

**Wednesday 16 May 2012 – Morning**

**A2 GCE MATHEMATICS (MEI)**

**4758/01** Differential Equations

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4758/01
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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1 Some differential equations of the form

$$\frac{d^2y}{dx^2} + 6 \frac{dy}{dx} + 9y = f(x)$$

are to be solved.

First consider the case  $f(x) = x^2$ .

(i) Find the general solution for  $y$  in terms of  $x$ . [9]

(ii) Find the particular solution subject to the conditions  $y = 0$ ,  $\frac{dy}{dx} = 0$  when  $x = 0$ . [5]

Now consider the case  $f(x) = e^{-3x}$ .

(iii) Explain why neither  $ae^{-3x}$  nor  $axe^{-3x}$  will be a particular integral for the differential equation. [1]

(iv) State an appropriate form for a particular integral and hence find the general solution. [7]

(v) State with reasons whether it is possible to have particular solutions for which

(A)  $y$  is positive for all values of  $x$ ,

(B)  $y$  is negative for all values of  $x$ . [2]

2 A parachutist of mass  $m$  kg falls vertically from rest. After she has fallen  $x$  m, her speed is  $v$   $\text{m s}^{-1}$ . The forces acting on her are her weight and a resistance force of magnitude  $mkv^2$  N, where  $k$  is a constant.

(i) Show that her motion is modelled by the differential equation

$$v \frac{dv}{dx} = g - kv^2$$

and solve this to show that  $v^2 = \frac{g}{k}(1 - e^{-2kx})$ . [8]

(ii) Given that her terminal speed is  $55 \text{ m s}^{-1}$ , calculate  $k$ . [1]

When her speed is  $54 \text{ m s}^{-1}$ , she opens her parachute. The motion is now modelled by assuming that the magnitude of the resistance force instantaneously changes to  $0.1mgv$  N. The time from the parachute opening is  $t$  seconds.

(iii) Formulate and solve a differential equation to find  $v$  in terms of  $t$ . [8]

(iv) Calculate the time it takes for her speed to reduce to  $12 \text{ m s}^{-1}$ . [1]

(v) Calculate the distance she falls from the point at which she opens her parachute to the point at which her speed is  $12 \text{ m s}^{-1}$ . [6]

3 The differential equation  $x \frac{dy}{dx} - 2y = x^3 \sin x$  is to be solved.

(i) Find the general solution for  $y$  in terms of  $x$ . [8]

(ii) Find the particular solution subject to the condition  $y = 0$  when  $x = \pi$ . Sketch the solution curve for  $0 \leq x \leq 4\pi$ . [5]

Now consider the differential equation  $x \frac{dy}{dx} - 2y^2 = 0$ .

(iii) Find the general solution for  $y$  in terms of  $x$ . [5]

Now consider the differential equation  $x \frac{dy}{dx} - 2y^2 = x^3 \sin x$ .

This is to be solved numerically using Euler's method. The algorithm is given by  $x_{r+1} = x_r + h, y_{r+1} = y_r + hy'_r$  with  $(x_0, y_0) = (3.14, 0)$ .

(iv) Use a step length of 0.01 to estimate  $y$  when  $x = 3.16$ . [5]

(v) How could this estimate be improved? [1]

4 The simultaneous differential equations

$$\frac{dx}{dt} = -2x - y + 6,$$

$$\frac{dy}{dt} = x - 2y + 7,$$

are to be solved.

(i) Eliminate  $y$  to obtain a second order differential equation for  $x$  in terms of  $t$ . Hence find the general solution for  $x$ . [12]

(ii) Find the corresponding general solution for  $y$ . [3]

Initially  $x = 7$  and  $y = 0$ .

(iii) Find the particular solutions. [4]

As  $t \rightarrow \infty, \frac{y}{x} \rightarrow k$ .

(iv) State the value of  $k$  and show that  $y = kx$  for infinitely many values of  $t$ . [5]

**THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE.**



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