OCR
RECOGNISING ACHIEVEMENT

Wednesday 16 May 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4758/01 Differential Equations

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4758/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

* 4 7 1 5 7 6 0 6 1 2 *

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Some differential equations of the form

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = f(x)$$

are to be solved.

First consider the case $f(x) = x^2$.

(i) Find the general solution for y in terms of x. **[9]**

(ii) Find the particular solution subject to the conditions $y = 0$, $\frac{dy}{dx} = 0$ when $x = 0$. **[5]**

Now consider the case $f(x) = e^{-3x}$.

(iii) Explain why neither ae^{-3x} nor axe^{-3x} will be a particular integral for the differential equation. **[1]**

(iv) State an appropriate form for a particular integral and hence find the general solution. **[7]**

(v) State with reasons whether it is possible to have particular solutions for which

(A) y is positive for all values of x,

(B) y is negative for all values of x. **[2]**

2 A parachutist of mass m kg falls vertically from rest. After she has fallen x m, her speed is v m s^{-1}. The forces acting on her are her weight and a resistance force of magnitude mkv^2 N, where k is a constant.

(i) Show that her motion is modelled by the differential equation

$$v\frac{dv}{dx} = g - kv^2$$

and solve this to show that $v^2 = \frac{g}{k}(1 - e^{-2kx})$. **[8]**

(ii) Given that her terminal speed is 55 m s^{-1}, calculate k. **[1]**

When her speed is 54 m s^{-1}, she opens her parachute. The motion is now modelled by assuming that the magnitude of the resistance force instantaneously changes to $0.1mgv$ N. The time from the parachute opening is t seconds.

(iii) Formulate and solve a differential equation to find v in terms of t. **[8]**

(iv) Calculate the time it takes for her speed to reduce to 12 m s^{-1}. **[1]**

(v) Calculate the distance she falls from the point at which she opens her parachute to the point at which her speed is 12 m s^{-1}. **[6]**

3 The differential equation $x\dfrac{dy}{dx} - 2y = x^3 \sin x$ is to be solved.

(i) Find the general solution for y in terms of x. **[8]**

(ii) Find the particular solution subject to the condition $y = 0$ when $x = \pi$. Sketch the solution curve for $0 \leqslant x \leqslant 4\pi$. **[5]**

Now consider the differential equation $x\dfrac{dy}{dx} - 2y^2 = 0$.

(iii) Find the general solution for y in terms of x. **[5]**

Now consider the differential equation $x\dfrac{dy}{dx} - 2y^2 = x^3 \sin x$.

This is to be solved numerically using Euler's method. The algorithm is given by
$x_{r+1} = x_r + h$, $y_{r+1} = y_r + hy'_r$ with $(x_0, y_0) = (3.14, 0)$.

(iv) Use a step length of 0.01 to estimate y when $x = 3.16$. **[5]**

(v) How could this estimate be improved? **[1]**

4 The simultaneous differential equations

$$\frac{dx}{dt} = -2x - y + 6,$$

$$\frac{dy}{dt} = x - 2y + 7,$$

are to be solved.

(i) Eliminate y to obtain a second order differential equation for x in terms of t. Hence find the general solution for x. **[12]**

(ii) Find the corresponding general solution for y. **[3]**

Initially $x = 7$ and $y = 0$.

(iii) Find the particular solutions. **[4]**

As $t \to \infty$, $\dfrac{y}{x} \to k$.

(iv) State the value of k and show that $y = kx$ for infinitely many values of t. **[5]**

THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE.