OCR
RECOGNISING ACHIEVEMENT

Friday 1 June 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4764 Mechanics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4764
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (24 marks)

1 A rocket in deep space has initial mass m_0 and is moving in a straight line at speed v_0. It fires its engine in the direction opposite to the motion in order to increase its speed. The propulsion system ejects matter at a constant mass rate k with constant speed u relative to the rocket. At time t after the engines are fired, the speed of the rocket is v.

(i) Show that while mass is being ejected from the rocket, $(m_0 - kt)\frac{\mathrm{d}v}{\mathrm{d}t} = uk$. **[6]**

(ii) Hence find an expression for v at time t. **[5]**

2 A light elastic string AB has stiffness k. The end A is attached to a fixed point and a particle of mass m is attached at the end B. With the string vertical, the particle is released from rest from a point at a distance a below its equilibrium position. At time t, the displacement of the particle below the equilibrium position is x and the velocity of the particle is v.

(i) Show that

$$mv\frac{\mathrm{d}v}{\mathrm{d}x} = -kx.$$ **[4]**

(ii) Show that, while the particle is moving upwards and the string is taut,

$$v = -\sqrt{\frac{k}{m}(a^2 - x^2)}.$$ **[5]**

(iii) Hence use integration to find an expression for x at time t while the particle is moving upwards and the string is taut. **[4]**

Section B (48 marks)

3 A uniform rigid rod AB of length $2a$ and mass m is smoothly hinged to a fixed point at A so that it can rotate freely in a vertical plane. A light elastic string of modulus λ and natural length a connects the midpoint of AB to a fixed point C which is vertically above A with AC = a. The rod makes an angle 2θ with the upward vertical, where $\frac{1}{3}\pi \leqslant 2\theta \leqslant \pi$. This is shown in Fig. 3.

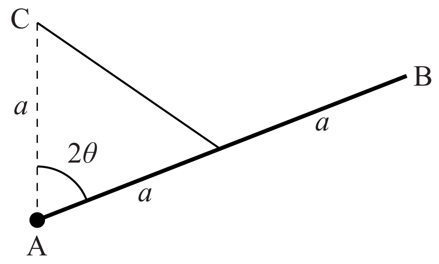

Fig. 3

(i) Find the potential energy, V, of the system relative to A in terms of m, λ, a and θ. Show that

$$\frac{dV}{d\theta} = 2a\cos\theta\,(2\lambda\sin\theta - 2mg\sin\theta - \lambda). \quad (*)$$ **[7]**

Assume now that the system is set up so that the result (*) continues to hold when $\pi < 2\theta \leqslant \frac{5}{3}\pi$.

(ii) In the case $\lambda < 2mg$, show that there is a stable position of equilibrium at $\theta = \frac{1}{2}\pi$. Show that there are no other positions of equilibrium in this case. **[9]**

(iii) In the case $\lambda > 2mg$, find the positions of equilibrium for $\frac{1}{3}\pi \leqslant 2\theta \leqslant \frac{5}{3}\pi$ and determine for each whether the equilibrium is stable or unstable, justifying your conclusions. **[7]**

4 **(i)** Show by integration that the moment of inertia of a uniform circular lamina of radius a and mass m about an axis perpendicular to the plane of the lamina and through its centre is $\frac{1}{2}ma^2$. **[6]**

A closed hollow cylinder has its curved surface and both ends made from the same uniform material. It has mass M, radius a and height h.

(ii) Show that the moment of inertia of the cylinder about its axis of symmetry is $\frac{1}{2}Ma^2\left(\frac{a+2h}{a+h}\right)$. **[6]**

For the rest of this question take the cylinder to have mass 8 kg, radius 0.5 m and height 0.3 m.

The cylinder is at rest and can rotate freely about its axis of symmetry. It is given a tangential impulse of magnitude 55 N s at a point on its curved surface. The impulse is perpendicular to the axis.

(iii) Find the angular speed of the cylinder after the impulse. **[3]**

A resistive couple is now applied to the cylinder for 5 seconds. The magnitude of the couple is $2\dot{\theta}^2$ N m, where $\dot{\theta}$ is the angular speed of the cylinder in rad s^{-1}.

(iv) Formulate a differential equation for $\dot{\theta}$ and hence find the angular speed of the cylinder at the end of the 5 seconds. **[7]**

The cylinder is now brought to rest by a constant couple of magnitude 0.03 N m.

(v) Calculate the time it takes from when this couple is applied for the cylinder to come to rest. **[3]**