

Mathematics (MEI)

Advanced GCE

Unit **4754A**: Applications of Advanced Mathematics: Paper A

Mark Scheme for January 2013

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.















All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations

Annotation in scoris	Meaning
 and 	
	Benefit of doubt
	Follow through
	Ignore subsequent working
 	Method mark awarded 0, 1
 	Accuracy mark awarded 0, 1
 	Independent mark awarded 0, 1
	Special case
	Omission sign
	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

- a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

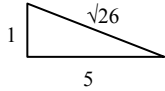
Question	Answer	Marks	Guidance
1	$\frac{2x}{x+1} - \frac{1}{x-1} = 1$ $\Rightarrow 2x(x-1) - (x+1) = (x+1)(x-1)$ $\Rightarrow 2x^2 - 3x - 1 = x^2 - 1$ $\Rightarrow x^2 - 3x = 0 = x(x-3)$ $\Rightarrow x = 0 \text{ or } 3$	<p>M1</p> <p>A1</p> <p>DM1</p> <p>A1</p> <p>[4]</p>	<p>mult throughout by $(x+1)(x-1)$ or combining fractions and mult up oe (can retain denominator throughout). Condone a single computational error provided that there is no conceptual error. Do not condone omission of brackets unless it is clear from subsequent work that they were assumed.</p> <p>any fully correct multiplied out form (including say, if 1's correctly cancelled) soi</p> <p>dependent on first M1. For any method leading to both solutions. Collecting like terms and forming quadratic ($= 0$) and attempting to solve *(provided that it is a quadratic and $b^2 - 4ac \geq 0$). Using either correct quadratic equation formula (can be error when substituting), factorising (giving correct x^2 and constant terms when factors multiplied out), completing the square oe soi.*</p> <p>for both solutions www.</p> <p>SC B1 for $x = 0$ (or $x = 3$) without any working SC B2 for $x = 0$ (or $x = 3$) without above algebra but showing that they satisfy equation SC M1A1M0 SCB1 for first two stages followed by stating $x = 0$ SC M1A1M0 SCB1 for first two stages and cancelling x to obtain $x = 3$ only.</p>

Question	Answer	Marks	Guidance
2	$\sqrt[3]{1-2x} = (1-2x)^{1/3}$ $= 1 + \frac{1}{3}(-2x) + \frac{\frac{1}{3}(-\frac{2}{3})}{2!}(-2x)^2 + \frac{\frac{1}{3}(-\frac{2}{3})(-\frac{5}{3})}{3!}(-2x)^3 + \dots$ $= 1 - \frac{2}{3}x - \frac{4}{9}x^2 - \frac{40}{81}x^3 + \dots$ <p>Valid for $-\frac{1}{2} < x < \frac{1}{2}$ or $x < \frac{1}{2}$</p>	<p>B1</p> <p>M1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>[6]</p>	<p>$n = 1/3$ only. Do not MR for $n \neq 1/3$</p> <p>all four correct unsimplified binomial coeffs (not nCr) soi</p> <p>condone absence of brackets only if it is clear from subsequent work that they were assumed</p> <p>$1 - \frac{2}{3}x$ www in this term</p> <p>$\dots - \frac{4}{9}x^2$ www in this term (not if used $2x$ for $(-2x)$ throughout)</p> <p>$\dots - \frac{40}{81}x^3$ www in this term</p> <p>If there is an error in say the third coeff of the expansion then M0 B1B0B1 is possible.</p> <p>Independent of expansion Allow \leq's (valid in this case) or a combination. Condone also, say, $-\frac{1}{2} < x < \frac{1}{2}$ but not $x < \frac{1}{2}$ or $-1 < 2x < 1$ or $-\frac{1}{2} > x > \frac{1}{2}$</p>

