5) **(i)** Most scored full marks on this part. Those that did not failed to differentiate a product. A few rewrote the function as the fraction $\frac{\sin x}{e^x}$ which was, of course, perfectly acceptable but gave extra opportunities for algebraic errors.

(ii) Most candidates confirmed that each side of the equality was $-2e^{-x}\cos x$. Having done so, a few failed to work out f″(0).

(iii) A significant number only found f‴(0). A few others made a guess as to what f‴(x) might look like. [–2f″(x) – 2f′(x) – 2f(x) was the most popular, closely followed by –3f″(x) – 3f′(x).] Most candidates failed to realise that all that was required was to differentiate f″(x) from part (ii).

(iv) Most obtained the method mark for using the Maclaurin series, albeit with incorrect values from the earlier parts. Just a few candidates ignored all the work in the first three parts and wrote down, and multiplied, the series expansions for $\sin x$ and e^x from the *List of Formulae*. This was, of course, acceptable but added time.

6) This was another standard question for which most scored full marks. A number of candidates failed to write down the standard form correctly. This is given in the *List of Formulae*. Candidates should be discouraged from relying on memory when it is not necessary!

7) **(i)** A number failed to note the precise demands of the question and drew curves for $q > \frac{1}{2}p$, or failed to state the equation of the line of symmetry or the coordinates of *P*. The sketch was also expected to show that candidates understood that $q = 0$ and $q = \frac{1}{2}p$ were tangents to the curve at the pole.

(ii) Most candidates scored well here, adopting the correct way to carry out the integration.

(iii) This part defeated many candidates. Because *P* was the point furthest from *O*, some candidates thought that $\frac{dr}{dq}$ had to be involved and even set equal to zero.

8) **(i)(a)** In order to confirm the root correct to 3 decimal places it was necessary to work to at least 4 decimal places. Some candidates failed to do so and lost marks needlessly.

(i)(b) This was another sketch requirement where some rather sloppy drawing meant that marks were lost. It was expected that initial values of x should be greater and less than ∂ but this was not always clearly seen.

(ii)(a) A large number of candidates failed to score full marks here because of incorrect algebra. If an error is made which leads to an incorrect result, candidates should check carefully for their error and not just write down the result as given in the question. Some candidates also lost marks for the reasons outlined before: the final result was written down too soon, missing out essential algebraic steps.

(ii)(b) Most candidates scored well on this part.

4727 Further Pure Mathematics 3

General comments

As usual at the January series, only a small number of candidates sat this paper. Overall the paper was found to be slightly less demanding than the paper of a year ago. Almost all candidates were able to gain some marks on each question and there was less evidence of candidates who had completely omitted study on one section of the specification (something observable last year). Some candidates again lacked the level of confidence in Core 1 to Core 4 techniques that this paper presupposes. This was demonstrated by a lack of ability to differentiate products (Q6(i)), find the sum of a geometric series (Q7(i)(a)) and in a general lack of ease when manipulating algebra or working with radians.

Well-prepared candidates were able to tackle several of the questions which were standard (textbook) types. Some candidates were also able to demonstrate their problem solving skills and their ability to produce well-written mathematical arguments; questions 7(ii) and (iii) and 8(iii) targeted the former, while question 2(iii) and much of 7 and 8 assessed the latter. There did not appear to be any problem with the length of the paper with all candidates appearing to have sufficient time. Compared to last year, there were fewer scripts that were difficult to decipher because of poor handwriting.

Comments on individual questions

1) This first question should have been straightforward for any properly prepared candidate since both parts are listed as techniques required in the vectors section of the specification. So it was surprising how many candidates did not answer this with ease.

(i) Most candidates used the scalar product method to find an angle, although one or two, perfectly acceptably, chose to use the cross product. It was disappointing to find that some candidates thought that $|a||b|\sin q$ was the value of the scalar product. A few were unfamiliar with the description 'acute angle' or did not know the relationship between the angle between planes and the angle between their normals.

(ii) Most candidates knew that they needed to find the direction of the line by using the vector product of the normals to the planes; a few made errors in actually finding it. Finding a point on the line proved more of a challenge for weaker candidates with some fixing values for two of the variables before trying to find the third. Those candidates who attempted to find the line solely through simultaneous equations were far more likely to introduce errors.

2) **(i)** This question was generally answered well, particularly when it came to stating the identity. A common error with the order of the group was to say that it was 5 rather than 25.

(ii) A significant number of candidates, erroneously, gave the answer as $-2 - 4$i, failing to realise that this was not an element of the group. A smaller number, for some reason, thought that the inverse had to be the complex conjugate.

(iii) This part of the question allowed candidates to demonstrate the thoroughness of their mathematical arguments. Some, however tended not to realise that $5(a + b\text{i}) = e$, whilst a necessary condition, was not a sufficient one without giving further reasoning. The strongest candidates either used Lagrange's theorem to help them complete the demonstration, or were exhaustive in their exploration of the multiples of all possible $a + b$i, considering $a \neq 0$ or $b \neq 0$ (or both).

3) This was another standard question from which most candidates were able to gain high marks. Common errors which caused progress to be impossible were not recognising the need to initially divide throughout by *x* or neglecting the negative sign in the initial equation. Where candidates successfully heeded these two issues, they could usually work through to a general solution. Some of them, however, lost the final mark by neglecting to multiply their constant by the x^3 term when isolating *y*. Candidates could pick up the final method mark, even when they had been unsuccessful in finding the general solution, by demonstrating that they knew how to find the constant using the boundary condition.

4) (i) Many candidates knew a standard formula for this request and most were then able to use that formula successfully. Others found success by working from first principles; once they had compared the vector between two general points with the perpendicular to both lines, these candidates either used the dot product to simplify or solved the three equations in three unknowns. Marks were sometimes lost through sign errors, with some candidates being unaware of the sign for the **j** component of the vector product.

(ii) Finding the cartesian equation of the plane was generally answered well by candidates who had found the direction of the perpendicular in part (i). Some, however, left their answer in vector equation form.

5) (i) Most candidates were familiar with this standard problem and many gained full marks. A variety of notation can be described as 'polar form', so it was acceptable to write the answer in exponential form, CiS form or in polar coordinates with argument in the interval $[0, 2\pi)$ or $(-\pi, \pi]$. Candidates were, however, penalised for using degrees, omitting i (commonly noted in exponential form attempts) or for roots of the form $\cos\theta - \mathrm{i}\sin\theta$.

(ii) Although many candidates were able to tackle this question, few of them recognised that, with a quartic equation, there would only be four roots; consequently most candidates did not spot the fact that the value corresponding to $k = 0$ should be rejected.

6) Parts (ii) and (iii), which assessed the interpretation of a solution to a differential equation, were answered well only by the very strongest candidates.

(i) Although this question covered another standard technique, there was a significant minority of candidates who made little progress towards a solution. Some demonstrated a lack of knowledge of the standard form of solution resulting from complex roots to the auxiliary equation. Others took insufficient care when finding derivatives of the particular integral. As in previous sessions, there were some whose work resulted in "constants" which were functions of *x* and others who failed to see any problem with a final solution which contained complex coefficients.

(ii) Marks were gained by some candidates for a recognition that the $x\cos 2x$ term would dominate as *x* tended to infinity, but only a few were able to fully describe the behaviour of this function. Good answers often used a diagram to flesh out a verbal explanation.

