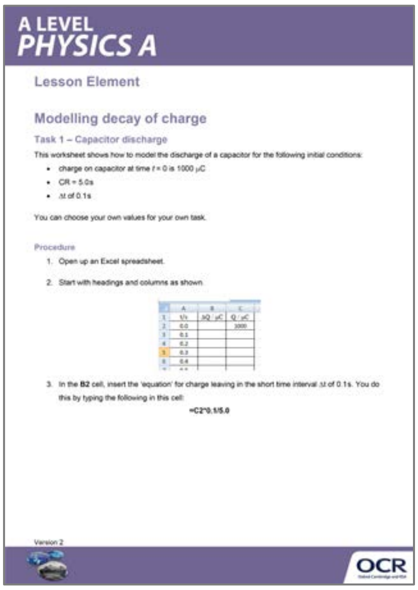
A LEVEL PHYSICS A

Lesson Element

Modelling decay of charge

Instructions and answers for teachers

These instructions should accompany the OCR resource 'Modelling decay of charge' activity *which supports OCR A Level Physics A.*


A LEVEL PHYSICS A

Lesson Element

Modelling decay of charge

Task 1 – Capacitor discharge

This worksheet shows how to model the discharge of a capacitor for the following initial conditions:

- charge on capacitor at time t = 0 is 1000 μC
- CR = 5.0s
- Δt of 0.1s

You can choose your own values for your own task.

Procedure

1. Open up an Excel spreadsheet.
2. Start with headings and columns as shown.
3. In the **B2** cell, insert the 'equation' for charge leaving in the short time interval Δt of 0.1s. You do this by typing the following in this cell:

=C2*0.1/5.0

Version 2

The Activity:

This resource comprises of 1 task

The simple, small-scale practical activity described here provides students with the opportunity to explore the precipitation reactions between solutions of halide salts and silver nitrate before going on to investigate the rates of hydrolysis of 1-chlorobutane, 1-bromobutane and 1-iodobutane.

This activity offers an opportunity for English skills development.

This activity offers an opportunity for maths skills development.

Associated materials:

'Modelling decay of charge' Lesson Element learner activity sheet.

Introduction

In the revised A level Physics A specification, there is a new learning outcome **6.1.3(d)** that requires students to model the discharge of a capacitor in a C-R circuit using a spreadsheet.

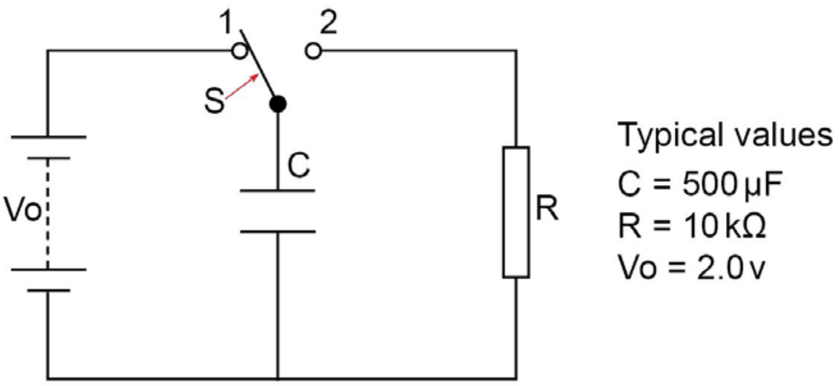
This lesson element provides a teacher with all the necessary resources and strategies to communicate the key ideas inherent in this learning outcome.

6.1.3(d) graphical methods and spreadsheet modelling of the equation $\frac{\Delta Q}{\Delta t} = -\frac{Q}{CR}$ for a discharging capacitor.

Background information

It is important to understand the origin of the equation $\frac{\Delta Q}{\Delta t} = -\frac{Q}{CR}$ given in the learning outcome.

Figure 1 shows the basic circuit for investigating the discharge of a capacitor through a resistor.



The capacitor has capacitance *C* and the resistor has resistance *R.* The capacitor is charged by connecting the switch **S** to position **1**. When **S** is moved to position **2**, the capacitor starts to discharge through the resistor. The charge stored on the capacitor and the potential difference (p.d.) across it both begins to fall.

It is worth noting that the capacitor and the resistor are both in **parallel**, hence the p.d. V across the capacitor is the same as the p.d. across the resistor. At any instant, the charge Q on the capacitor and the current I in the circuit are given by the equations:

$Q = \frac{V}{C}$ and $I = \frac{V}{R}$

Current is the rate of flow of charge from the capacitor. Therefore

$$I = -\frac{\Delta Q}{\Delta t}.$$

The negative sign is necessary because it shows that the charge on the capacitor decreases as time increases.

