A LEVEL
PHYSICS A

Checkpoint Task

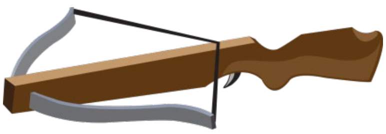
Mechanics

Instructions and answers for teachers

This Checkpoint Task should be used in conjunction with the KS5–HE Transition Guide – Mechanics.

Task 1 – Finding the range

(Convention that all downward acting vectors are negative)



Apparatus

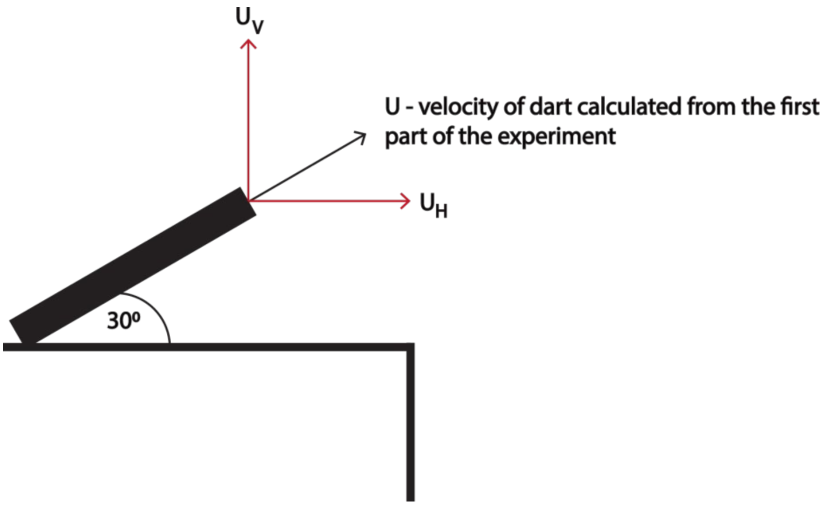
- toy crossbow and dart
- lead foil
- stop-clock
- 10m tape measure
- retort stand and clamps (x2)
- inclinometer
- cardboard box to act as target

The velocity of the dart shot by the crossbow can be found by firing the crossbow vertically and either measuring the vertical height the dart reaches (the dart can be weighted using lead foil wrapped around it so that it reaches a convenient height) or by timing the time of flight.

$v^2 = u^2 + 2as$ Since v = 0 at the maximum height, u can be calculated.

$v = u + at$ Again v = 0 and we half the measured time, so u can be calculated.

The crossbow is then angled at 30° to the horizontal, using lab stands and clamps, and a large flat cardboard box positioned according to the pupils' calculations. Don't forget the prizes!

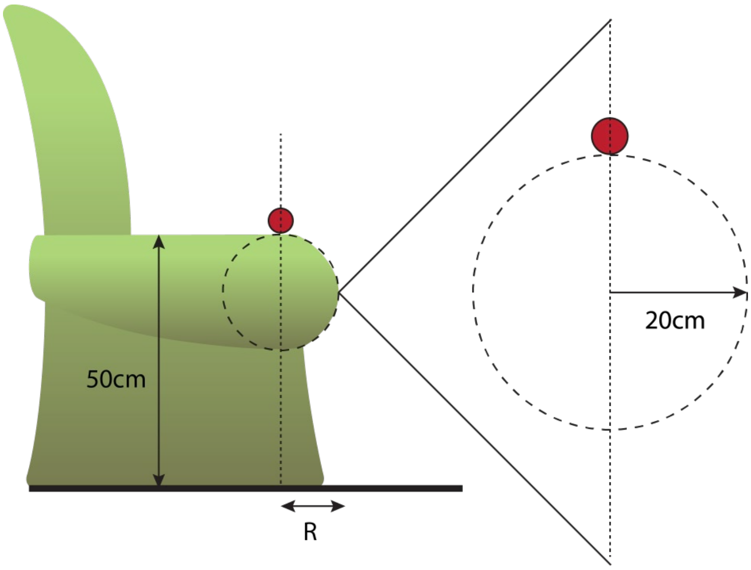


Use u_V to calculate the time of flight of the dart.

Use u_H to calculate the horizontal range.

Task 2 – The armchair problem

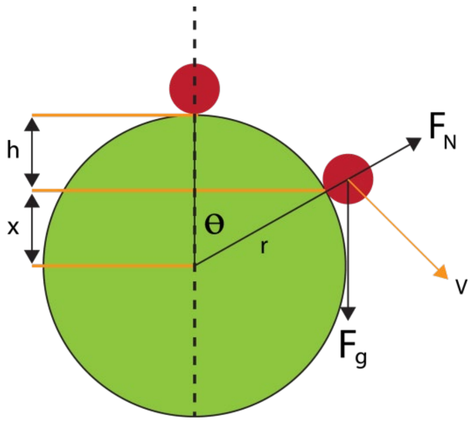
A person sitting in an armchair rolls a small ball off the chair as shown in the diagram below.



The part of the arm that the ball rolls down is circular in shape with a radius of 20cm and the ball is released from the position shown. If the ball has a mass of 25 grams calculate the horizontal distance from the release point that the ball will hit the floor.

Part 1
As long as the ball is in contact with the arm of the chair we can say three things:

1. there must be a centripetal force acting on the ball to move it in a circular path
2. there must be a normal force from the surface acting on the ball
3. the centre acting force must be formed from a combination of $\mathbf{F_g}$ and **N**



$$F_g = mg$$

$$mg\cos\theta = N$$

$$\therefore mg\ \cos\theta - N = \frac{mv^2}{r}$$

At the point shown in the diagram the ball is just in contact with the surface. It has moved through an angle of **θ** and fallen a vertical height '**h**'.

Distance $x = r\ cos\theta$

$$mgh = \frac{1}{2}mv^2$$

$$h = r - x$$

$$\therefore h = r - r\ \cos\theta$$

$$\therefore h = r(1 - \cos\theta)$$

$$\therefore mgr(1 - \cos\theta) = \frac{1}{2}mv^2$$

$$\therefore 2mgr(1 - \cos\theta) = mv^2$$

We can now combine the equations related to energy transfer and circular motion:

$$\frac{2mgr(1 - \cos\theta)}{r} - mg\ \cos\theta - N$$

$$2mgr(1 - \cos\theta) = mg\cos\theta - N$$

But when the ball leaves contact with the arm $N = 0$

$$\therefore 2mg(1 - \cos\theta) = mg\cos\theta$$

$$\therefore 2(1 - \cos\theta) = \cos\theta$$

$$\therefore 2 - 2\cos\theta = \cos\theta$$

$$\therefore 2 = 3\cos\theta$$

$$\therefore \cos\theta = \frac{2}{3}$$

$$\therefore \theta = 48.2°$$

The ball will leave contact with the arm at an angle of about 48°. This angle is irrespective of the mass of the ball or the radius of the arm.

BUT – HAVE WE MISSED ANYTHING OUT?

The solution above assumes that the ball is sliding. It will probably roll. As such:

Loss in gpe = Gain in linear KE + Gain in rotational KE

$\therefore mgh = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$

For a solid sphere $I = \frac{2}{5} mr^2$

$\therefore mgh = \frac{7}{10} mv^2$

Since $mgh = mgr(1 - \cos\theta)$

$\therefore mgr(1 - \cos\theta) = \frac{7}{10} mv^2$

$\therefore \frac{10}{7} mgr(1 - \cos\theta) = mv^2$

For the rolling ball moving in a circular path:

$\frac{mv^2}{r} = mg\cos\theta - F_N$

But when the ball leaves the surface $F_N = 0$

$\therefore \frac{mv^2}{r} = mg\cos\theta$

$\therefore mv^2 = mgr\cos\theta$

$\therefore \frac{10}{7} mgr(1 - \cos\theta) = mgr\cos\theta$

$\therefore \cos\theta = \frac{10}{17}$

$\therefore \theta = 54°$

Part 2

We can assume the direction of the ball is tangential to the surface of the arm as it leaves contact, which means that this direction must be at right-angle to the radius ‘r’. The assumption is that the ball is treated as a particle.

We can now calculate the vertical component of the velocity of the ball as it leaves contact with the arm and as such calculate the time taken to fall vertically to the floor.

From the time of flight and the horizontal component of velocity of the ball we can calculate the horizontal range.