(iii) This question was, in part, a way to assess candidates understanding of how to select the particular integral since this is the change that affects the shape of the graph as *x* tends to infinity. Candidates who could do part (ii) and who spotted this usually gained full marks.

7) (i) It was pleasing, in part (a), to see that most candidates spotted that they were dealing with a geometric progression and were able to gain the first two marks for this question. Converting from $\frac{a(1-r^n)}{1-r}$ (given in the *List of Formulae* MF1) to the final form was beyond some candidates. Part (b) was often attempted by using the formula in part (a) instead of realising the necessity to refer back to the original definition of *S*.

(ii) Those candidates who used the real part of (a) plus de Moivre's theorem could, with care concerning i in the denominator, produce the right-hand side from the left with relative ease. Some candidates, however, chose to use $\cos nq = \frac{z^n + z^{-n}}{2}$ for each term. If they were adept at algebraic manipulation, they could still work their way through to the solution by summing two geometric series. Some candidates, though, tended to make errors such as believing, mistakenly, that there was now one geometric series with $r = z - z^{-1}$.

(iii) Many candidates did not realise that it is insufficient to use (inexact) decimal calculator values for functions when trying to show the exact value of a root. Because they were not trying to solve the equation $\sin\frac{21q}{2} = \sin\frac{q}{2}$, they were then unable to make progress when it came to finding another root. Some candidates availed themselves of their knowledge of the identities $\sin x = \sin(p - x) = \sin(x + 2p)$ to produce neat, well argued solutions.

8) **(i)** The most elegant answers started from the given identity and worked from there to the new identity in clear steps with reference to the use of $a^4 = e$ or $w^2 = e$. Other well-written arguments worked from one side of the new identity to the other. Where candidates started with the identity to be shown and operated on both sides of the identity till both sides were shown to equate, they still gained the marks on this occasion. However, this practice should be discouraged.

(ii) There were a few candidates who both here - and sometimes in part (i) - falsely assumed commutativity. Most others recognised what was required and made a good attempt to simplify the squared terms they produced.

(iii) While many candidates recognised the need to use only elements of order 2 (plus *e*) in their subgroups, some of these had clearly not considered ensuring that their subset was closed. So it was common to see sets such as $\{e, aw, aw^2, aw^3\}$ alongside one correct cyclic subgroup. The strongest candidates often demonstrated a familiarity not only with general group theory, but also with the structure of both groups of order 4.

Overview – Mechanics

General comments

A high standard of work was apparent across all units this session, and in each the number of poor scripts was small. For the most part scripts contained accurate and well organised solutions to the questions posed. In each module however there were items which caused problems for many candidates, and which were – for the best – the only cause of loss of marks.

In Mechanics 1, it was apparent that that the force exerted by a string on a pulley, Q5 (iv), was an unfamiliar topic. Other problems arose from unusual situations, specifically Q3, parts (i) and (ii), and the final part of Q7.

Difficulties arising from context were apparent also in Mechanics 2, where problems were seen in many scripts with Q3 and Q8. In both questions situations normally tested on horizontal surfaces were instead set on slopes, and strategic thinking was required.

Motion in a circle also proved problematic in Mechanics 3, where it featured the less usual two particle configuration. Towards the end of that paper, the novel configuration of rods in Q6 and the use of SHM equations for a simple pendulum in Q7 proved taxing.

It is of course impossible for students to learn mechanics without reference to specific situations and contexts. It did however seem that even some good candidates were unable to use their knowledge effectively when faced by an unfamiliar situation. The switch from using specific situations to illustrate general principles, to using those principles to confront unusual questions, is perennial, and a difficulty not to be underestimated.

4728 Mechanics 1

General comments

Candidates were well prepared for the examination, a large majority displaying an excellent knowledge of the specification, and how to use that knowledge in the context of the paper. One feature of some wrong answers was that they seldom made a candidate look for an error, as noted in Q5 (ii) and Q7 (i).

There were no significant general areas of weakness. Particular questions which gave difficulty are described below. Misreading of the questions, and a somewhat cavalier attitude to sign, were a source of significant mark loss to some candidates. The choice of positive sense determines whether vector quantities will be evaluated as positive or negative. An unexplained change of sign in an answer may well lose marks, as the value given is a wrong solution to the candidate's equation.

Comments on individual questions

1) Most frequently candidates started the paper by gaining full marks, though 3 out of 5 was a common total among candidates who did not target the "bearing".
Some scripts contained a confusion between bearings and polar angles.

A small minority of candidates began by finding the resultant of the 5 N and 12 N forces, and then combining it with the 14 N force. This gives a diagram which should contain an obtuse angle and calculation which incorporates the ambiguous case of the sine rule. This was not done.

2) (i) This question was very well done, with constant acceleration formulae almost entirely absent.

(ii) Again this was done well, with nearly all candidates demonstrating the appropriate substitution.

3) (i) Though most scripts contained solutions based on resolving parallel to the plane, a significant minority resolved perpendicular to it, getting $T = 6.20$ as the answer. In this question, candidates having a clear diagram with forces were at a significant advantage. However, having 0.2 as the mass, or $T\cos 30$ as the component of T, were common misreads.

(ii) Correct solutions were common. Some candidates who had used correct data in (i) used incorrect values in (ii). It was also quite usual for resolving perpendicular to the plane to be used again in (ii) after it had been (wrongly) used in (i).

(iii) Candidates who made errors in (i) or (ii) were able to gain full marks here, and most did. In this simple context candidates were able to avoid gross sign errors.

4) (i) Nearly all candidates gained both marks, getting their answers via factorisation.

(ii) Attempts to use constant acceleration were rare. Nearly all candidates, correctly, went beyond their value of the integration constant, explicitly finding v at time zero.

(iii) While most candidates started well, many did not make explicit the link between the change of sign for v and a change in direction of motion. Finding a value of t when v was zero was not regarded as showing a change of direction.

5 **(i)** A minority of scripts included the assertion that $u = 1.4$, but most contained a correct calculation of acceleration. Some candidates failed to calculate the tension in the string, while others erred in confusing the signs of their terms, and found $T = 4.41$ N.

(ii) Again this was answered well, though sign errors led to some wrong answers. Finding $m = 0.3$ seldom seemed to surprise a candidate

(iii) In only a few scripts did candidates demonstrate the upward velocity of *Q* at the instant when *P* struck the ground, or assume that the deceleration of *P* would continue to be 4.9. Fully correct work was often seen.

(iv)(a) This was the first major area of difficulty for many candidates, who used particle masses rather than the tension in the string.

(iv)(b) Though some candidates made this a complex situation, for many the answer was simple.

6 **(i)** There were few incorrect solutions.

(ii)(a) Though this was an uncommon request, most scripts showed correct solutions. Some indicated faulty logic: "As *P* has the greater momentum, it will be moving after the collision". A significant minority of candidates believed *P* to be in motion after the collision, with speed 0.667 m s^{-1}.

(ii)(b) This question took candidates away from formal specification topics (such as constant acceleration), and many were unsure how to proceed. There were however many correct answers gaining full marks.

(ii)(c) Speed-time diagrams were fairly common, as was a failure to label which line segment related to which particle. Many diagrams showed *P* and *Q* with zero initial velocity, and graph lines which sloped.

