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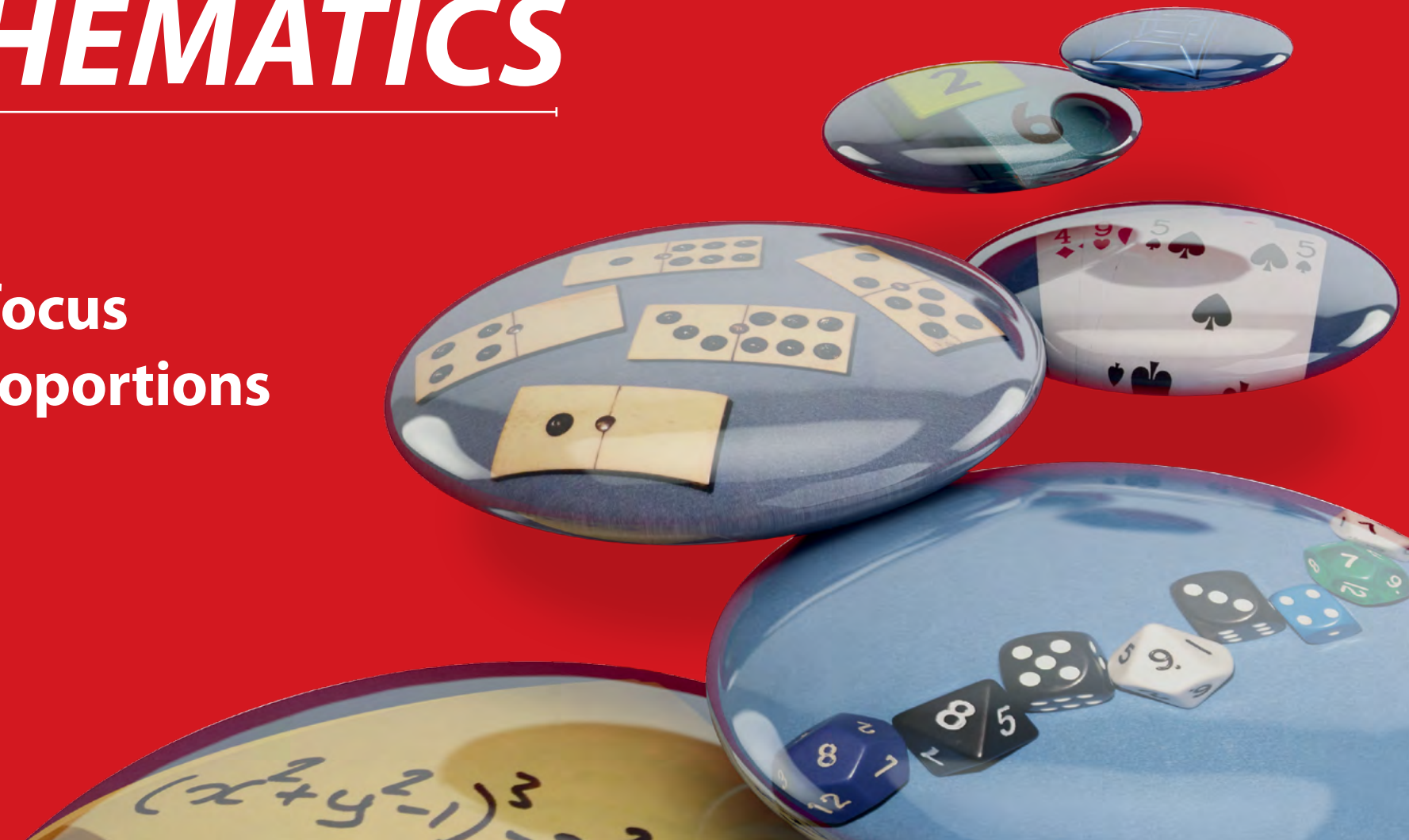
GCSE (9–1)
Transition Guide

MATHEMATICS

J560
For first teaching in 2015

KS3–KS4 focus
Theme: Proportions

Version 2



www.ocr.org.uk/maths

GCSE (9–1) MATHEMATICS

Key Stage 3 to 4 Transition Guides focus on how a particular topic is covered at the different key stages and provide information on:

- Differences in the demand and approach at the different levels;
- Useful ways to think about the content at Key Stage 3 which will help prepare students for progression to Key Stage 4;
- Common student misconceptions in this topic.

