OCR
RECOGNISING ACHIEVEMENT

Monday 28 January 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4758/01 Differential Equations

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4758/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR
- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The differential equation

$$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 6y = \sin x$$

is to be solved.

(i) Show that 2 is a root of the auxiliary equation. Find the other two roots and hence find the general solution of the differential equation. **[10]**

When $x = 0$, $y = 1$ and $\frac{dy}{dx} = 0$. Also, y is bounded as $x \to \infty$.

(ii) Find the particular solution. **[6]**

(iii) Write down an approximate solution for large positive values of x. Calculate the amplitude of this approximate solution and sketch the solution curve for large positive x. **[4]**

Suppose instead that a solution is required that is bounded as $x \to -\infty$.

(iv) Determine whether there is a solution for which $y = 1$ and $\frac{dy}{dx} = 0$ when $x = 0$. **[4]**

2 A ball of mass m kg falls vertically from rest through a liquid. At time t s, the velocity of the ball is $v\,\text{m s}^{-1}$ and the ball has fallen a distance x m. The forces on the ball are its weight and a total upwards force of R N. A student investigates three models for R.

In the first model $R = mkv$, where k is a positive constant.

(i) Show that $\frac{dv}{dt} = 9.8 - kv$ and hence find v in terms of t and k. **[7]**

The terminal velocity of the ball is observed to be $7\,\text{m s}^{-1}$.

(ii) Find k. **[1]**

In the second model, $R = 0.2mv^2$.

(iii) Find v in terms of t. Show that your solution is consistent with a terminal velocity of $7\,\text{m s}^{-1}$. **[10]**

In the third model, $R = 0.529mv^{\frac{3}{2}}$. Euler's method is to be used to solve for v numerically.

The algorithm is given by $t_{r+1} = t_r + h$, $v_{r+1} = v_r + h\dot{v}_r$ with $(t_0, v_0) = (0, 0)$.

(iv) Show that $\frac{dv}{dt} = 9.8 - 0.529v^{\frac{3}{2}}$ and find v when $t = 0.2$ using Euler's method with a step length of 0.1. **[5]**

(v) Show that this model is consistent with a terminal velocity of approximately $7\,\text{m s}^{-1}$. **[1]**

3 **(a)** Solve the differential equation

$$\frac{dy}{dx} - y\tan x = \sin x$$

to find y in terms of x subject to the condition $y = 1$ when $x = 0$. **[9]**

(b) Consider the differential equations

$$\frac{dy}{dx} + f(x)y = g(x), \qquad (1)$$

$$\frac{dy}{dx} + f(x)y = 0. \qquad (2)$$

Show that if $y = p(x)$ satisfies (1) and $y = c(x)$ satisfies (2), then $y = p(x) + Ac(x)$ satisfies (1), where A is an arbitrary constant. **[5]**

(c) The differential equation

$$\frac{dy}{dx} + \frac{2y}{x} = 2e^{x^2}\left(\frac{x^2+1}{x}\right) \qquad (3)$$

is to be solved.

(i) Verify that $y = e^{x^2}$ satisfies (3). **[3]**

(ii) Find the general solution of $\frac{dy}{dx} + \frac{2y}{x} = 0$, giving y in terms of x. **[4]**

(iii) Use the result of part **(b)** to find a solution of (3) for which $y = 1$ when $x = 1$. **[3]**

4 The simultaneous differential equations

$$\frac{dx}{dt} = -\frac{1}{2}x - \frac{3}{2}y + t$$

$$\frac{dy}{dt} = \frac{3}{2}x - \frac{1}{2}y + 2t$$

are to be solved.

(i) Eliminate y to obtain a second order differential equation for x in terms of t. Hence find the general solution for x. **[13]**

(ii) Find the corresponding general solution for y. **[4]**

When $t = 0$, $x = 1$ and $y = 0$.

(iii) Find the particular solutions. **[3]**

(iv) Show that in this case $x + y$ tends to a finite limit as $t \to \infty$ and state its value. Determine whether $x + y$ is equal to this limit for any values of t. **[4]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.