7 **(i)** This was well answered in the main, but candidates who found that *P* had an acceleration down the plane greater than *g* did not look for an error in their calculation. There were a significant number of scripts in which only one of *v* or *t* were found.

(ii)(a) There was a noticeable reluctance on the part of candidates to create a Newton's Second Law equation in which all the force terms were negative. This lead to the sly insertion of a minus sign into a calculation of the distance travelled by *P* before coming to rest.

(ii)(b) This proved too demanding for most candidates, hinging as it did on using the acceleration found in (i), and a distance calculated in (ii)(a). Some candidates who did this introductory work correctly then overlooked the vector nature of momentum. A significant minority used either the velocity calculated in (i) or the acceleration found in (ii)(a) in this part.

4729 Mechanics 2

General comments

A large number of very good candidates were entered for this module. There were many excellent scripts which showed thorough understanding and some of which scored full marks. Only a very small minority showed a lack of preparation for the rigours of the paper. Generally, marks were lost through candidates not answering fully the questions asked, particularly when more than one request was made. However, many candidates could improve their performance by taking greater care over presentation, notation and their diagrams. Q 3 and Q8(ii) proved to be the most challenging.

Comments on individual questions

1) **(i)** This question was well answered. Only a small minority of candidates failed to resolve the force.

(ii) Again well answered. Candidates either used their answer to (i) divided by time or used force component multiplied by speed with equal success.

2) **(i)** The majority of candidates knew how to convert the power to a driving force and then use it in a Newton's second law equation successfully.

(ii) This question was well done by the majority. Only a small minority attempted to use the acceleration given for part (i).

3) **(i)** Setting the motion on an inclined plane caused problems for a significant number of candidates. Solutions in which the speed of A before the collision was taken to be zero or found from using an acceleration of g were all too common. Many were able to use the coefficient of restitution correctly with only a few examples of wrong signs with velocities being used.

(ii) The usual approach seen was to use constant acceleration equations using their acceleration found in (i). It was surprising though that a significant number, having found acceleration as $g\sin 30$ in (i), used g in this part.

(iii) Most recognised the need to use conservation of momentum to solve this request and the signs of the individual terms were usually correct. A few candidates used the masses with the wrong velocities.

4) **(i)** The majority of candidates used a correct approach for this question. It was surprising the number who did not find the centre of mass correctly from using the wrong formula, using a radius of 6 cm, or using an angle of 90° instead of $/_2$. Other causes of lost marks were finding the area of a semicircle incorrectly and adding rather than finding the difference of the area of the square and semicircle to find the total area.

(ii) Most scored well in this part as candidates used their wrong value from part (i) correctly without penalty.

5) Candidates should be advised that the most profitable way of attempting this type of question is to take moments once and resolve twice. It is of course possible to have more than one moments equation, but candidates who do this are usually not as successful.

(i) Most candidates approached this by taking moments about *A*. Most problems arose from having the force at *P* in the wrong direction, or finding wrong distances.

(ii) Resolving in two directions was by far the more successful approach. Those who tried to take moments, although a valid method, were generally less successful. Examiners reported that the use of normal reaction as weight was seen as often as in previous series.

6) (i) This proved to be the most accessible question on the paper. Only a small minority were unfamiliar with the relationship between impulse and momentum.

(ii) Most candidates used the intended energy approach. This was the most successful method with only a minority having the change in kinetic energy with a wrong sign. It was possible to use constant acceleration to solve the problem if a general angle of the slope was used. Unfortunately many, wrongly, used acceleration as *g* and displacement as 0.3 m.

(iii) Two common methods were seen by examiners. The energy approach was usually well done, with common errors seen as using conservation of energy with wrong signs and also the potential energy of the particle used twice. The distance of 0.2/sin30 m in the work done by the frictional force was usually attempted. Candidates, who used constant acceleration combined with Newton's second law, commonly thought the displacement to be used was 0.2 m rather than 0.2/sin30 m.

7) (i) Many good solutions were seen. There was very little evidence of attempts to 'fudge' the given answer.

(ii) The connection between this and the previous part was not always appreciated and quite a few candidates started again. The most common error was to take $y = 2.1$ and some thought the trig identity to be used was $\sec^2\theta = 1 - \tan^2\theta$ but nevertheless still, wrongly, obtained the required result! Some candidates who could not show the given result were sensible enough to use it to find the angle.

(iii) Although the simple method using the expression for the *x* displacement was often seen, many chose to work vertically, often unsuccessfully since they failed to consider all stages of the motion. Some correctly used their expression for *y* displacement, a few simply used 22/14.

8) (i)(a) Some good solutions were seen. Some candidates either got a wrong angle or got a correct angle and resolved incorrectly, but amazingly still managed to get the correct result.

(i)(b) Finding a second appropriate equation proved difficult for quite a few. A remarkable number of combinations of values of *F* and *R* appear to produce $\mu = 0.336$! Some candidates lost marks here by eliminating both *R* and *F* from their equations to find just μ and so failing to find the values of *R* and *F* as required.

(ii) This proved to be the most difficult question on the paper with very few correct solutions seen by examiners. Only a minority realised the direction of *F* had to change in this situation and most simply used the same equations as before, or used one equation with the values of *R* and *F* found in (ii), or solved equations in which *F* (or *R*) was zero.

4730 Mechanics 3

General comments

Many of the candidates for this unit were very competent and very well prepared; these candidates found the paper well within their grasp. There were a number of others who struggled, either with the whole paper or with certain questions. There were many cases of candidates doing some questions completely correctly but making no attempt, or no attempt worth any credit at other questions – the questions they could and could not do varied. However, the questions found most difficult were Questions 4, 6 and 7 – especially the later parts. It may be that some candidates were having an initial attempt at the unit before having a more serious and better prepared attempt in June.

The presentation of the scripts was extremely good in many cases, but there were rather a lot of scripts where the writing was hard to read and the mathematical argument very hard to follow. An examiner can only give credit for work that can be read – and some of the work this series was very nearly totally illegible.

Comments on individual questions

1) Most candidates found this question straightforward, with about half using the cosine rule to find I and then the sine rule to find θ and about half finding expressions for $I\sin\theta$ and $I\cos\theta$ and solving these. A small number of candidates completed the diagram wrongly, and so ended up with wrong answers. Others made small errors, like omitting the mass from one or more term.

2) Almost all candidates did this question very well, working methodically through the three parts, with only an occasional sign error, or a confusion of sine and cosine. However, a small number of candidates thought that, after the collision, the motion of *B* was at right angles to the line of centres; the question clearly states that the direction is at right angles to *B*'s original direction of motion. These candidates were restricted to a method mark in each part.

3) Most candidates correctly used $v\frac{\mathrm{d}v}{\mathrm{d}x}$ for acceleration in part (i), with a very small number making a sign error, missing the mass of 0.3 kg or using equations of motion for constant acceleration. Almost all of those who had the correct start went on to correctly establish the given expression. Most candidates established that the arbitrary constant was 0, though not doing this was not penalised in part (i). In part (ii) some candidates worked from the given answer in part (i) by expressing v as $\frac{\mathrm{d}x}{\mathrm{d}t}$, others started with $\frac{\mathrm{d}v}{\mathrm{d}t}=5x$ and, provided they managed to write $5x$ in terms of v they made progress; however, some candidates attempted to solve the differential equation written in terms of the 3 variables t, v and x. In this part candidates were required to either find the arbitrary constant, or show that it was 0 – depending on their method of solution. A number of candidates made mistakes involving the $\sqrt{5}$ factor, either losing it or putting it in the wrong place in an equation.

4) Many candidates did this question completely correctly. In part (i) a few made things more difficult than necessary for themselves by using a zero level for potential energy that was not the initial position, others made a sign error, or forgot about the kinetic energy of one of the particles. Quite a number of candidates made a sign error in the equation $0.4g\sin\theta - R = \frac{0.4v^2}{a}$ and so did not find a correct expression for R; a small number omitted the weight term. Almost all candidates knew that part (ii) depended on putting $R = 0$, and those who had part (i) correct invariably gained both marks in part (ii). A considerable number of candidates omitted this part.

5) Most candidates correctly established the extension of the string in part (i) and went on to find the speed of *Q* when it reached *P* and the common speed after that. A few candidates did not know how to use energy to find the speed of *Q*, and some tried to use energy to find the common speed. Part (ii) was done extremely well, with only a few candidates going wrong by forgetting about the initial elastic energy or by making errors in working out the kinetic or potential energy terms. A small number of candidates used a wrong '*X*', though some of those managed to correct for this later and arrive at the correct equation.

6) Although the working was often rather complicated in part (i), most candidates did manage to show that the force on *CD* at *D* is $\frac{1}{2}W$. While most then went on to find the force acting on *AB* at *B* a small number missed out this demand and others did very complicated and wrong work. Part (ii) was obvious to quite a number of candidates, but a large number of candidates did very complicated and usually wrong work to arrive at quite exotic wrong answers. Part (iii) was found quite challenging – only candidates with the correct answer to part (ii) were able to gain full marks, but other candidates were given method marks for correct working.

7) Part (i) was done well by many candidates. In part (ii), some candidates missed out the length of the pendulum, and a few others only quoted the SHM result; a few failed to work out the period of the motion. Part (iii) proved more challenging, though most candidates did tackle it by means of the equations for velocity and displacement in terms of cos ωt and sin ωt. Those who chose to use $\dot{\theta}^2 = \omega^2(a^2 - \theta^2)$ were able to find the angular displacement but were not usually able to correctly find the time, since this required some manipulation involving the period of motion. For both methods, some candidates used a linear speed of -0.2 m s^{-1} when they should have used an angular speed of -0.25 rad s^{-1}.

Overview – Probability & Statistics

This was the last January sitting for these units. As usual the standards were high on S2 and S3, and in all three units many candidates were able to answer standard numerical questions well. Answers to hypothesis test questions are also improving.

There is some concern about candidates who use calculators with a wide range of statistical functions. Such calculators are of course permitted in examinations, but their use does not excuse candidates from the duty of showing full working. Candidates who write down an answer obtained from a calculator without showing the level of working that would be expected from a candidate without such a calculator (for instance, the appropriate standardisation formula in S2) risk losing several marks if their answer is wrong, even if it is nearly right.

The attention of Centres is particularly drawn to the desirability of explicit teaching of the use of the Formulae Book MF1 (see S1) and to the misunderstandings concerning the nature of the variables in a probability density function (see S2 and S3).

4732 Probability & Statistics 1

General comments

There were a few questions that were found more difficult than might have been expected, particularly questions 4 and 6. In addition to this, the question on Spearman's rank correlation coefficient (7) was not the usual "turn the handle" calculation but required some thought. As a result fewer candidates than usual scored high marks overall. The questions that required an answer given in words were fairly well attempted, except for question 7(ii) which required logical thought rather than interpretation.

A few candidates lost marks by premature rounding (eg in questions 3 and 5) or by giving their answer to fewer than three significant figures without having previously given an exact or a longer version of their answer. It is important to note that although an intermediate answer may be rounded to three significant figures, this rounded version should not be used in subsequent working. The safest approach is to use exact figures (in fraction form) or to keep intermediate answers correct to several more significant figures.

Question 7(i) required some elementary algebraic manipulation which some candidates were unable to cope with.

Few candidates appeared to run out of time.

In order to understand more thoroughly the kinds of answers which are acceptable in the examination context, centres should refer to the published mark scheme.

Use of statistical formulae and tables

The formulae booklet, MF1, was useful in questions 3, 5 (for binomial tables) and 7. In question 3 a few candidates quoted their own (usually incorrect) formulae for b and/or r, rather than using the ones from MF1. Some thought that, eg, $S_{xy} = \Sigma xy$ or $\Sigma x^2 = (\Sigma x)^2$. Others tried to use the less convenient versions, $r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\{\Sigma(x-\bar{x})^2\}\{\Sigma(y-\bar{y})^2\}}}$ and $b = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$ from MF1, despite the fact that the individual data values are not given in this question. These candidates completely misunderstood the formulae, interpreting them as, for example, $\frac{(\Sigma x-\bar{x})(\Sigma y-\bar{y})}{\sqrt{\{(\Sigma x-\bar{x})^2\}\{(\Sigma y-\bar{y})^2\}}}$ and $\frac{(\Sigma x-\bar{x})(\Sigma y-\bar{y})}{(\Sigma x-\bar{x})^2}$.

In question 7(i), Σd^2 was sometimes misinterpreted as $(\Sigma d)^2$ and the formula was sometimes misquoted as $\frac{6\times\Sigma d^2}{n(n^2-1)}$ or $\frac{1-6\times\Sigma d^2}{n(n^2-1)}$ or $1-\frac{6\times\Sigma d^2}{n}$ or $1-\frac{6\times\Sigma d^2}{n^2(n-1)}$.

In question 5, some candidates' use of the binomial tables showed that they understood the entries to be individual, rather than cumulative, probabilities.

It is worth noting yet again, that candidates would benefit from direct teaching on the proper use of the formulae booklet, particularly in view of the fact that text books give statistical formulae in a huge variety of versions. Much confusion could be avoided if candidates were taught to use exclusively the versions given in MF1 (except in the case of b, the regression coefficient). They need to understand which formulae are the simplest to use, where they can be found in MF1 and also how to use them.

Comments on individual questions

1) (i) Many candidates argued in a circle, using the given answer $P(X = 6) = \frac{3}{10}$ to find k and then using this value of k to derive the answer $\frac{3}{10}$. Others simply verified that if $P(X = 6) = \frac{3}{10}$, and the other probabilities are in proportion, then their sum is 1. Neither of these methods scored any marks. Many started with $\frac{1}{4}$ and continued with $\frac{1}{4}k = \frac{3}{10} \Rightarrow k = \frac{6}{5} \Rightarrow P(X = 6) = \frac{6}{5} \times \frac{1}{4} = \frac{3}{10}$. A few found $\frac{1}{3}(1 - \frac{3}{10}) = \frac{7}{30}$ as the probability for each of the other three values.

(ii) Most candidates answered this standard question well. Incorrect probabilities from (i) were allowed, so long as their sum was 1. In the calculation of Var(X) a few candidates omitted to subtract $(E(X))^2$ and others subtracted E(X). The usual error of dividing by 4 was occasionally seen. A few candidates assumed that the probabilities were each 0.25. Some used a formula for the mean and/or variance of the binomial distribution or the geometric distribution.

