some calculators.

| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| MECHANICS: FORCES (2) |  |  |  |  |  |
| Vector treatment of forces | MF6 | Be able to resolve a force into components and be able to select suitable directions for resolution. Be able to find the resultant of several concurrent forces by resolving and adding components. | e.g. Horizontally and vertically, or parallel and perpendicular to an inclined plane. |  |  |
|  | F7 | Know that a particle is in equilibrium if and only if the resultant of the forces acting on it is zero. Know that a body is in equilibrium under a set of concurrent forces if and only if their resultant is zero. |  |  |  |
|  | F8 | Know that vectors representing a set of forces in equilibrium sum to zero. Know that a closed figure may be drawn to represent the addition of the forces on an object in equilibrium. |  |  |  |
|  | F9 | Be able to formulate and solve equations for a particle in equilibrium: by resolving forces in suitable directions; by drawing and using a polygon of forces. | For example, a triangle of forces. |  | Non-coplanar forces. |
| Frictional force and normal contact force | F10 | Understand that the overall contact force between surfaces may be expressed in terms of a frictional force and a normal contact force and be able to draw an appropriate force diagram. <br> Understand that the normal contact force cannot be negative. | Understand the following modelling assumptions. <br> - Smooth is used to mean that friction may be ignored. <br> - Rough indicates that friction must be taken into account. | Normal reaction. |  |
|  | F11 | Understand that the frictional force may be modelled by $F \leq \mu R$ and that friction acts in the direction to oppose sliding. Model friction using $F=\mu R$ when sliding occurs. |  | Coefficient of friction $=\mu$ Limiting friction, static equilibrium | The term angle of friction. |
|  | F12 | Be able to apply Newton's Laws to problems involving friction. |  |  |  |


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| MECHANICS: NEWTON'S LAWS OF MOTION (1) |  |  |  |  |  |
| Newton's laws for a particle | Mn1 | Know and understand the meaning of Newton's three laws. | Includes applying the laws to problems. |  |  |
|  | n2 | Understand the term equation of motion. |  |  |  |
|  | n3 | Be able to formulate the equation of motion for a particle moving in a straight line when the forces acting are parallel or in two perpendicular directions or in simple cases of forces given as 2-D vectors in component form. | Including motion under gravity. | $F=m a$ where $F$ is the resultant force. $\mathbf{F}=m \mathbf{a}$ where $\mathbf{F}$ is the resultant force. | Variable mass. |
| Connected particles | n4 | Be able to model a system as a set of connected particles. | e.g. simple smooth pulley systems, trains. Internal and external forces for the system. |  |  |
|  | n5 | Be able to formulate the equations of motion for the individual particles within the system. |  |  |  |
|  | n6 | Know that a system in which none of its components have any relative motion may be modelled as a single particle with the mass of the system. | e.g. Train. |  |  |
| MECHANICS: NEWTON'S LAWS OF MOTION (2) |  |  |  |  |  |
| Newton's laws for a particle | n7 | Be able to formulate the equation of motion for a particle moving in a straight line or in a plane. | Including motion under gravity. | $F=m a$ where $F$ is the resultant force. $\mathbf{F}=m \mathbf{a}$ where $\mathbf{F}$ is the resultant force. | Variable mass. |
| Newton's laws of motion |  |  |  |  |  |
| I An object continues in a state of rest or uniform motion in a straight line unless it is acted on by a resultant force. <br> II A resultant force $\mathbf{F}$ acting on an object of fixed mass $m$ gives the object an acceleration a given by $\mathbf{F}=m \mathbf{a}$. <br> III When one object exerts a force on another, there is always a reaction which is equal in magnitude and opposite in direction to the acting force. |  |  |  |  |  |


| Specification | Ref. | Learning outcomes | Notes | Notation | Exclusions |
| :--- | :--- | :--- | :--- | :--- | :--- |

## MECHANICS: RIGID BODIES (2)

This section is an introduction to moments in static contexts. The only situations considered are bodies that may be modelled as (possibly non-uniform) rods and rectangular laminas. The only forces considered are coplanar, and act perpendicular to the rod or to an edge of the lamina. The learning outcome should be read in the light of this restriction.

In more advanced work, moments are described as acting about an axis, and learners should be aware of this. Given the restrictions on the situations considered, however, moments may be described as acting about a point, with an implied axis perpendicular to the plane in which the forces are acting. This is consistent with the approach used to describe rotations in 2-D.

| Rigid bodies in equilibrium | MF13 | Be able to calculate the moment of a force about a point or axis. |  | Units of moment are N m. | Vector treatment. |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | F14 | Understand that a rigid body is in equilibrium when the resultant force is zero and the sum of the moments about any one point is zero. |  |  |  |
|  | F15 | Understand that a system of forces can have a turning effect on a rigid body. |  | Moment |  |
|  | F16 | Know that, for the purpose of calculating its moment, the weight of a body can be taken as acting through a point. | The point is the centre of mass of the body. <br> Questions will be restricted to cases where the centre of mass is given or can be found using symmetry or can be found from consideration of moments. | Uniform | Finding the centre of mass of a composite body. |

## 2g. Prior knowledge, learning and progression

- It is assumed that learners are familiar with the content of GCSE (9-1) Mathematics for first teaching from 2015.
- A Level Mathematics B (MEI) provides the framework within which a large number of young people continue the subject beyond GCSE level. It supports their mathematical needs across a broad range of other subjects at this level and provides a basis for subsequent quantitative work in a very wide range of higher education courses and in employment. It also supports the study of AS and A Level Further Mathematics B (MEI).
- A Level Mathematics B (MEI) builds from GCSE Level Mathematics and introduces calculus and its applications. It emphasises how mathematical ideas are interconnected and how mathematics can be applied to help make sense of data, to understand the physical world and to solve problems in a variety of contexts, including social sciences and business. It prepares students for further study and
employment in a wide range of disciplines involving the use of mathematics.
- $\quad$ Some learners may wish to follow a mathematics course only up to AS level, in order to broaden their curriculum, and to develop their interest and understanding of different areas of the subject. Others may follow a co-teachable route, completing a one year AS course and then continuing to complete the second year of the two year A level course, developing a deeper knowledge and understanding of mathematics and its applications.
- Learners who wish to specialise in mathematics or STEM subjects such as physics or engineering can further extend their knowledge and understanding of mathematics and its applications by taking Further Mathematics B (MEI) AS or A Level.