Transition guides also contain links to a range of teaching activities that can be used to deliver the content at Key Stage 3 and 4 and are designed to be of use to teachers of both key stages. Central to the transition guide is a Checkpoint task, which is specifically designed to help teachers determine whether students have developed deep conceptual understanding of the topic at Key Stage 3 and assess their 'readiness for progression' to Key Stage 4 content on this topic. This checkpoint task can be used as a summative assessment at the end of Key Stage 3 teaching of the topic or by Key Stage 4 teachers to establish their students' conceptual starting point.

Key Stage 3 to 4 Transition Guides are written by experts with experience of teaching at both key stages.

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Key Stage 3 Content

Key Stage 3 National Curriculum Content*

- solve problems involving direct and inverse proportion, including graphical and algebraic representations
- use compound units such as speed, unit pricing and density to solve problems

* https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/239058/SECONDARY_national_curriculum_-_Mathematics.pdf

Key Stage 4 Content

GCSE (9–1) content Ref.	Subject content	Initial learning for this qualification will enable learners to…	Foundation tier learners should also be able to…	Higher tier learners should additionally be able to…	DfE Ref.
5.02	**Direct and inverse proportion**				
5.02a	Direct proportion	Solve simple problems involving quantities in direct proportion including algebraic proportions. e.g. Using equality of ratios, if $y \propto x$, then $\frac{y_1}{y_2} = \frac{x_1}{x_2}$ or $\frac{y_1}{x_1} = \frac{y_2}{x_2}$. Currency conversion problems. *[see also Similar shapes, 9.04c]*	Solve more formal problems involving quantities in direct proportion (i.e. where $y \propto x$). Recognise that if $y = kx$, where k is a constant, then y is proportional to x.	Formulate equations and solve problems involving a quantity in direct proportion to a power or root of another quantity.	R7, R10, R13

Key Stage 4 Content contd

GCSE (9–1) content Ref.	Subject content	Initial learning for this qualification will enable learners to…	Foundation tier learners should also be able to…	Higher tier learners should additionally be able to…	DfE Ref.
5.02	**Direct and inverse proportion**				
5.02b	Inverse proportion	Solve simple word problems involving quantities in inverse proportion or simple algebraic proportions. e.g. speed–time contexts (if speed is doubled, time is halved).	Solve more formal problems involving quantities in inverse proportion (i.e. where $y \propto \frac{1}{x}$). Recognise that if $y = \frac{k}{x}$, where k is a constant, then y is inversely proportional to x.	Formulate equations and solve problems involving a quantity in inverse proportion to a power or root of another quantity.	R10, R13

Comment

Difference between the level of demand and KS3 and KS4

The essential difference between Key Stage 3 and Key Stage 4 is the move from proportion of quantities to algebraic representations of proportion.

At Key Stage 3 many areas of the curriculum are connected by proportion. At this level proportion is introduced through fractions, ratios, in numerical calculations, through pie charts as a visual presentation and in simple word problems, such as converting a recipe from "serving 4" to "serving 6".

Students at this level should understand the equivalence of the proportionality statements of the form $\frac{a}{b} = \frac{c}{d}$, $\frac{a}{c} = \frac{b}{d}$, a : b = c : d and a : c = b : d. The fact that knowing three of the four quantities is sufficient to find the fourth is a simple way to introduce *algebra* to find an *unknown* quantity. This is easily extended to finding unknown lengths in similar shapes.