2) (i) Most candidates answered this part correctly. A few omitted the probability of success at either the first attempt or the third attempt. Others thought that the probability of success at the third attempt was $\frac{1}{4} \times \frac{5}{8} \times \frac{13}{16}$ instead of $\frac{1}{4} \times \frac{5}{8} \times \frac{3}{16}$. Only a few chose the more elegant method using the complement.

(ii) Many good answers were seen. Some candidates appeared not to understand the difference between P(he passes on the 2nd attempt) and P(he passes on the 2nd attempt, given that he failed on the first), giving an answer of 0.58 – 0.4 = 0.18. A few formed an equation, but with the term "0.4p" instead of "0.6p". Some found the correct answer of 0.3 but then unnecessarily continued by using the formula $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.6 \times 0.3}{0.6} = 0.3$. Others, having found the correct value of 0.3, continued with 0.6 × 0.3 = 0.18. If candidates did not make clear that this last line was only a check, rather than an answer, they were likely to lose the final mark. Many candidates gave incorrect attempts based on misunderstandings of conditional probability, such as 0.4 ÷ 0.58 = 0.690.

3) (i) This part was answered very well on the whole. A few candidates made a sign error when substituting b (which is negative) in order to find a. Some simply lost the minus sign in b. Some found b and stopped. Others found r instead of what was asked. A few found r and then used this as their value of b.

(ii) Many candidates ignored the instruction to "use the regression line in the diagram" and used their equation from (i). A few candidates misinterpreted the situation, giving an answer such as 71 000.

(iii) Most candidates gave one correct answer, using the word "extrapolation" or some equivalent wording. However, many either gave no second reason or gave one that was, in effect, equivalent to their first reason (eg "The IMR will become negative"). Some candidates gave the valid second reason, namely that the diagram does not show good linear correlation.

(iv) This part was answered well by most candidates. A few used the original totals, just changing the value of n from 6 to 7. Others made the opposite error, finding new totals, but with $n = 6$. Sensibly, most wrote down their new totals such as Σxy, but some were incorrect and, without any indication of method, these lost a method mark.

(v) A few candidates thought that r would decrease because the values used in the formula would decrease, but most stated correctly that r would be unchanged.

4) (i)(a) Most candidates answered this part correctly.

(i)(b) Some candidates answered this very simple part correctly, although some used complicated (though correct) methods such as considering all possible cases: with 3 repetitions or 2 or none ($3 + {}^3C_2\times2\times3 + 3!$). Others used all manner of incorrect methods, such as ${}^3P_3$, ${}^9P_3$, $3!\times3!$, $3!\times3!\times3!$, $3!\times2+3 + 3\times2 + 3\times3$ and ${}^2P_2\times3+{}^3P_3+3$.

(i)(c) A few candidates recognised that this was simply (i)(b) – 3 and gained both marks even if (i)(b) had been incorrect. Most candidates, however, did not use this method but started from scratch, giving either correct methods such as $3! + 6\times3$ or, more commonly, incorrect methods such as $3!\times3!\times2$, ${}^6C_3$, ${}^6P_3$, $6! \div 3!$, $5!$, $6 + 9$, $3!\times3!\div2!$, $3!\times2$ and $2\times3! +2\times3! + 2\times3! + 6$.

(ii) Very few candidates scored marks on this question. This is surprising in the light of the explicit phrase in the specification "solve problems about arrangements . . . . including those involving repetition". Only a few candidates correctly found the number of numbers with one pair of repeated digits ($\frac{4!}{2!}\times 3$). Even fewer found the number of numbers with two pairs if repeated digits ($\frac{4!}{2!2!}\times 3$). Some candidates attempted to list numbers in categories, e.g. those beginning with 1 or those containing 1 and 2, but the lists were frequently incomplete or included repetitions. A common incorrect method was $3\times3\times2\times2$ which fails to distinguish between cases in which the first two digits are the same and those in which they are different. Out of the vast number of incorrect methods seen, some examples are these: $3\times3\times2$, $3!\times4\times2$, ${}^4P\times{}^3C_2$, $3^4 - 4\times3\times3$, $2\times4 + 2\times4 + 2\times4$, $4!\times3!\div12$ and $4!\times3!\times3! - 3!\times3!\times3! - 3!\times23 - 3!\times2^3$. Some incorrect methods resulted in answers in the millions.

5) (i)(a) This part was answered correctly by almost all candidates. A few unnecessarily attempted to use Σxp, sometimes successfully.

(i)(b) Most candidates answered this correctly, using either the tables or the formula. A few just read 0.8965 from the table without subtracting 0.6328.

(ii) Some correct solutions were seen. A small number of candidates gave an elegant method, using B(10, 0.25). However, most candidates gave partially correct solutions. Common such attempts were those which omitted $(P(X = 0))^2$ or omitted one case of P(sum = 0)×(P(sum = 1) or doubled $(P(X = 0))^2$ as well as doubling P(sum = 0)×(P(sum = 1). Some candidates found P(sum ≤ 2) or P(sum = 2). Others just added $P(X = 0) + P(X = 1)$.

(iii) Some candidates used B(5, 0.25). Others understood that a different value of p was required, but chose the answer to (ii) instead of (i) for that value. Others misread the question to read "10 such sums are chosen . . ." and found P(exactly three values of $(X_1 + X_2)$ are 2).

6) (i) This question gave rise to many different approaches, only some of which were valid. Many candidates appreciated the need to find the total area, but many did not appear to understand what units they were using. Some used cm^2 or small squares but others took the scale from the x–axis and assumed a scale for the y–axis. In many cases it appeared that candidates were not aware that they were actually choosing a scale in their calculation. Marking was generous and many candidates scored two marks for attempting to find the total area (in any units) and relating this correctly to the total frequency of 800. However, because of the muddle over scales, few knew how to take the final step and find a. Some candidates considered only one block without considering the total area of the histogram. Others only considered the heights of the blocks rather than their areas. Many candidates used the range (80) and found 800 ÷ 80, which led to an incorrect method in almost every case.

(ii) Few fully correct answers were seen. Most candidates found half the total area or frequency. Many identified the correct class (50–56) but some of these just gave the midpoint of this class as the median. Others tried to find exactly where in the class the median was situated, but only some of these could handle the necessary proportion calculation. A few candidates found the mid–point of the range, giving an answer of 60.

7) (i) Most candidates correctly attempted to form an equation by equating the formula for r_s to $\frac{63}{65}$. Many did not know what to do with the Σd^2. Some substituted $\Sigma d^2 = 6$ (or 8) by looking at the table. Others recognised $d = 1$ but thought that $\Sigma d^2 = 1$. Some understood that $d = 1$ for n pairs and while a few put $\Sigma d^2 = 1^n = 1$, many found the correct value of $\Sigma d^2 = n$. From then on the problem was one of algebra. Some cancelled the n immediately and went on to solve the equation correctly. Others did not cancel and found themselves faced with a cubic equation. Some cancelled at that stage although some did so incorrectly. Many algebraic errors crept in.

A few candidates simply tried a few values of n and generally found the correct value in the end.

(ii)(a) Many candidates answered this part correctly, often referring to a straight line or to perfect correlation. A few thought that the statement is false because r and r_s are independent of each other.