There are a number of Mathematics specifications at OCR. Find out more at www.ocr.org.uk

## 3a. Forms of assessment

OCR A Level in Mathematics B (MEI) consists of three components that are externally assessed. The three externally assessed components (01-03) contain some synoptic assessment, some extended response questions and some stretch and challenge questions.

Stretch and challenge questions are designed to allow the most able learners the opportunity to demonstrate the full extent of their knowledge and skills.

Stretch and challenge questions will support the awarding of A* grade at $^{*}$ level, addressing the need for greater differentiation between the most able learners.

## Pure Mathematics and Mechanics (Component 01)

This component is worth $36.4 \%$ of the total A level. All questions are compulsory and there are 100 marks in total. The examination paper has two sections: A and $B$.

Section A consists of shorter questions with minimal reading and interpretation; the aim of this is to ensure that all students feel as though they can do some of the questions on the paper. Section B includes longer questions and more problem solving. Section B has a gradient of difficulty. 41 to 47 of the marks are for mechanics; mechanics questions can be in either section $A$ or $B$.

Section A will have 20 to 25 marks with the remainder of the marks allocated to section $B$.

## Pure Mathematics and Statistics (Component 02)

This component is worth $36.4 \%$ of the total A level. All questions are compulsory and there are 100 marks
in total. The examination paper has two sections: A and $B$.

Section A consists of shorter questions with minimal reading and interpretation; the aim of this is to ensure that all students feel as though they can do some of the questions on the paper. Section B includes longer questions and more problem solving. Section B has a gradient of difficulty. 50 to 60 of the marks are for statistics with some of these being for questions based on the pre-release data set; statistics questions can be in either section $A$ or $B$.

Section A will have 20 to 25 marks with the remainder of the marks allocated to section B.

## Pure Mathematics and Comprehension (Component 03)

This component is worth $27.3 \%$ of the total A level. All questions are compulsory and there are 75 marks in total. The examination paper has two sections: A and B .

Section A will have 60 marks on the pure mathematics content.

Section B will have 15 marks worth of questions on a previously unseen comprehension passage based on the pure mathematics content of the specification, rather than on mechanics or statistics content, to ensure that mechanics and statistics are not overassessed in some years. The passage may include examples of applications of the pure content (other than those which are specified in the mechanics and statistics sections). The mechanics and statistics content of the specification is assumed knowledge for component 03 but this assumed knowledge will not be the focus of any of the questions.

## 3b. Assessment objectives (AO)

There are 3 Assessment Objectives in OCR AS Level Mathematics B (MEI). These are detailed in the table below.

| Assessment objectives |  | Weightings |
| :---: | :---: | :---: |
|  |  | A level |
| A01 | Use and apply standard techniques <br> Learners should be able to: <br> - select and correctly carry out routine procedures; and <br> - accurately recall facts, terminology and definitions. | $\begin{gathered} 50 \% \\ ( \pm 2 \%) \end{gathered}$ |
| A02 | Reason, interpret and communicate mathematically <br> Learners should be able to: <br> - construct rigorous mathematical arguments (including proofs); <br> - make deductions and inferences; <br> - assess the validity of mathematical arguments; <br> - explain their reasoning; and <br> - use mathematical language and notation correctly. <br> Where questions/tasks targeting this assessment objective will also credit Learners for the ability to 'use and apply standard techniques' (AO1) and/or to 'solve problems within mathematics and other contexts' (AO3) an appropriate proportion of the marks for the question/task will be attributed to the corresponding assessment objective(s). | $\begin{gathered} 25 \% \\ ( \pm 2 \%) \end{gathered}$ |
| A03 | Solve problems within mathematics and in other contexts <br> Learners should be able to: <br> - translate problems in mathematical and non-mathematical contexts into mathematical processes; <br> - interpret solutions to problems in their original context, and, where appropriate, evaluate their accuracy and limitations; <br> - translate situations in context into mathematical models; <br> - use mathematical models; and <br> - evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them. <br> Where questions/tasks targeting this assessment objective will also credit Learners for the ability to 'use and apply standard techniques' (AO1) and/or to 'reason, interpret and communicate mathematically' (AO2) an appropriate proportion of the marks for the question/task will be attributed to the corresponding assessment objective(s). | $\begin{gathered} 25 \% \\ ( \pm 2 \%) \end{gathered}$ |

## AO weightings in A Level in Mathematics B(MEI)

The relationship between the assessment objectives and the components are shown in the following table:

| Component | AO marks per component |  |  |
| :--- | :--- | :--- | :--- |
|  | AO1 | AO2 | AO3 |
| Pure Mathematics and Mechanics (H640/01) | $47-53$ marks | $18-24$ marks | $26-32$ marks |
| Pure Mathematics and Statistics (H640/02) | $47-53$ marks | $22-28$ marks | $22-28$ marks |
| Pure Mathematics and Comprehension (H640/03) | $35-39$ marks | $21-25$ marks | 13-17 marks |
| \% of overall A level in Mathematics B (MEI) (H640) | $48-52 \%$ | $23-27 \%$ | $23-27 \%$ |

Across all three papers combined in any given series, AO totals will fall within the stated percentages for the qualification. More variation is allowed per component, however, to allow for flexibility in the design of items.