The three equations above can now be combined as follows:

$$I = \frac{V}{R} = \frac{Q}{CR}$$

Hence

$$\frac{\Delta Q}{\Delta t} = -\frac{Q}{CR}.$$

The product *CR* in this equation is known as the time constant of the circuit. How quickly or slowly a capacitor discharges depends on the time constant.

The **Resource A** summarises the key ideas above. It may be used to show your students how the equation $\frac{\Delta Q}{\Delta t} = -\frac{Q}{CR}$ comes about.

The basics of the task

The equation $\frac{\Delta Q}{\Delta t} = -\frac{Q}{CR}$ can be used be determine the variation of charge *Q* on the capacitor with time *t.* The technique outlined here is known iterative modelling.

- Start with a known initial value for the charge on the capacitor and the time constant *CR* of the circuit.
- Choose a time interval $\Delta t \ll CR$. In doing so, the predictions made by the modelling process will match closely to the experimental results.
- Calculate the charge leaving the capacitor in the time interval Δt using

  $$\Delta Q = \frac{Q}{CR} \times \Delta t.$$
- Determine the charge left on the capacitor at the end of the Δt period.
- Repeat the whole process for the next interval of time.

This process is quite laborious to do by hand. This is where a spreadsheet can take over. The table of results can also be used to effortlessly plot Q-t graphs for different initial conditions.

Instructions for the task

Once you have used **Resources A** and **B**, your students should almost be ready to carry out some investigative tasks. Depending on your class size, students can either work individually, in pairs or in small groups on any of the tasks below.

Each student should be given **Worksheet 1**; which has all the necessary instructions to carry out this activity. No details are given on how to the plot Q-t graph. The guidance for this can come either from the teacher or an expert student in the class. Here are some possible ‘investigations’ that can be carried out by the students.

Investigate the discharge of a capacitor for a total time of about 10.0 s

- with initial charge = 1000 μC, *CR* = 5.0 s and Δt = 0.1 s
- with initial charge = 2000 μC, *CR* = 5.0 s and Δt = 0.1 s
- with initial charge = 1000 μC, *CR* = 10.0 s and Δt = 0.1 s
- with initial charge = 1000 μC, *CR* = 2.5 s and Δt = 0.1 s
- with initial charge = 1000 μC, *CR* = 5.0 s from *t* = 0 to *t* = 6.0 s and *CR* = 1.0 s for *t* > 6.0 s and Δt of 0.1 s

The students can either report their findings to the class or the teacher can summarise the key ideas through discussion. The important conclusions are:

1. The initial charge on the capacitor does not affect its rate of discharge.
2. After a time equal to CR, the charge on the capacitor is e^{-1} or about 36.8% of its initial charge.
3. The charge on the capacitor can also be found directly using $Q = Q_0 e^{-\frac{t}{CR}}$.
4. The Q-t graphs show exponential decay.

Some typical results for the teacher

The sample results below show how they can be the catalyst to summarise the key ideas.

1 Table of results when initial charge = 1000 μC, *CR* = 5.0 s and Δ*t* = 0.1 s

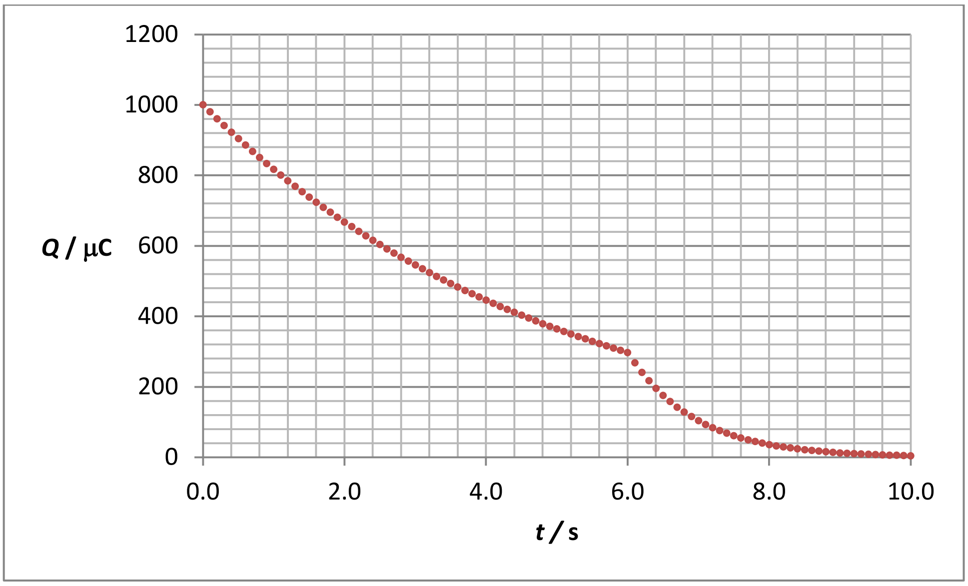
t/s	ΔQ / μC	Q / μC
0.0	20	1000
0.1	19.6	980.0
0.2	19.2	960.4
0.3	18.8	941.2
0.4	18.4	922.4
0.5	18.1	903.9
0.6	17.7	885.8
0.7	17.4	868.1
0.8	17.0	850.8
0.9	16.7	833.7
1.0	16.3	817.1
1.1	16.0	800.7
1.2	15.7	784.7
1.3	15.4	769.0
1.4	15.1	753.6
1.5	14.8	738.6
1.6	14.5	723.8
etc...		