(ii)(b) Candidates struggled to give a convincing explanation here. Some unwisely ignored the instruction to “use a diagram” thus compounding their difficulties. A few clearly understood the point but drew a diagram in which one point was out of position, thus invalidating their argument. Others drew a diagram with $r \approx -1$ and $r_s = -1$. Some drew incorrect diagrams, showing apparently randomly scattered points, which they claimed showed cases in which $r_s = 1$ but $r \neq 1$. Some gave separate diagrams for r and r_s. Some candidates thought that the statement is true for a reason such as “r_s is r for ranks”.

8) (i)(a) Most candidates answered this correctly, although a few gave $0.9^5 \times 0.1$.

(i)(b) Geometric distribution questions involving “before” or “after” often cause problems. Candidates are confused as to whether a “1 –” is needed. Others think that since it is a geometric situation, “$\times p$” must be included. Also sometimes there is confusion over the power. In fact most candidates answered this question correctly, with a few giving 0.9^4 or $1 - 0.9^5$ or $0.9^5 \times 0.1$. Some used the long method (ie the complement method), but (as usual) a few of these omitted a term or added an extra term.

(i)(c) Only a few candidates used the simplest method which involves SS, FSS, SFS. Few candidates answered this question totally correctly although many gave partially correct answers. Some gave only $0.1^2 \times 0.9$. Many gave $3 \times 0.1^2 \times 0.9$ but omitted $+ 0.1^3$. Many included terms such as 0.1×0.9^2. Some used the complement method, but most of these only gave $1 - 0.9^3$, omitting to subtract $3 \times 0.9^2 \times 0.1$ also.

(ii)(a) This question was well answered by most candidates. A few misread and thought Jill went first. Others included success for the wrong girl or for both girls.

(ii)(b) Many candidates were confused as to how many failures were necessary for each girl. Others included success for the wrong girl or for both girls.

4733 Probability & Statistics 2

General comments

This was a challenging paper in several ways, but there were many candidates who were able to display a very good ability to handle techniques and who could understand most of the finer points involved. The proportion of correct continuity corrections was gratifyingly high. Answers to hypothesis tests were of a high standard, with most candidates stating hypotheses and conclusions carefully and correctly.

Candidates who rely on calculators, rather than tables, for probabilities need to ensure that they show full details of working. As this paper does not assume the use of calculators with an inbuilt normal probability function, it is expected that all candidates show the standardisation of normal variables. Candidates who merely say, for instance, P(> 29.98) = 0.3274 risk losing *all* subsequent marks (even as many as 5) if their answer is wrong – even if it is close. The same is true for calculations involving the Poisson formula.

As mentioned in previous reports, modelling assumptions for the Poisson distribution continue to be poorly understood, and the same is plainly true for the concept of the probability density function.

Comments on individual questions

1) Most knew what to do, though many lost marks by failure to spell out the answers for the critical region and the significance level. Some wrongly attempted a right-hand tail.

2) (i) Almost everyone scored full marks here.

(ii) Many got this right, but almost as many used a standard deviation of $\sigma/\sqrt{10}$, which is wrong.

3) (i) Many scored full marks here, but some failed to give sufficient detail. DVDs should be numbered “sequentially”, or “from 1 to 9000”, or similar; it should be stated that numbers falling outside the range 1 to 9000 are rejected; and “select numbers randomly” is not an acceptable alternative to “select using random numbers”.

3) (ii) This was invariably answered well, with a very large number of candidates choosing the correct continuity correction.

4) (i) This simple request revealed many misunderstandings. It is clear that the letter *x* had no meaning in this context for many candidates. The correct answer is that *x* represents a value, or values, taken by the random variable *X*.

(ii) Almost always correct.

(iii) Generally done very well, but some failed to square the mean (even when they had written down a formula involving μ^2), and some omitted it altogether. Examiners find that candidates who try to use a single formula, involving the subtraction of the square of an integral, are more likely to get the wrong answer; this formula seems to be too complicated for the majority of candidates.

A number of candidates failed to realise that the final answer had to be given in terms of *a* only, and not *k*.

5) (i) Generally very well done. The continuity correction was very often correct; with these numbers it makes little difference, but it should still be included.

(ii) Full marks here were common, either by using the intended Poisson approximation (as used by a large majority) or the exact binomial.

6) (i) This was generally very well handled. Few confused $\bar{x}$ and *m*, or *p* and *z*. The most common mistake was to take 12 as the standard deviation rather than the variance.

It was also pleasing that so many candidates realised that the necessary assumption was that the standard deviation remained the same. Those who said “all other factors must remain the same” also gained full credit.

A continuity correction (of –1/60) is correct in this context, but those who did not use it (the vast majority) were not penalised. Almost nobody attempted to use the total rather than the average score.

Candidates whose final conclusion was that “there was significant evidence that Gordon’s mean score had not improved” lost the final mark.

(ii) There remains confusion between whether use of the CLT is *necessary* (because you are not told that the parent distribution is normal) and whether it is *possible* (as *n* is large). It is baffling that so many candidates answer that the parent distribution was known to be normal, when there had been no mention of the normal distribution.

Some candidates continue to have a wrong idea of what the CLT says, often seeming to think that the CLT refers to the *n* divisor in the standard error.

7) (i) Most candidates could get under way with this, correctly using *z*-values. Those whose calculators gave them answers to more than 4 SF made life very difficult for themselves; it was expected that the tabular values *z* = 2 and –1 would be used, although of course a final answer such as *s* = 5.0003Ö*n* would gain full marks.

Examiners continue to feel that candidates would benefit from being taught how best to set out, and solve, two equations of this sort, especially as this is an absolutely standard type of question. Far too many make very heavy weather of substitution, when elimination is far preferable.

(ii) This question caused many candidates to think hard. Although there are three unknowns, only two probabilities are needed. Some could see this at once; many attempted to use Aidan’s view and abandoned it only when they found that it didn’t work.

8) (i) Very few candidates knew what the word “random” means. It simply means “not following a predictable pattern”, and does not imply either “independent” or “constant average rate”. (The term “random *sample*” does have extra implications of this sort, but that is not the term used here.)

(ii) As usual this question was poorly answered. Most candidates rightly said that traffic light failures had to be independent, and could get a further mark either by saying that it would be true because traffic lights were not on the same circuit, or that it wouldn’t because they were – or similar arguments. Examiners were not testing candidates’ knowledge of traffic lights but of their understanding of the assumptions.

The second condition is that failures must occur at constant average rate. Many omitted the word “average”, and inadvertently showed the necessity for it by saying things like “it will not hold as you will not have exactly the same number of failures in each hour”. Many candidates wrote comments that addressed independence rather than constant average rate, such as “in a storm there would be more failures”. Further, the issue is whether the average rate is constant within one day, and not from day to day, so it was wrong to say “it wouldn’t happen because failures would be more common in winter”.

As has been repeatedly stated in these Reports, “singly” should not be used as a condition (despite its appearance in textbooks). This context shows clearly that if it is not part of the “independence” condition it is meaningless, as the probability that two lights fail at *exactly* the same instant is vanishingly small. As this is inherent in the scenario, it is wrong to state it as an assumption.

(iii) Pleasingly, this question was found very straightforward by many candidates, and full marks were common. Some made life exceptionally difficult for themselves by not cancelling the factor of $e^{-/}$, often taking logs or attempting to apply the laws of indices, and almost always incorrectly.

9) (i) Almost always right.