Question	Answer	Marks	Guidance
3 (i)	$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2\cos 2\theta}{\cos \theta}$ <p>When $\theta = \pi/6$ $= \frac{dy}{dx} = \frac{2\cos(\pi/3)}{\cos(\pi/6)}$</p> $= \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$ <p>.....</p> <p>OR</p> $y = 2x\sqrt{1-x^2}$ $\frac{dy}{dx} = -2x^2(1-x^2)^{-1/2} + 2(1-x^2)^{1/2}$ <p>at $\theta = \pi/6, \sin \pi/6 = 1/2$</p> $\frac{dy}{dx} = \frac{-2}{4} \left(1 - \frac{1}{4}\right)^{-1/2} + 2\left(\frac{3}{4}\right)^{1/2} = \frac{2}{\sqrt{3}}$	<p>M1 A1</p> <p>DM1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>DM1</p> <p>A1</p> <p>[4]</p>	<p>their $dy/d\theta$ / their $dx/d\theta$ www correct (can isw)</p> <p>subst $\theta = \pi/6$ in theirs</p> <p>oe exact only, www (but not $1/\sqrt{3}/2$)</p> <p>.....</p> <p>full method for differentiation including product rule and function of a function oe oe cao (condone lack of consideration of sign)</p> <p>subst $\sin \pi/6 = 1/2$ in theirs</p> <p>oe ,exact only, www (but not $1/\sqrt{3}/2$)</p>
3 (ii)	$y = \sin 2\theta = 2 \sin \theta \cos \theta$ $\Rightarrow y^2 = 4 \sin^2 \theta \cos^2 \theta = 4x^2(1-x^2)$ $= 4x^2 - 4x^4 *$	<p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>using $\sin 2\theta = 2 \sin \theta \cos \theta$</p> <p>using $\cos^2 \theta = 1 - \sin^2 \theta$ to eliminate $\cos \theta$ AG need to see sufficient working or A0.</p>

Question		Answer	Marks	Guidance
4	(a)	$V = \int_0^2 \pi y^2 dx = \int_0^2 \pi(1 + e^{2x}) dx$ $= \pi \left[x + \frac{1}{2} e^{2x} \right]_0^2$ $= \pi(2 + \frac{1}{2} e^4 - \frac{1}{2})$ $= \frac{1}{2} \pi(3 + e^4)$	<p>M1</p> <p>B1</p> <p>DM1</p> <p>A1</p> <p>[4]</p>	<p>$\int_0^2 \pi(1 + e^{2x}) dx$ limits must appear but may be later</p> <p>condone omission of dx if intention clear</p> <p>$\left[x + \frac{1}{2} e^{2x} \right]$ independent of π and limits</p> <p>dependent on first M1. Need both limits substituted in their integral of the form $ax + b e^{2x}$, where a, b non-zero constants. Accept answers including e^0 for M1. Condone absence of π for M1 at this stage</p> <p>cao exact only</p>
4	(b) (i)	<p>$x = 0, y = 1.4142; x = 2, y = 7.4564$</p> $A = 0.5/2 \{ (1.4142 + 7.4564) + 2(1.9283 + 2.8964 + 4.5919) \}$ <p>$= 6.926$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>1.414, 7.456 or better</p> <p>correct formula seen (can be implied by correct intermediate step eg 27.7038../4)</p> <p>6.926 or 6.93 (do not allow more dp)</p>
4	(b) (ii)	<p>8 strips: 6.823, 16 strips: 6.797</p> <p>Trapezium rule overestimates this area, but the overestimate gets less as the no of strips increases.</p>	<p>B1</p> <p>[1]</p>	<p>oe</p>