## 3c. Total qualification time

Total qualification time (TQT) is the total amount of time, in hours, expected to be spent by a learner to achieve a qualification. It includes both guided learning hours and hours spent in preparation, study,
and assessment. The total qualification time for $A$ Level in Mathematics B (MEI) is 360 hours. The total guided learning time is 360 hours.

## 3d. Qualification availability outside of England

This qualification is available in England. For Wales and Northern Ireland please check the Qualifications in Wales Portal (QIW) or the Northern Ireland Department of Education Performance Measures /

Northern Ireland Entitlement Framework
Qualifications Accreditation Number (NIEFQAN) list to see current availability.

## 3e. Language

This qualification is available in English only. All assessment materials are available in English only and all candidate work must be in English.

## 3f. Assessment availability

There will be one examination series available each year in May/June to all learners.

All examined components must be taken in the same examination series at the end of the course.

This specification will be certificated from the June 2018 examination series onwards.

## 3g. Retaking the qualification

Learners can retake the qualification as many times as they wish. They must retake all components of the qualification.

## 3h. Assessment of extended response

The assessment materials for this qualification provide learners with the opportunity to demonstrate their ability to construct and develop a sustained and coherent line of reasoning and marks for extended
responses are integrated into the marking criteria. Tasks which offer this opportunity will be found across all three components.

## 3i. Synoptic assessment

- Synoptic assessment is the learner's understanding of the connections between different elements of the subject. It involves the explicit drawing together of knowledge, skills and understanding within different parts of the A level course.
- The emphasis of synoptic assessment is to encourage the understanding of Mathematics as a discipline.
- Synoptic assessment allows learners to demonstrate the understanding they have acquired from the course as a whole and their ability to integrate and apply that understanding. This level of understanding is needed for successful use of the knowledge and skills from this course in future life, work and study.
- Learners are required to know and understand the content of all the pure mathematics and to be able to apply the overarching themes, along with associated mathematical thinking and understanding, in all the assessment components of A Level Mathematics B (MEI).
- In all the examination papers, learners will be required to integrate and apply their understanding in order to address problems which require both breadth and depth of understanding in order to reach a satisfactory solution.
- Learners will be expected to reflect on and interpret solutions, drawing on their understanding of different aspects of the course.


## 3j. Calculating qualification results

A learner's overall qualification grade for A Level in Mathematics $B$ (MEI) will be calculated by adding together their marks from the three components taken to give their total mark. This mark will then be
compared to the qualification level grade boundaries for the relevant exam series to determine the learner's overall qualification grade.

## 4 Admin: what you need to know

The information in this section is designed to give an overview of the processes involved in administering this qualification so that you can speak to your exams officer. All of the following processes require you to submit something to OCR by a specific deadline.

More information about the processes and deadlines involved at each stage of the assessment cycle can be found in the Administration area of the OCR website. OCR's Admin overview is available on the OCR website at http://www.ocr.org.uk/administration.

## 4a. Pre-assessment

## Estimated entries

Estimated entries are your best projection of the number of learners who will be entered for a qualification in a particular series. Estimated entries
should be submitted to OCR by the specified deadline. They are free and do not commit your centre in any way.

## Final entries

Final entries provide OCR with detailed data for each learner, showing each assessment to be taken. It is essential that you use the correct entry code, considering the relevant entry rules.

Final entries must be submitted to OCR by the published deadlines or late entry fees will apply.

All learners taking an A Level in Mathematics B (MEI) must be entered for H640.

| Entry code | Title | Component code | Component title | Assessment type |
| :---: | :---: | :---: | :---: | :---: |
| H640 | Mathematics B (MEI) | 01 | Pure Mathematics and Mechanics | External Assessment |
|  |  | 02 | Pure Mathematics and Statistics | External Assessment |
|  |  | 03 | Pure Mathematics and Comprehension | External Assessment |

## Collecting evidence of student performance to ensure resilience in the qualifications system

Regulators have published guidance on collecting evidence of student performance as part of long-term contingency arrangements to improve the resilience of the qualifications system. You should review and consider this guidance when delivering this qualification to students at your centre.

For more detailed information on collecting evidence of student performance please visit our website at: https://www.ocr.org.uk/administration/generalqualifications/assessment/

## 4b. Special consideration

Special consideration is a post assessment adjustment to marks or grades to reflect temporary injury, illness or other indisposition at the time the assessment was taken.

Detailed information about eligibility for special consideration can be found in the JCQ publication $A$ guide to the special consideration process.

## 4c. External assessment arrangements

Regulations governing examination arrangements are contained in the JCQ Instructions for conducting examinations.

Learners are permitted to use a scientific or graphical calculator for all components. Calculators are subject to the rules in the document Instructions for Conducting Examinations published annually by JCQ (www.jcq.co.uk).

## Head of centre annual declaration

The Head of centre is required to provide a declaration to the JCQ as part of the annual NCN update, conducted in the autumn term, to confirm that the centre is meeting all of the requirements detailed in the specification. Any failure by a centre
to provide the Head of Centre Annual Declaration will result in your centre status being suspended and could lead to the withdrawal of our approval for you to operate as a centre.

## Private candidates

Private candidates may enter for OCR assessments.

A private candidate is someone who pursues a course of study independently but takes an examination or assessment at an approved examination centre. A private candidate may be a part-time student, someone taking a distance learning course, or someone being tutored privately. They must be based in the UK.

Private candidates need to contact OCR approved centres to establish whether they are prepared to host them as a private candidate. The centre may charge for this facility and OCR recommends that the arrangement is made early in the course.

Further guidance for private candidates may be found on the OCR website: http://www.ocr.org.uk.

## 4d. Results and certificates

## Grade Scale

A level qualifications are graded on the scale: $A^{*}, A$, $B, C, D, E$, where $A^{*}$ is the highest. Learners who fail to reach the minimum standard for E will be

Unclassified (U). Only subjects in which grades A* to E are attained will be recorded on certificates.

## Results

Results are released to centres and learners for information and to allow any queries to be resolved before certificates are issued.

Centres will have access to the following results information for each learner:

- the grade for the qualification
- the raw mark for each component
- the total weighted mark for the qualification.

The following supporting information will be available:

- raw mark grade boundaries for each component
- weighted mark grade boundaries for the qualification.

Until certificates are issued, results are deemed to be provisional and may be subject to amendment.

A learner's final results will be recorded on an OCR certificate. The qualification title will be shown on the certificate as 'OCR Level 3 Advanced GCE in Mathematics B (MEI)'.

## 4e. Post-results services

A number of post-results services are available:

- Review of marking - If you are not happy with the outcome of a learner's results, centres may request a review of marking. Full details of the post-results services are provided on the OCR website.
- Missing and incomplete results - This service should be used if an individual subject result for a learner is missing, or the learner has been omitted entirely from the results supplied.
- Access to scripts - Centres can request access to marked scripts.


## 4f. Malpractice

Any breach of the regulations for the conduct of examinations and non-exam assessment work may constitute malpractice (which includes maladministration) and must be reported to OCR as soon as it is detected.

Detailed information on malpractice can be found in the JCQ publication Suspected Malpractice in Examinations and Assessments: Policies and Procedures.

## 5 <br> Appendices

## 5a. Overlap with other qualifications

The content of this specification overlaps with AS Mathematics B (MEI) and with other specifications in A Level Mathematics and AS Mathematics.

## 5b. Accessibility

Reasonable adjustments and access arrangements allow learners with special educational needs, disabilities or temporary injuries to access the assessment and show what they know and can do, without changing the demands of the assessment. Applications for these should be made before the examination series. Detailed information about eligibility for access arrangements can be found in the

## JCQ Access Arrangements and Reasonable Adjustments.

The A level qualification and subject criteria have been reviewed in order to identify any feature which could disadvantage learners who share a protected Characteristic as defined by the Equality Act 2010. All reasonable steps have been taken to minimise any such disadvantage.

## 5c. Mathematical notation

The tables below set out the notation that may be used by AS and A level mathematics specifications. Students will be expected to understand this notation without need for further explanation.

| 1 | Set Notation |  |
| :--- | :--- | :--- |
| 1.1 | $\in$ | is an element of |
| 1.2 | $\notin$ | is not an element of |
| 1.3 | $\subseteq$ | is a subset of |
| 1.4 | $\subset$ | is a proper subset of |
| 1.5 | $\left\{x_{1}, x_{2}, \ldots\right\}$ | the set with elements $x_{1}, x_{2}, \ldots$ |
| 1.6 | $\{x: \ldots\}$ | the set of all $x$ such that $\ldots$ |
| 1.7 | $\mathrm{n}(A)$ | the number of elements in set $A$ |
| 1.8 | $\varnothing$ | the empty set |
| 1.9 | $\varepsilon$ | the universal set |
| 1.10 | $A^{\prime}$ | the complement of the set $A$ |
| 1.11 | $\mathbb{N}$ | the set of natural numbers, $\{1,2,3, \ldots\}$ |
| 1.12 | $\mathbb{Z}$ | the set of integers, $\{0, \pm 1, \pm 2,3, \ldots\}$ |
| 1.13 | $\mathbb{Z}^{+}$ | the set of positive integers, $\{1,2,3, \ldots\}$ |


| 1.14 | $\mathbb{Z}_{0}^{+}$ | the set of non-negative integers, $\{0,1,2,3, \ldots\}$ |
| :---: | :---: | :---: |
| 1.15 | $\mathbb{R}$ | the set of real numbers |
| 1.16 | $\mathbb{Q}$ | the set of rational numbers, $\left\{\frac{p}{q}: p \in \mathbb{Z}, q \in \mathbb{Z}^{+}\right\}$ |
| 1.17 | U | union |
| 1.18 | $\bigcirc$ | intersection |
| 1.19 | $(x, y)$ | the ordered pair $x, y$ |
| 1.20 | $[a, b]$ | the closed interval $\{x \in \mathbb{R}: a \leq x \leq b\}$ |
| 1.21 | $[a, b)$ | the interval $\{x \in \mathbb{R}: a \leq x<b\}$ |
| 1.22 | ( $a, b$ ] | the interval $\{x \in \mathbb{R}: a<x \leq b\}$ |
| 1.23 | ( $a, b$ ) | the open interval $\{x \in \mathbb{R}: a<x<b\}$ |
| 2 | Miscellaneous Symbols |  |
| 2.1 | $=$ | is equal to |
| 2.2 | \# | is not equal to |
| 2.3 | 三 | is identical to or is congruent to |
| 2.4 | $\approx$ | is approximately equal to |
| 2.5 | $\infty$ | infinity |
| 2.6 | $\propto$ | is proportional to |
| 2.7 | $\therefore$ | therefore |
| 2.8 | $\because$ | because |
| 2.9 | < | is less than |
| 2.10 | $\leqslant, \leq$ | is less than or equal to, is not greater than |
| 2.11 | > | is greater than |
| 2.12 | $\geqslant, \geq$ | is greater than or equal to, is not less than |
| 2.13 | $p \Rightarrow q$ | $p$ implies $q$ (if $p$ then $q$ ) |
| 2.14 | $p \Leftarrow q$ | $p$ is implied by $q$ (if $q$ then $p$ ) |
| 2.15 | $p \Leftrightarrow q$ | $p$ implies and is implied by $q$ ( $p$ is equivalent to $q$ ) |
| 2.16 | $a$ | first term for an arithmetic or geometric sequence |
| 2.17 | $l$ | last term for an arithmetic sequence |
| 2.18 | $d$ | common difference for an arithmetic sequence |
| 2.19 | $r$ | common ratio for a geometric sequence |
| 2.20 | $S_{n}$ | sum to $n$ terms of a sequence |
| 2.21 | $S_{\infty}$ | sum to infinity of a sequence |