The table can be used to show the constant-ratio property associated with exponential decay, learning outcome **6.1.3(e)**. For a time interval of 0.4 s, the ratios of the charges are the same.

$$\frac{922.4}{1000} \approx \frac{850.8}{922.4} \approx \frac{784.7}{850.8} \approx 0.922$$

You can try another time interval; the conclusion will still be the same.

2 Graph when initial charge = 1000 μC, CR = 5.0 s from t = 0 to t = 6.0 s and CR = 1.0 s for t > 6.0 s.



After a time equal to the time constant of 5.0 s, the charge $Q \approx 364$ μC according to the table of results. This is very close to the expected value of 368 μC. The difference is simply due to the fact that the time interval was not small enough.

The sudden change in the gradient of the graph at t = 6.0 s is due to the change in the time constant of the circuit from 5.0 s to 1.0 s.

Task 1 – Capacitor discharge

This worksheet shows how to model the discharge of a capacitor for the following initial conditions:

- charge on capacitor at time t = 0 is 1000 μC
- CR = 5.0 s
- Δt of 0.1 s

You can choose your own values for your own task.

Procedure

1. Open up an Excel spreadsheet.

2. Start with headings and columns as shown.

	A	B	C
1	t/s	ΔQ / μC	Q / μC
2	0.0		1000
3	0.1		
4	0.2		
5	0.3		
6	0.4		
7	0.5		

3. In the **B2** cell, insert the ‘equation’ for charge leaving in the short time interval Δt of 0.1 s. You do this by typing the following in this cell:

   =C2*0.1/5.0

   This will give the following results:

	A	B	C
1	t/s	ΔQ / μC	Q / μC
2	0.0	20	1000
3	0.1		
4	0.2		
5	0.3		
6	0.4		
7	0.5		

4. Now in the **C3** cell, insert the charge left on the capacitor after an interval of 0.1 s. You do this by typing the following in this cell:

=C2-B2

This will give the following results:

	A	B	C
1	t/s	ΔQ / μC	Q / μC
2	0.0	20	1000
3	0.1		980
4	0.2		
5	0.3		
6	0.4		
7	0.5		

5. Just two more steps now and you will have all the values you require for your task. Click on the **B2** cell and then drag the bottom right hand side corner all the way down the B column. This will copy the equation in all the B cells. Carry out the same procedure with the **C3** cell and the C column.

   If all goes well, you will end up with the following:

	A	B	C
1	t/s	ΔQ / μC	Q / μC
2	0.0	20	1000
3	0.1	19.6	980.0
4	0.2	19.2	960.4
5	0.3	18.8	941.2
6	0.4	18.4	922.4
7	0.5	18.1	903.9

6. Use your table to plot a graph of Q against t.

7. If possible, try different values for the time constant and observe what happens to the results in the table and to the Q-t graph. (See the list below of the things you could try.)

8. The charge on the capacitor decays exponentially. How can you use the table of results or your graph to confirm this?

9. The charge Q on the capacitor after a time *t* is given by the equation $Q = Q_0 e^{-\frac{t}{CR}}$, where Q_0 is the initial charge and CR is the time constant. Do you results agree with this equation?

Things to try:

Investigate the discharge of a capacitor for a total time of about 10.0 s

- with initial charge = 1000 μC, CR = 5.0 s and Δt = 0.1 s
- with initial charge = 2000 μC, CR = 5.0 s and Δt = 0.1 s
- with initial charge = 1000 μC, CR = 10.0 s and Δt = 0.1 s
- with initial charge = 1000 μC, CR = 2.5 s and Δt = 0.1 s
- with initial charge = 1000 μC, CR = 5.0 s from t = 0 to t = 6.0 s and CR = 1.0 s for t > 6.0 s and Δt of 0.1 s