Question	Answer	Marks	Guidance
5	$2\sec^2 \theta = 5 \tan \theta$ $\Rightarrow 2(1 + \tan^2 \theta) = 5 \tan \theta$ $\Rightarrow 2\tan^2 \theta - 5 \tan \theta + 2 = 0$ $\Rightarrow (2\tan \theta - 1)(\tan \theta - 2) = 0$ $\Rightarrow \tan \theta = \frac{1}{2} \text{ or } 2$ $\Rightarrow \theta = 0.464,$ 1.107 <p>.....</p> <p>OR</p> $2/\cos^2 \theta = 5 \sin \theta / \cos \theta$ $\Rightarrow 2 \cos \theta = 5 \sin \theta \cos^2 \theta, \cos \theta \neq 0$ $\Rightarrow \cos \theta (2 - 5 \sin \theta \cos \theta) = 0$ $\Rightarrow \cos \theta = 0, \text{ or } \sin 2\theta = 0.8$ $\Rightarrow \sin 2\theta = 0.8$ $\Rightarrow 2\theta = 0.9273 \text{ or } 2.2143$ $\Rightarrow \theta = 0.464,$ 1.107	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>.....</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>[6]</p>	<p>$\sec^2 \theta = 1 + \tan^2 \theta$ used</p> <p>correct quadratic oe</p> <p>solving their quadratic for $\tan \theta$ (follow rules for solving as in Question 1 [*,*])</p> <p>www</p> <p>first correct solution (or better)</p> <p>second correct solution (or better) and no others in the range</p> <p>Ignore solutions outside the range.</p> <p>SC A1 for both 0.46 and 1.11</p> <p>SC A1 for both 26.6° and 63.4° (or better)</p> <p>Do not award SCs if there are extra solutions in range.</p> <p>.....</p> <p>using both $\sec = 1/\cos$ and $\tan = \sin/\cos$</p> <p>correct one line equation $2 - 5 \sin \theta \cos \theta = 0$ or $2 \cos \theta = 5 \sin \theta \cos^2 \theta$ oe (or common denominator). Do not need $\cos \theta \neq 0$ at this stage.</p> <p>using $\sin 2\theta = 2 \sin \theta \cos \theta$ oe eg $2 = 5 \sin \theta \sqrt{1 - \sin^2 \theta}$ and squaring</p> <p>$\sin 2\theta = 0.8$ or, say, $25 \sin^4 \theta - 25 \sin^2 \theta + 4 = 0$</p> <p>first correct solution (or better)</p> <p>second correct solution (or better) and no others in range</p> <p>Ignore solutions outside the range</p> <p>SCs as above</p>

Question	Answer	Marks	Guidance
6 (i)	$AC = \operatorname{cosec} \theta$ $\Rightarrow AD = \operatorname{cosec} \theta \sec \varphi$	M1 A1 [2]	or $1/\sin \theta$ oe but not if a fraction within a fraction
6 (ii)	$DE = AD \sin(\theta + \varphi)$ $= \operatorname{cosec} \theta \sec \varphi \sin(\theta + \varphi)$ $= \operatorname{cosec} \theta \sec \varphi (\sin \theta \cos \varphi + \cos \theta \sin \varphi)$ $= \frac{\sin \theta \cos \varphi + \cos \theta \sin \varphi}{\sin \theta \cos \varphi}$ $= 1 + \frac{\cos \theta \sin \varphi}{\sin \theta \cos \varphi}$ $= 1 + \tan \varphi / \tan \theta^*$ OR equivalent, eg from $DE = CB + CD \cos \theta$ $= 1 + CD \cos \theta$ $= 1 + AD \sin \varphi \cos \theta$ $= 1 + \operatorname{cosec} \theta \sec \varphi \sin \varphi \cos \theta$ $= 1 + \tan \varphi / \tan \theta^*$	M1 M1 A1 M1 M1 A1 [3]	AD $\sin(\theta + \varphi)$ with substitution for their AD correct compound angle formula used Do not award both M marks unless they are part of the same method. (They may appear in either order.) simplifying using $\tan = \sin/\cos$. A0 if no intermediate step as AG from triangle formed by using X on DE where CX is parallel to BE to get $DX = CD \cos \theta$ and $CB = 1$ (oe trigonometry) substituting for both $CD = AD \sin \varphi$ and their AD oe to reach an expression for DE in terms of θ and φ only (M marks must be part of same method) AG simplifying to required form
7 (i)	$DE = \sqrt{(-5)^2 + 0^2 + 1^2} = \sqrt{26}$  $\cos \theta = 5/\sqrt{26}$ oe $\Rightarrow \theta = 11.3^\circ$	M1 A1 M1 A1 [4]	oe oe using scalar products eg $-5\mathbf{i} + \mathbf{k}$ with \mathbf{i} oe or better (or 168.7°). Allow radians.

Question	Answer	Marks	Guidance
7 (ii)	$\overline{AE} = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}, \overline{ED} = \begin{pmatrix} 5 \\ 0 \\ -1 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -4 \\ 5 \end{pmatrix} = 1 - 16 + 15 = 0$ $\begin{pmatrix} 5 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -4 \\ 5 \end{pmatrix} = 5 + 0 - 5 = 0$ <p>$\Rightarrow \mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$ is normal to AED</p> <p>\Rightarrow eqn of AED is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -4 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ -4 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -4 \\ 5 \end{pmatrix}$</p> <p>$\Rightarrow x - 4y + 5z = 16$ B lies in plane if $8 - 4(-a) + 5 \cdot 0 = 16$ $\Rightarrow a = 2$</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[7]</p>	<p>two relevant direction vectors (or $6\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$ oe)</p> <p>scalar product with a direction vector in the plane (including evaluation and $= 0$) (OR M1 forms vector cross product with at least two correct terms in solution)</p> <p>scalar product with second direction vector, with evaluation. (following OR above, A1 all correct ie a multiple of $\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$) (NB finding only one direction vector and its scalar product is B1 only.)</p> <p>for $x - 4y + 5z = c$ oe</p> <p>M1A0 for $\mathbf{i} - 4\mathbf{j} + 5\mathbf{k} = 16$ allow M1 for subst in their plane or if found from say scalar product of normal with vector EB can also get M1A1 For first five marks above SC1, if states, ‘if $\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$ is normal then of form $x - 4y + 5z = c$’ and substitutes one coordinate gets M1A1, then substitutes other two coordinates A2 (not A1,A1). Then states so $\begin{pmatrix} 1 \\ -4 \\ 5 \end{pmatrix}$ is normal can get B1 provided that there is a clear argument ie M1A1A2B1. Without a clear argument this is B0. SC2, if finds two relevant vectors, B1 and then finds equation of the plane from vector form, $r = a + \mu b + \lambda c$ gets B1. Eliminating parameters B1cao. If then states ‘so $\begin{pmatrix} 1 \\ -4 \\ 5 \end{pmatrix}$ is normal’ can get B1 (4/5).</p>

Question	Answer	Marks	Guidance
7 (iii)	D: $6 + 2 = 8$ B: $8 + 0 = 8$ C: $8 + 0 = 8$ \Rightarrow plane BCD is $x + z = 8$ Angle between $\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$ and $\mathbf{i} + \mathbf{k}$ is θ $\Rightarrow \cos \theta = (1 \times 1 + (-4) \times 0 + 5 \times 1) / \sqrt{42} \sqrt{2} = 6 / \sqrt{84}$ $\Rightarrow \theta = 49.1^\circ$	B2,1,0 M1 M1 A1 A1 [6]	or any valid method for finding $x + z = 8$ gets M1A1 between two correct relevant vectors complete method (including cosine) (for M1 ft their normal(s) to their plane(s)) allow correct substitution or $\pm 6 / \sqrt{84}$, correct only or 0.857 radians (or better) acute only
8 (i)	$h = 20$, stops growing	B1 [1]	AG need interpretation
(ii)	$h = 20 - 20e^{-t/10}$ $dh/dt = 2e^{-t/10}$ $20e^{-t/10} = 20 - 20(1 - e^{-t/10}) = 20 - h$ $= 10dh/dt$ when $t = 0$, $h = 20(1 - 1) = 0$ OR verifying by integration $\int \frac{dh}{20-h} = \int \frac{dt}{10}$ $\Rightarrow -\ln(20-h) = 0.1t + c$ $h = 0, t = 0, \Rightarrow c = -\ln 20$ $\Rightarrow \ln(20-h) = -0.1t + \ln 20$ $\Rightarrow \ln\left(\frac{20-h}{20}\right) = -0.1t$ $\Rightarrow 20-h = 20e^{-0.1t}$ $\Rightarrow h = 20(1 - e^{-0.1t})$	M1A1 M1 A1 B1 M1 A1 B1 M1 A1 [5]	differentiation (for M1 need $ke^{-t/10}$, k const) oe eg $20 - h = 20 - 20(1 - e^{-t/10}) = 20e^{-t/10}$ $= 10dh/dt$ (showing sides equivalent) initial conditions sep correctly and intending to integrate correct result (condone omission of c , although no further marks are possible) condone $\ln(h - 20)$ as part of the solution at this stage constant found from expression of correct form (at any stage) but B0 if say $c = \ln(-20)$ (found using $\ln(h - 20)$) combining logs and anti-logging (correct rules) correct form (do not award if B0 above)