| 3 | Operations |  |
| :---: | :---: | :---: |
| 3.1 | $a+b$ | $a$ plus $b$ |
| 3.2 | $a-b$ | $a$ minus $b$ |
| 3.3 | $a \times b, a b, a . b$ | $a$ multiplied by $b$ |
| 3.4 | $a \div b, \frac{a}{b}$ | $a$ divided by $b$ |
| 3.5 | $\sum_{i=1}^{n} a_{i}$ | $a_{1}+a_{2}+\ldots+a_{n}$ |
| 3.6 | $\prod_{i=1}^{n} a_{i}$ | $a_{1} \times a_{2} \times \ldots \times a_{n}$ |
| 3.7 | $\sqrt{a}$ | the non-negative square root of $a$ |
| 3.8 | $\|a\|$ | the modulus of $a$ |
| 3.9 | $n$ ! | $n$ factorial: $n!=n \times(n-1) \times \ldots \times 2 \times 1, n \in \mathbb{N} ; 0!=1$ |
| 3.10 | $\binom{n}{r},{ }^{n} \mathrm{C}_{r,}{ }_{n} \mathrm{C}_{r}$ | the binomial coefficient $\frac{n!}{r!(n-r)!}$ for $n, r \in \mathbb{Z}_{0}^{+}, r \leqslant n$ or $\frac{n(n-1) \ldots(n-r+1)}{r!}$ for $n \in \mathbb{Q}, r \in \mathbb{Z}_{0}^{+}$ |
| 4 |  | Functions |
| 4.1 | $\mathrm{f}(x)$ | the value of the function f at $x$ |
| 4.2 | $\mathrm{f}: x \mapsto y$ | the function f maps the element $x$ to the element $y$ |
| 4.3 | $\mathrm{f}^{-1}$ | the inverse function of the function $f$ |
| 4.4 | gf | the composite function of f and g which is defined by $\operatorname{gf}(x)=\mathrm{g}(\mathrm{f}(x))$ |
| 4.5 | $\lim _{x \rightarrow a} \mathrm{f}(x)$ | the limit of $\mathrm{f}(x)$ as $x$ tends to $a$ |
| 4.6 | $\Delta x, \delta x$ | an increment of $x$ |
| 4.7 | $\frac{\mathrm{d} y}{\mathrm{~d} x}$ | the derivative of $y$ with respect to $x$ |
| 4.8 | $\frac{\mathrm{d}^{n} y}{\mathrm{~d} x^{n}}$ | the $n^{\text {th }}$ derivative of $y$ with respect to $x$ |
| 4.9 | $\mathrm{f}^{\prime}(x), \mathrm{f}^{\prime \prime}(x), \ldots, \mathrm{f}^{(n)}(x)$ | the first, second, $\ldots, n^{\text {th }}$ derivatives of $\mathrm{f}(x)$ with respect to $x$ |
| 4.10 | $\dot{x}, \ddot{x}, \ldots$ | the first, second, ... derivatives of $x$ with respect to $t$ |
| 4.11 | $\int y \mathrm{~d} x$ | the indefinite integral of $y$ with respect to $x$ |
| 4.12 | $\int_{a}^{b} y \mathrm{~d} x$ | the definite integral of $y$ with respect to $x$ between the limits $x=a$ and $x=b$ |


| 5 | Exponential and Logarithmic Functions |  |
| :---: | :---: | :---: |
| 5.1 | e | base of natural logarithms |
| 5.2 | $\mathrm{e}^{x}, \exp x$ | exponential function of $x$ |
| 5.3 | $\log _{a} x$ | logarithm to the base $a$ of $x$ |
| 5.4 | $\ln x, \log _{\mathrm{e}} x$ | natural logarithm of $x$ |
| 6 | Trigonometric Functions |  |
| 6.1 | $\left.\begin{array}{l} \text { sin, cos, } \tan , \\ \text { cosec, } \\ \text { sec }, \\ \text { cot } \end{array}\right\}$ | the trigonometric functions |
| 6.2 | $\left.\begin{array}{l} \sin ^{-1}, \cos ^{-1}, \tan ^{-1} \\ \arcsin , \arccos , \arctan \end{array}\right\}$ | the inverse trigonometric functions |
| 6.3 | - | degrees |
| 6.4 | rad | radians |
| 9 | Vectors |  |
| 9.1 | a, $\underline{\text { a }}$, ${ }_{\sim}$ | the vector a, a, a; these alternatives apply throughout section 9 |
| 9.2 | $\overrightarrow{\mathrm{AB}}$ | the vector represented in magnitude and direction by the directed line segment AB |
| 9.3 | â | $a \mathrm{unit}$ vector in the direction of a |
| 9.4 | $\mathbf{i}, \mathbf{j}, \mathrm{k}$ | unit vectors in the directions of the cartesian coordinate axes |
| 9.5 | $\|\mathbf{a}\|, a$ | the magnitude of $\mathbf{a}$ |
| 9.6 | $\|\overrightarrow{\mathrm{AB}}\|, \mathrm{AB}$ | the magnitude of $\overrightarrow{\mathrm{AB}}$ |
| 9.7 | $\binom{a}{b}, a \mathbf{i}+b \mathbf{j}$ | column vector and corresponding unit vector notation |
| 9.8 | r | position vector |
| 9.9 | s | displacement vector |
| 9.10 | v | velocity vector |
| 9.11 | a | acceleration vector |
| 11 | Probability and Statistics |  |
| 11.1 | $A, B, C$, etc. | events |
| 11.2 | $A \cup B$ | union of the events $A$ and $B$ |
| 11.3 | $A \cap B$ | intersection of the events $A$ and $B$ |