The ideas and concepts can be extended at Key Stage 3 by considering, for example, the gradient of straight line graphs for *quantities* in direct proportion and by encouraging investigation of the link between the gradient of such a line and the proportional relationship between the two *quantities*. Simple conversion graphs could be used for this purpose and the unitary method of finding the gradient would work well here.

Students will experience working with, and should be able to identify, proportional relationships in formulae they are given or have derived from simple practical word problems. Using the relationships between units in measures is an excellent way to introduce inverse proportion, for example comparing the relationship between distance and time for constant speed with that for speed and time for constant distance.

At Key Stage 4, as well as being expected to handle more demanding problems concerning proportionally related *quantities*, students will be expected to understand and interpret relationships between *variables*. These relationships may involve simple powers or roots of the independent *variable*. The language used in such questions will be more rigorous. Higher-achieving students will be expected to form equations describing proportional relationships, and to use and apply these in solving problems. This application to problem solving may involve sketching or drawing graphs of the equations formed.

The understanding of the gradient of a straight line as a rate of change in algebraic format should be considered at this stage. For higher-achieving students this may lead into work on interpreting the gradient at a point on a curve as the instantaneous rate of change.

Pre-requisite knowledge for KS4

By the end of Key Stage 3, more able students should have a good understanding of proportional relationships, having experienced them in many different contexts and content areas. In preparation for Key Stage 4 these students may need brief revision of equivalent fractions, simple ratio and the difference between direct and inverse proportion in practical scenarios. Study of the general equation of a straight line and sketching graphs of simple quadratic and cubic functions as well as the graphs of reciprocal (and possibly square root) functions would be very beneficial, if not already done. Dynamic graphing software could be a useful tool here. Direct variation could be approached by considering pairs of coordinates such as (2, 4), (3, 6), (4, 8), (5, 10) and determining that $\frac{y}{x}$ is constant. This could then lead to a derivation of the general relationship $y = kx$.

Less able students may need to spend more time revisiting equivalent fractions and simple ratios. They may need to revise the difference between direct and inverse proportion between quantities in practical scenarios before being introduced to algebraic representations.

Common misconceptions

A common false generalisation is that direct proportion means "as one quantity increases so does the other". This may cause confusion between a decreasing rate of change and inverse proportion. Students should be aware that the constant of proportionality could be negative and it is better to describe a relationship as being directly proportional if the corresponding quantities, when divided, produce the same value (the constant of proportionality).

Another common misconception is that all straight lines represent direct proportion. Students should have this clarified when working with the equation of a straight line, i.e. that while $y = 2x + 3$ has constant gradient, y and x are not in direct proportion.

Activities

STEM Learning: Department for Education – Developing Proportional Reasoning N6
https://www.stem.org.uk/elibrary/resource/26924

In this resource from the DfE Standards Unit (2000 – 2009), students reflect on the reasoning they currently use when solving proportion problems, examine proportion problems, appreciate their multiplicative structure and create their own variants of proportion problems. A wealth of extra supporting materials, can be found on the STEM Learning website: https://www.stem.org.uk/resources/search?=Search&resource_query=ratio+and+proportion

Centre for Innovation in Maths Teaching, Plymouth: Direct Proportion
http://www.cimt.org.uk/projects/mepres/book8/bk8i7/bk8_7i2.htm

A great quiz-style webpage with some practice questions. Each question has interactive worked answers and then a quiz to try, with a "How you did" grid at the bottom.

GeoGebraTube: Proportions in similar triangles
https://www.geogebra.org/m/Dms6d3DJ

This dynamic geometry resource allows students to investigate different pairs of similar triangles to see how they are connected and acts as a good visual reinforcement.

GeoGebraTube: Missing lengths in similar triangles
https://www.geogebra.org/m/jSuh3X9r

This would work as a good follow up to the previous activity. It uses proportionality to find unknowns in similar triangles and is another good interactive activity with excellent visual reinforcement.