(ii) Often right, though some used 1 – P(³ 12) rather than 1 – P(£ 13), and others failed to subtract from 1.

(iii) This was a challenging final question. Only one or two candidates saw the short method: the only way that $p = 0.5$ for the second test is if there is a Type II error on the first test. Most attempted to divide the scenario up into separate cases, and a pleasingly large number of correct answers were seen, although too many considered only two cases instead of three or four, and many could not deal with the factors of 0.2 and 0.8.

4734 Probability & Statistics 3

General comments

There were 43 candidates, roughly the same as other January sessions. 5 candidates scored 70 or more out of 72. 27 candidates scored 50 or more.

Most candidates have learned to give their conclusion to significance tests in context, and not to be over-assertive.
Most candidates used the normal distribution correctly in questions 1(ii) and 7(iii).

Comments on individual questions

1) (i) Most candidates answered the first part correctly. In the second part, a few made the common error of saying Var(S) = 25Var(X) + 4Var(Y) + 362 = 408, but most answered correctly.

(ii) Several candidates failed to recognise that X is a continuous distribution and made the error $\mathrm{P}(X \geq 2) = 1 - \mathrm{P}(X \leq 1)$.

2) Usually well-answered, but a few lost marks for premature rounding. A few candidates did not use a paired t-test and were able to score a maximum of 2 marks.

3) (i) Many candidates scored full marks. Of those who did not, many failed to obtain a pooled estimate of variance and/or used an incorrect value of t, or even a z-value.

(ii) Most knew that the population variances should be equal, but some did not use the word population, or mention 'scores' or 'schemes' in their answers and thus did not gain this mark.

4) (i) Most candidates gained most of the marks in this part. The most common error was obtaining an incorrect range for y. In recent series candidates have learned to answer this type of question well, understanding the difference between Y and y.

(ii) Those who tried to evaluate $\int_0^1 \frac{3}{2\sqrt{x}}dx$ usually scored better than those using $\int y^2 g(y)\,\mathrm{d}y$, many having the wrong limits obtained in (i).

5) (i) As usual, most candidates answered the question on confidence limits for a proportion well.

(ii) Most gave an acceptable answer to this part, realising that consideration of a large number of intervals was necessary.

(iii) Most candidates obtained the correct answer to this part. A few used an incorrect z value.

6) Well answered by most candidates. About one-quarter of the candidates did not recognise that a χ^2 test was necessary and tried to carry out various tests based on proportions. Most of these scored no marks.

7) (i) Most answered this part correctly. Of those who did not, some used an incorrect z value, others used $\frac{1}{16}$ or 1 instead of $\frac{1}{4}$.

(ii) More candidates obtained the correct variance than the correct mean. Common errors were $\mu - \mu$ and $\mu_x - \mu_y$.

(iii) This was the most difficult question on the paper. Many considered $\bar{x}+1.163<\bar{y}+1.234$ and the equivalent statement with a – sign, or only one inequality.

8) (i) Most scored well on this part. Some did not use Yates' correction, others did not take the modulus of the difference between *O* and *E* before subtracting 0.5.

(ii) Many candidates did not pool the samples. A special ruling allowing 5/7 was in place for these candidates. Otherwise a good discriminator, better candidates scoring at least 6/7 weaker ones scoring from 0 to 2.

4736 Decision Mathematics 1

General comments

Most candidates were able to attempt every question and they were generally able to fit their answers into the spaces provided in the Printed Answer Book. Those candidates who used extension sheets usually labelled their answers to show which part of which question the work referred to. The quality of written answers was rather better than in previous sessions.

Comments on individual questions

1) (i) Most candidates carried out the shuttle sort correctly and recorded the comparisons and swaps accurately. A few candidates used bubble sort or sorted into increasing order and one or two shuttled from the right hand end of the list. The comparisons and swaps should be recorded in figures, not as tally marks. Inevitably there were a few misreads and some candidates who miscopied their own values or lost values in the course of carrying out the algorithm.

(ii) Generally answered correctly, with the sacks appearing in the correct boxes in the correct order.

(iii) Several candidates were able to answer this appropriately, often by first trying $W = 128/4 = 32$ and then realising why it had to be 33. Those who started with an arbitrary higher value often suggested $W = 35$, not realising that the 4th and 5th weights (in decreasing order) could be packed together.

2) (i) Most candidates were able to draw suitable graphs, often these preserved the shape of the original tetromino, although any topologically equivalent graphs (in the correct answer spaces) were acceptable.

(ii) Many candidates thought that (1) was not possible, this seemed to have arisen because they had assumed that the vertices corresponded to the centres of the faces in the relative positions as drawn. Graph (1) could, however, have represented a line of squares, as in picture (A) in the diagram for part (i), for example. It was generally appreciated that (2) and (4) did represent tetrominoes while (3), (5) and (6) did not. The explanations of why these did not represent tetrominoes often referred back to the mistaken idea that the vertices showed the relative positions of the centres of the faces.

(iii) This question asked candidates to identify a tetromino from the ones shown. They needed to choose a letter A to D, from the first set of diagrams, or possibly draw a picture of the tetromino that they had chosen.
In part (a), C had a graph that was not a tree. In part (b), A and D had graphs that were the same tree structure.

(iv) There were several candidates who found two or three of the appropriate graphs and some who found all four. Some candidates repeated graphs they had already drawn but with a different orientation, for example the graph that looks like W and the graph that looks like L are the same. Some candidates carefully drew the four pentominoes with distinct graphs but did not actually draw the graphs.

3) (i) Nearly all the candidates were able to apply Dijkstra's algorithm accurately to find the required path and its weight.

(ii) There were several correct answers. Some candidates forgot to subtract the weight of the arc that had been removed from the given total. A few candidates started from scratch and wrote down a route and then added up its weight, these usually made a slip at some point.

(iii) Again, there were several correct answers. Some candidates used 230 again as the total weight, and some assumed that the least weight route from *A* to *G* was the same as in part (ii). The point here was that the route of weight 41 was no longer available and an alternative route had to be found, such a route must either start *AD* or *AF* and must then use the least weight route to get from there to *G*, this essentially meant checking just the possibilities: *ADEG*, *ADFG* and *AFG*, of these *AFG* = 47 is the best.

4) (i) The question had asked for a cycle (a closed path). Many candidates were able to apply the nearest neighbour method correctly to get a path from Pam through all the other employees, but they did not always close the cycle by returning to Pam at the end. Some candidates only showed their working on the table and did not actually write down their cycle (or their incomplete cycle), where the order of selection was apparent candidates were given credit for the method, but to achieve full marks the cycle needed to be written down: PBAGCFEDG.

(ii) There were several requests in this part, and some candidates did not do all of them, they were asked to apply Prim's algorithm to the table (starting by crossing out the row for P), to list the arcs in the order they were chosen (note, this asked for arcs not vertices), to draw the spanning tree and to write down its weight.

(iii) The lower bound was for Pam and the seven employees and candidates were asked to start by finding a mst with Gita removed. Some candidates removed Pam, or sometimes Alan, instead of Gita. Other candidates constructed the mst on the reduced network but then did not reinsert Gita by using the two least weight arcs from G.