| 11.4 | $\mathrm{P}(A)$ | probability of the event $A$ |
| :---: | :---: | :---: |
| 11.5 | $A^{\prime}$ | complement of the event $A$ |
| 11.6 | $\mathrm{P}(A \mid B)$ | probability of the event $A$ conditional on the event $B$ |
| 11.7 | $X, Y, R$, etc. | random variables |
| 11.8 | $x, y, r$, etc. | values of the random variables $X, Y, R$ etc. |
| 11.9 | $x_{1}, x_{2}, \ldots$ | values of observations |
| 11.10 | $f_{1}, f_{2}, \ldots$ | frequencies with which the observations $x_{1}, x_{2}, \ldots$ occur |
| 11.11 | $\mathrm{p}(x), \mathrm{P}(X=x)$ | probability function of the discrete random variable $X$ |
| 11.12 | $p_{1}, p_{2}, \ldots$ | probabilities of the values $x_{1}, x_{2}, \ldots$ of the discrete random variable $X$ |
| 11.13 | $\mathrm{E}(X)$ | expectation of the random variable $X$ |
| 11.14 | $\operatorname{Var}(X)$ | variance of the random variable $X$ |
| 11.15 | $\sim$ | has the distribution |
| 11.16 | $\mathrm{B}(n, p)$ | binomial distribution with parameters $n$ and $p$, where $n$ is the number of trials and $p$ is the probability of success in a trial |
| 11.17 | $q$ | $q=1-p$ for binomial distribution |
| 11.18 | $\mathrm{N}\left(\mu, \sigma^{2}\right)$ | Normal distribution with mean $\mu$ and variance $\sigma^{2}$ |
| 11.19 | $Z \sim \mathrm{~N}(0,1)$ | standard Normal distribution |
| 11.20 | $\phi$ | probability density function of the standardised Normal variable with distribution $\mathrm{N}(0,1)$ |
| 11.21 | $\Phi$ | corresponding cumulative distribution function |
| 11.22 | $\mu$ | population mean |
| 11.23 | $\sigma^{2}$ | population variance |
| 11.24 | $\sigma$ | population standard deviation |
| 11.25 | $\bar{x}$ | sample mean |
| 11.26 | $s^{2}$ | sample variance |
| 11.27 | $s$ | sample standard deviation |
| 11.28 | $\mathrm{H}_{0}$ | null hypothesis |
| 11.29 | $\mathrm{H}_{1}$ | alternative hypothesis |
| 11.30 | $r$ | product moment correlation coefficient for a sample |
| 11.31 | $\rho$ | product moment correlation coefficient for a population |


| $\mathbf{1 2}$ | Mechanics |  |
| :--- | :--- | :--- |
| 12.1 | kg | kilograms |
| 12.2 | m | metres |
| 12.3 | km | kilometres |
| 12.4 | $\mathrm{~m} / \mathrm{s}, \mathrm{m} \mathrm{s}^{-1}$ | metres per second (velocity) |
| 12.5 | $\mathrm{~m} / \mathrm{s}^{2}, \mathrm{~m} \mathrm{~s}^{-2}$ | metres per second per second (acceleration) |
| 12.6 | $F$ | Force or resultant force |
| 12.7 | N | newton |
| 12.8 | N m | newton metre (moment of a force) |
| 12.9 | $t$ | time |
| 12.10 | $s$ | displacement |
| 12.11 | $u$ | initial velocity |
| 12.12 | $v$ | velocity or final velocity |
| 12.13 | $a$ | acceleration |
| 12.14 | $g$ | acceleration due to gravity |
| 12.15 | $\mu$ | coefficient of friction |

## 5d. Mathematical formulae and identities

Learners must be able to use the following formulae and identities for A Level mathematics, without these formulae and identities being provided, either in these forms or in equivalent forms. These formulae and identities may only be provided where they are the starting point for a proof or as a result to be proved.