Question	Answer	Marks	Guidance
8 (iii)	$\frac{200}{(20+h)(20-h)} = \frac{A}{20+h} + \frac{B}{20-h}$ $\Rightarrow 200 = A(20-h) + B(20+h)$ $h = 20 \Rightarrow 200 = 40B, B = 5$ $h = -20 \Rightarrow 200 = 40A, A = 5$ $200 \frac{dh}{dt} = 400 - h^2$ $\Rightarrow \int \frac{200}{400-h^2} dh = \int dt$ $\Rightarrow \int \left(\frac{5}{20+h} + \frac{5}{20-h} \right) dh = \int dt$ $\Rightarrow 5 \ln(20+h) - 5 \ln(20-h) = t + c$ <p>When $t = 0, h = 0 \Rightarrow 0 = 0 + c \Rightarrow c = 0$</p> $\Rightarrow 5 \ln \frac{20+h}{20-h} = t$ $\Rightarrow \frac{20+h}{20-h} = e^{t/5}$ $\Rightarrow 20+h = (20-h)e^{t/5} = 20e^{t/5} - he^{t/5}$ $\Rightarrow h + he^{t/5} = 20e^{t/5} - 20$ $\Rightarrow h(e^{t/5} + 1) = 20(e^{t/5} - 1)$ $\Rightarrow h = \frac{20(e^{t/5} - 1)}{e^{t/5} + 1}$ $\Rightarrow h = \frac{20(1 - e^{-t/5})}{1 + e^{-t/5}} *$	<p>M1 A1 A1</p> <p>M1</p> <p>A1 B1</p> <p>M1</p> <p>DM1</p> <p>A1</p> <p>[9]</p>	<p>cover up, substitution or equating coeffs</p> <p>separating variables and intending to integrate (condone sign error)</p> <p>substituting partial fractions</p> <p>fit their A, B, condone absence of c. Do not allow $\ln(h-20)$ for A1. cao need to show this. c can be found at any stage. NB $c = \ln(-1)$ (from $\ln(h-20)$) or similar scores B0.</p> <p>anti-logging an equation of the correct form. Allow if $c = 0$ clearly stated (provided that $c = 0$) even if B mark is not awarded, but do not allow if c omitted. Can fit their c.</p> <p>making h the subject, dependent on previous mark NB method marks can be in either order, in which case the dependence is the other way around. (In which case, $20+h$ is divided by $20-h$ first to isolate h).</p> <p>AG must have obtained B1 (for c) in order to obtain final A1.</p>
8 (iv)	As $t \rightarrow \infty, h \rightarrow 20$. So long-term height is 20m.	B1 [1]	www
8 (v)	1 st model $h = 20(1 - e^{-0.1}) = 1.90..$ 2 nd model $h = 20(e^{1/5} - 1)/(e^{1/5} + 1) = 1.99..$ so 2 nd model fits data better	B1 B1 B1 dep [3]	Or 1 st model $h = 2$ gives $t = 1.05..$ 2 nd model $h = 2$ gives $t = 1.003..$ dep previous B1s correct

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