(iv) The candidates who had found the correct minimum spanning tree usually realised who was in which team. Sometimes they just redrew the tree without any indication of how this related to the teams. The minimum elapsed time to contact all employees should have been 40 minutes, this is achieved by Pam contacting Caz, then while Caz rings each of her team, Pam can contact Bob and Bob can contact each of his team. There was no requirement in this part for anyone to ring back to Pam at the end.

5) (i) The profit was £1 on each box of cupcakes, so the expression to be maximised was $x + y + z$. Several candidates just picked out the values 24, 20 and 12 from the stem. This would have led to an appropriate expression if the profit had been £1 on each cupcake (although this seems a little excessive).

(ii) Most candidates were able to convincingly achieve the given expression, a few played around with the given values in a less than convincing way. The best answers were those that included both numerical values and some written explanation.

(iii) Many candidates were able to get the second constraint as $48x + 60y + 48z \leq 3000$ and then reduce this to $4x + 5y + 4z \leq 250$. Some candidates achieved the constraint with 3000 on the rhs but then only divided by 4.

(iv) The tableau should have 3 rows and 7 columns. Some candidates omitted the *P* column, the Simplex tableau is a shorthand for a set of matrix operations really and so the *P* column does need to be included.

(v) The question asked candidates to choose a pivot from the *x*-column, despite this some chose to use the *y*-column. Candidates should show the values of the ratios, 69.44 and 62.5, rather than giving a general description of the method. Having chosen the entry on which to pivot, the row operation was required for each row, including the objective row and the pivot row. The operations could be given in abbreviated forms, for example R2-18pr, provided that (for the rows other than the pivot) they were of the form current row $\pm$ appropriate multiple of (original or new) pivot row. The numerical work of most candidates was accurate.

(vi) Candidates were asked to write down the values that their first iteration gave for x, y and z. This was a simple case of reading off from a tableau, there was no need for simultaneous equations. Some candidates did not know how to interpret the tableau and gave the coefficients from the objective row. A small, but not insignificant, number of candidates claimed that they still had $x=0$, $y= 0$, $z=0$.
Candidates should have achieved $x=62.5$, $y=0$, $z=0$; as only complete boxes could be sold, the maximum profit was £62, this meant that as well as excess topping there were also decorations left over. Some candidates were able to calculate these excess amounts, usually by working out how much topping and how many decorations had been used in making 62 boxes each containing 24 miniature cupcakes. The candidates who used the slack values generally did not scale the constraints back up to 5000 and 3000, and rarely allowed for the half box that could not be sold.

(vii) Some candidates were able to achieve full marks on this part, although fully correct solutions were rare. By rearranging $P = x+y+z$, the variable x can be eliminated from the second constraint to give an inequality involving just P and y. For P to be as large as possible we try setting $P = 62$, the values of x, y and z then follow.

4737 Decision Mathematics 2

General comments

Most candidates were able to attempt every question and they were generally able to fit their answers into the spaces provided in the Printed Answer Book. The quality of written answers was substantially better than in previous series.

Comments on individual questions

1) (i) The bipartite graph was usually correct, a few candidates missed out a single arc, often the arc joining *J* to *B*.

(ii) Many candidates found the alternating path *D*-*F*-*A*-*I* and drew the corresponding incomplete matching. Some candidates found a longer alternating path, such as *D*-*F*-*A*-*G*-*B*-*J*, this only got 1 mark because the question had asked for the shortest alternating path.

(iii) Most candidates were able to construct the complete matching, even if they had not found the alternating path in part (ii).

(iv) Candidates were usually able to explain why there is just one complete matching, generally they started at either *D* or *J* and followed the logic around the diagram from there.

2) (i) Candidates were able to complete the immediate predecessors.

(ii) Most candidates were able to complete the passes, including dealing with the dummy activities.

(iii) Candidates generally realised that *B* and *E* had 2 minutes of float and could be delayed without affecting the project completion time.

(iv) Here the delay was on a critical activity so it caused a 2 minute delay in the project completion time, as would a similar delay on any other critical activity.

(v) Some candidates assumed that a delay in *C* would have no effect on the project completion time as *C* is not a critical activity, however most candidates realised that there was only 1 minute of float on *C* and the other minute would be a delay in the completion time.

3) (i) There were very few arithmetic errors in subtracting the given values from 10, a few candidates subtracted 10 from each value and did not seem concerned that they had achieved negative values.

(ii) This was done well. Some candidates did not explain how the various tables were formed and some went into far too much detail, all that was needed was phrases like ‘reduce rows’, ‘augment by 2’, etc.

(iii) Some candidates assumed that the values found in part (i) must be the guesses, this is not what the question says however. Starting from the original table and deleting the row for L and the column for R enables an allocation with 23 guesses to be found, this cannot be bettered since any allocation with 24 would require K and M to both be the nurse.

4) (i) The supersource was usually correctly added to feed the two sources with appropriate arc weights.

(ii) Many candidates just wrote down values with no evidence of working. Several candidates had clearly not dealt with the arcs in which the flow across the cut was from *T* to *S*.

(iii) As in part (ii), there was often little evidence of the method used. The arcs that join vertices from set *X* to vertices from set *Y* were S_1B, S_2C, *AD*, *EB* and *EC*. Some candidates using the graphical method tried to include *AE*, but this arc was cut twice, if it was cut at all, so it cancelled itself out.

(iv) Most candidates were able to show an appropriate flow.

(v) Some candidates gave the largest individual arc capacity on the route and others gave the lowest flow in any of the arcs on the route. Most candidates were able to state that the flow of 13 could be augmented by another 6+7=13.

(vi) The maximum flow was 30, several candidates claimed 26 but this was usually because they had assumed that the flow in arc *DE* was from *D* to *E*. Undirected arcs may be dealt with by treating them as a pair of directed arcs. An example of a feasible flow of 30 litres per second is obtained by sending 9 along S_2CET, 2 along S_1BCET, 8 along S_1BET, 1 along S_1AET, 4 along S_1BEDT and 6 along S_1ADT.

5) (i) Most candidates answered this correctly, giving both the value 4 and the choice of the diamond card.

(ii) Candidates usually understood that the square card dominated over the diamond card for Rose, in the sense that she won more points by playing square than by playing diamond for each of Colin's choices. Most candidates were able to say that Colin should not play the triangle card and write out the reduced table. The question had made three requests and candidates needed to explicitly answer all three of these to get the marks in this part.

(iii) Most candidates understood what they needed to do here, although there were some arithmetic errors. Colin should have ended up with two play-safe strategies and candidates needed to write down both of these. Candidates should show the numerical values when demonstrating that the game is unstable.

(iv) Many candidates were able to explain that 6 had been added throughout the original matrix to make the entries non-negative, then the square column had been used to give the expression in the question.

(v) The value of *m* was usually correct and most candidates subtracted 6 to get the optimal value for *M*. In this particular case, all three expressions gave $m \leq 149/24$, it would be more usual to have at least two different values and then the smallest value would be the appropriate choice.

6) (i) Candidates often understood that this referred to there being no houses stored overnight on Thursday night, and could usually explain why there must be at least one house stored. A few candidates thought that it referred to Friday night.

(ii) There were several completely correct answers to this part, although other candidates were clearly very confused about what was happening.

(iii) The dynamic programming should have started at stage 4, some candidates tried to show (5; 0), but nothing has happened until there is travel from (5; 0) to (4; 1) or (4; 2).

(iv) Many candidates were able to find the appropriate production plan.

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