## Pure Mathematics

## Quadratic Equations

$a x^{2}+b x+c=0$ has roots $\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$

## Laws of Indices

$a^{x} a^{y}=a^{x+y}$
$a^{x} \div a^{y}=a^{x-y}$
$\left(a^{x}\right)^{y} \equiv a^{x y}$

## Laws of Logarithms

$x=a^{n} \Leftrightarrow n=\log _{a} x$ for $a>0$ and $x>0$
$\log _{a} x+\log _{a} y \equiv \log _{a}(x y)$
$\log _{a} x-\log _{a} y \equiv \log _{a}\left(\frac{x}{y}\right)$
$k \log _{a} x \equiv \log _{a}\left(x^{k}\right)$

## Coordinate Geometry

A straight line graph, gradient $m$ passing through $\left(x_{1}, y_{1}\right)$ has equation
$y-y_{1}=m\left(x-x_{1}\right)$
Straight lines with gradients $m_{1}$ and $m_{2}$ are perpendicular when $m_{1} m_{2}=-1$

## Sequences

General term of an arithmetic progression:
$u_{n}=a+(n-1) d$
General term of a geometric progression:

$$
u_{n}=a r^{n-1}
$$

## Trigonometry

In the triangle ABC

```
    Sine rule: \(\quad \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}\)
    Cosine rule: \(\quad a^{2}=b^{2}+c^{2}-2 b c \cos A\)
    Area \(=\frac{1}{2} a b \sin C\)
\(\cos ^{2} A+\sin ^{2} A \equiv 1\)
\(\sec ^{2} A \equiv 1+\tan ^{2} A\)
\(\operatorname{cosec}^{2} A \equiv 1+\cot ^{2} A\)
\(\sin 2 A \equiv 2 \sin A \cos A\)
\(\cos 2 A \equiv \cos ^{2} A-\sin ^{2} A\)
\(\tan 2 A \equiv \frac{2 \tan A}{1-\tan ^{2} A}\)
```


## Mensuration

Circumference and Area of circle, radius $r$ and diameter $d$ :
$C=2 \pi r=\pi d \quad A=\pi r^{2}$
Pythagoras' Theorem: In any right-angled triangle where $a, b$ and $c$ are the lengths of the sides and $c$ is the hypotenuse:
$c^{2}=a^{2}+b^{2}$

Area of a trapezium $=\frac{1}{2}(a+b) h$, where $a$ and $b$ are the lengths of the parallel sides and $h$ is their perpendicular separation.

Volume of a prism $=$ area of cross section $\times$ length
For a circle of radius $r$, where an angle at the centre of $\theta$ radians subtends an arc of length $s$ and encloses an associated sector of area $A$ :
$s=r \theta \quad A=\frac{1}{2} r^{2} \theta$

## Calculus and Differential Equations

## Differentiation

## Function

$x^{n}$
$\sin k x$
$\cos k x$
$\mathrm{e}^{k x}$
$\ln x$
$\mathrm{f}(x)+\mathrm{g}(x)$
$\mathrm{f}(x) \mathrm{g}(x)$
$\mathrm{f}(\mathrm{g}(x))$

Derivative
$n x^{n-1}$
$k \cos k x$
$-k \sin k x$
$k \mathrm{e}^{k x}$
$\frac{1}{x}$
$\mathrm{f}^{\prime}(x)+\mathrm{g}^{\prime}(x)$
$\mathrm{f}^{\prime}(x) \mathrm{g}(x)+\mathrm{f}(x) \mathrm{g}^{\prime}(x)$
$\mathrm{f}^{\prime}(\mathrm{g}(x)) \mathrm{g}^{\prime}(x)$

Integral
$\frac{1}{n+1} x^{n+1}+c, n \neq-1$
$\frac{1}{k} \sin k x+c$
$-\frac{1}{k} \cos k x+c$
$\frac{1}{k} \mathrm{e}^{k x}+c$
$\ln |x|+c, x \neq 0$
$\mathrm{f}(x)+\mathrm{g}(x)+c$
$\mathrm{f}(\mathrm{g}(x))+c$

Area under a curve $=\int_{a}^{b} y \mathrm{~d} x(y \geq 0)$

## Vectors

$|x \mathbf{i}+y \mathbf{j}+z \mathbf{k}|=\sqrt{x^{2}+y^{2}+z^{2}}$

## Mechanics

## Forces and Equilibrium

Weight $=$ mass $\times g$
Friction: $F \leq \mu R$
Newton's second law in the form: $F=m a$

## Kinematics

For motion in a straight line with variable acceleration:
$v=\frac{\mathrm{d} r}{\mathrm{~d} t} \quad a=\frac{\mathrm{d} v}{\mathrm{~d} t}=\frac{\mathrm{d}^{2} r}{\mathrm{~d} t^{2}}$
$r=\int v \mathrm{~d} t \quad v=\int a \mathrm{~d} t$

## Statistics

The mean of a set of data: $\bar{x}=\frac{\sum x}{n}=\frac{\sum f x}{\sum f}$
The standard Normal variable: $Z=\frac{X-\mu}{\sigma} \quad$ where $\quad X \sim \mathrm{~N}\left(\mu, \sigma^{2}\right)$

Learners will be given the following formulae sheet in each question paper:

## Formulae A Level Mathematics B (MEI) (H640)

## Arithmetic series

$S_{n}=\frac{1}{2} n(a+l)=\frac{1}{2} n\{2 a+(n-1) d\}$

## Geometric series

$S_{n}=\frac{a\left(1-r^{n}\right)}{1-r}$
$S_{\infty}=\frac{a}{1-r}$ for $|r|<1$

## Binomial series

$(a+b)^{n}=a^{n}+{ }^{n} \mathrm{C}_{1} a^{n-1} b+{ }^{n} \mathrm{C}_{2} a^{n-2} b^{2}+\ldots+{ }^{n} \mathrm{C}_{r} a^{n-r} b^{r}+\ldots+b^{n} \quad(n \in \mathbb{N})$,
where ${ }^{n} \mathrm{C}_{r}={ }_{n} \mathrm{C}_{r}=\binom{n}{r}=\frac{n!}{r!(n-r)!}$
$(1+x)^{n}=1+n x+\frac{n(n-1)}{2!} x^{2}+\ldots+\frac{n(n-1) \ldots(n-r+1)}{r!} x^{r}+\ldots \quad(|x|<1, n \in \mathbb{R})$