Checkpoint Task

The checkpoint task is a series of 4 activities, graded in difficulty, that aim to form a basis for formative assessment at the end of Key Stage 3, at the start of or during Key Stage 4. The activities cover different areas of the curriculum and demonstrate many of the various contexts that can be used to assess proportion.

Activity 1 (Basic)	**Supersaver Sid**	**(Number/Measure)**
Activity 2 (Explore)	**What's the Risk?**	**(Number/Probability)**
Activity 3 (Challenge)	**Finding Factors**	**(Geometry/Algebra)**
Activity 4 (Extend)	**Keep it in Proportion**	**(Algebra/Measure)**

Allow students time to understand and engage with each activity and to spend time reasoning with the problem. If they ask for support, after some thinking time, directive questioning can be used along the lines suggested in the Teacher Notes.

Formative assessment opportunities will arise through observation and questioning as each activity progresses. Once an activity is complete, if students have been working in groups, each could present their findings and self/peer assessment could be carried out. Alternatively, students' efforts could be teacher marked or peer marked.

Each assessment could be cloned and adapted so that different groups had slightly different or differentiated activities. Ideas for adapting each activity are given in the Teacher Notes.

Checkpoint Task:

http://www.ocr.org.uk/Images/170304-proportions-checkpoint-task.doc

Activities

STEM Learning: The Royal Institution, Calculating Colours
https://www.stem.org.uk/elibrary/resource/31677

This practical activity provides students with an understanding of the different ways in which proportion can be expressed. Through a mixture of explanation and practical work, students explore how it is possible to give an accurate quantitative description of colours.

Students mix varying quantities of red and blue water to create a range of different shades of purple, which are then described in terms of fractional and percentage composition. Through this activity students experience a functional use of ordering equivalent fractions.

The resource includes guidance for teachers, an overview of the activities, a presentation and activity sheet.

TES Teaching Resources: Direct/Inverse Proportion
http://www.tes.co.uk/ResourceDetail.aspx?storyCode=6386769

A PowerPoint and complementary worksheet covering graphical and algebraic content for more able students.

TES Teaching Resources: Direct/Inverse Proportion
http://www.tes.co.uk/teaching-resource/Direct-and-Inverse-Proportionality-6400608/

Four sets of differentiated questions on direct and inverse proportion with answers. An excellent resource for either group working/presentation or individual study.

Activities

Tyler Wallace (Creative Commons) Faculty:
Rational Expressions - Proportions
http://www.wallace.ccfaculty.org/book/7.6%20Proportions.pdf

This links to a pdf that extends the use of proportion to solving linear and quadratic equations. It then extends this further to forming the equations prior to solving.

There's a wealth of material here that can be adapted to suit a variety of needs.

STEM Learning: MEI – Simple Gears and Transmission
https://www.stem.org.uk/elibrary/resource/31823

This engineering resource, produced by Mathematics in Education and Industry for the Royal Academy of Engineering, asks the question: how are transmissions designed so that they provide the force, speed and direction required and how efficient is the design?

The mathematics covered in this activity is:

- evaluate expressions
- work with fractions
- solve problems involving ratio and proportion
- understand and work with percentages
- use scale drawings
- simplify and evaluate expressions involving the use of indices
- change the subject of the formula
- solve problems involving angular motion, converting between units for the speed of revolution.

Resources, Links and Support

Total Maths – Our termly update Total Maths is designed to keep you up-to-date with Mathematics here at OCR, as well as to share information, news and resources from the Maths community that you might find useful: http://www.ocr.org.uk/qualifications/by-subject/mathematics/total-maths/

Find resources and qualification information through our Maths page: www.ocr.org.uk/maths

Contact the team: maths@ocr.org.uk

Continue the discussion on the maths community forum: http://social.ocr.org.uk/

and follow us on Twitter, @OCR_Maths

To find out more about GCSE and A Level reform please visit: http://www.ocr.org.uk/qualifications/gcse-and-a-level-reform

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