## Differentiation

| $\mathrm{f}(x)$ | $\mathrm{f}^{\prime}(x)$ |
| :--- | :--- |
| $\tan k x$ | $k \sec ^{2} k x$ |
| $\sec x$ | $\sec x \tan x$ |
| $\cot x$ | $-\operatorname{cosec}^{2} x$ |
| $\operatorname{cosec} x$ | $-\operatorname{cosec} x \cot x$ |

Quotient Rule $y=\frac{u}{v}, \frac{\mathrm{~d} y}{\mathrm{~d} x}=\frac{v \frac{\mathrm{~d} u}{\mathrm{~d} x}-u \frac{\mathrm{~d} v}{\mathrm{~d} x}}{v^{2}}$

## Differentiation from first principles

$\mathrm{f}^{\prime}(x)=\lim _{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$
Integration
$\int \frac{\mathrm{f}^{\prime}(x)}{\mathrm{f}(x)} \mathrm{d} x=\ln |\mathrm{f}(x)|+c$
$\int \mathrm{f}^{\prime}(x)(\mathrm{f}(x))^{n} \mathrm{~d} x=\frac{1}{n+1}(\mathrm{f}(x))^{n+1}+c$
Integration by parts $\int u \frac{\mathrm{~d} v}{\mathrm{~d} x} \mathrm{~d} x=u v-\int v \frac{\mathrm{~d} u}{\mathrm{~d} x} \mathrm{~d} x$

## Small angle approximations

$\sin \theta \approx \theta, \quad \cos \theta \approx 1-\frac{1}{2} \theta^{2}, \quad \tan \theta \approx \theta$ where $\theta$ is measured in radians

## Trigonometric identities

$\sin (A \pm B)=\sin A \cos B \pm \cos A \sin B$
$\cos (A \pm B)=\cos A \cos B \mp \sin A \sin B$
$\tan (A \pm B)=\frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad\left(A \pm B \neq\left(k+\frac{1}{2}\right) \pi\right)$

## Numerical methods

Trapezium rule: $\int_{a}^{b} y \mathrm{~d} x \approx \frac{1}{2} h\left\{\left(y_{0}+y_{n}\right)+2\left(y_{1}+y_{2}+\ldots+y_{n-1}\right)\right\}$, where $h=\frac{b-a}{n}$
The Newton-Raphson iteration for solving $\mathrm{f}(x)=0$ : $x_{n+1}=x_{n}-\frac{\mathrm{f}\left(x_{n}\right)}{\mathrm{f}^{\prime}\left(x_{n}\right)}$

## Probability

$\mathrm{P}(A \cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A \cap B)$
$\mathrm{P}(A \cap B)=\mathrm{P}(A) \mathrm{P}(B \mid A)=\mathrm{P}(B) \mathrm{P}(A \mid B) \quad$ or $\quad \mathrm{P}(A \mid B)=\frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

## Sample variance

$s^{2}=\frac{1}{n-1} S_{x x}$ where $S_{x x}=\sum\left(x_{i}-\bar{x}\right)^{2}=\sum x_{i}^{2}-\frac{\left(\sum x_{i}\right)^{2}}{n}=\sum x_{i}^{2}-n \bar{x}^{2}$
Standard deviation, $s=\sqrt{\text { variance }}$

## The binomial distribution

If $X \sim \mathrm{~B}(n, p)$ then $P(X=r)={ }^{n} \mathrm{C}_{r} p^{r} q^{n-r}$ where $q=1-p \quad$ Mean of $X$ is $n p$

Hypothesis testing for the mean of a Normal distribution
If $X \sim \mathrm{~N}\left(\mu, \sigma^{2}\right)$ then $\bar{X} \sim \mathrm{~N}\left(\mu, \frac{\sigma^{2}}{n}\right)$ and $\frac{\bar{X}-\mu}{\sigma / \sqrt{n}} \sim \mathrm{~N}(0,1)$

## Percentage points of the Normal distribution

| $p$ | 10 | 5 | 2 | 1 |
| :---: | :---: | :---: | :---: | :---: |
| $z$ | 1.645 | 1.960 | 2.326 | 2.576 |



## Kinematics

Motion in a straight line
Motion in two dimensions
$v=u+a t$
$\mathbf{v}=\mathbf{u}+\mathbf{a} t$
$s=u t+\frac{1}{2} a t^{2}$
$\mathbf{s}=\mathbf{u} t+\frac{1}{2} \mathbf{a} t^{2}$
$s=\frac{1}{2}(u+v) t$
$\mathbf{s}=\frac{1}{2}(\mathbf{u}+\mathbf{v}) t$
$v^{2}=u^{2}+2 a s$
$s=v t-\frac{1}{2} a t^{2}$
$\mathbf{s}=\mathbf{v} t-\frac{1}{2} \mathbf{a} t^{2}$

## Summary of updates

| Date | Version | Section | Title of section | Change |
| :--- | :--- | :--- | :--- | :--- |
| June 2018 | 1.1 | Front cover | Disclaimer | Addition of Disclaimer |
| October 2018 | 2.0 | Multiple |  | Revised sections 1 and 2 with new <br> subsections focussing on key features and <br> command words. Correction of minor <br> typographical errors. No changes have <br> been made to any assessment <br> requirements. |
| May 2021 | 2.1 | Front and <br> back cover | NA | Delete reference to Social Community <br> and replace with Online Support Centre. <br> Update to specification covers to meet <br> digital accessibility standards. |
| February 2023 | 2.2 | 3c | 2f |  |
| February 2024 | 2.4 | Content of |  |  |
| Mathematics B |  |  |  |  |
| (MEI) |  |  |  |  |

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