OCR
RECOGNISING ACHIEVEMENT

Friday 17 May 2013 – Morning

LEVEL 3 CERTIFICATE MATHEMATICS FOR ENGINEERING

H860/01 Paper 1

Candidates answer on the Answer Booklet.

OCR supplied materials:
- 12 page Answer Booklet (OCR12) (sent with general stationery)
- List of Formulae (MF1)
- Graph paper

Other materials required:
- Scientific or graphical calculator

Duration: 2 hours

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INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the Answer Booklet. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a scientific or graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **60**.
- This document consists of **8** pages. Any blank pages are indicated.

1 Tests are carried out on 150 steel cables to determine their breaking strength. Each cable is pulled by an increasing force until it breaks. The force at the time of breaking is recorded for each test and the results are summarised in the cumulative frequency polygon shown in Fig. 1.

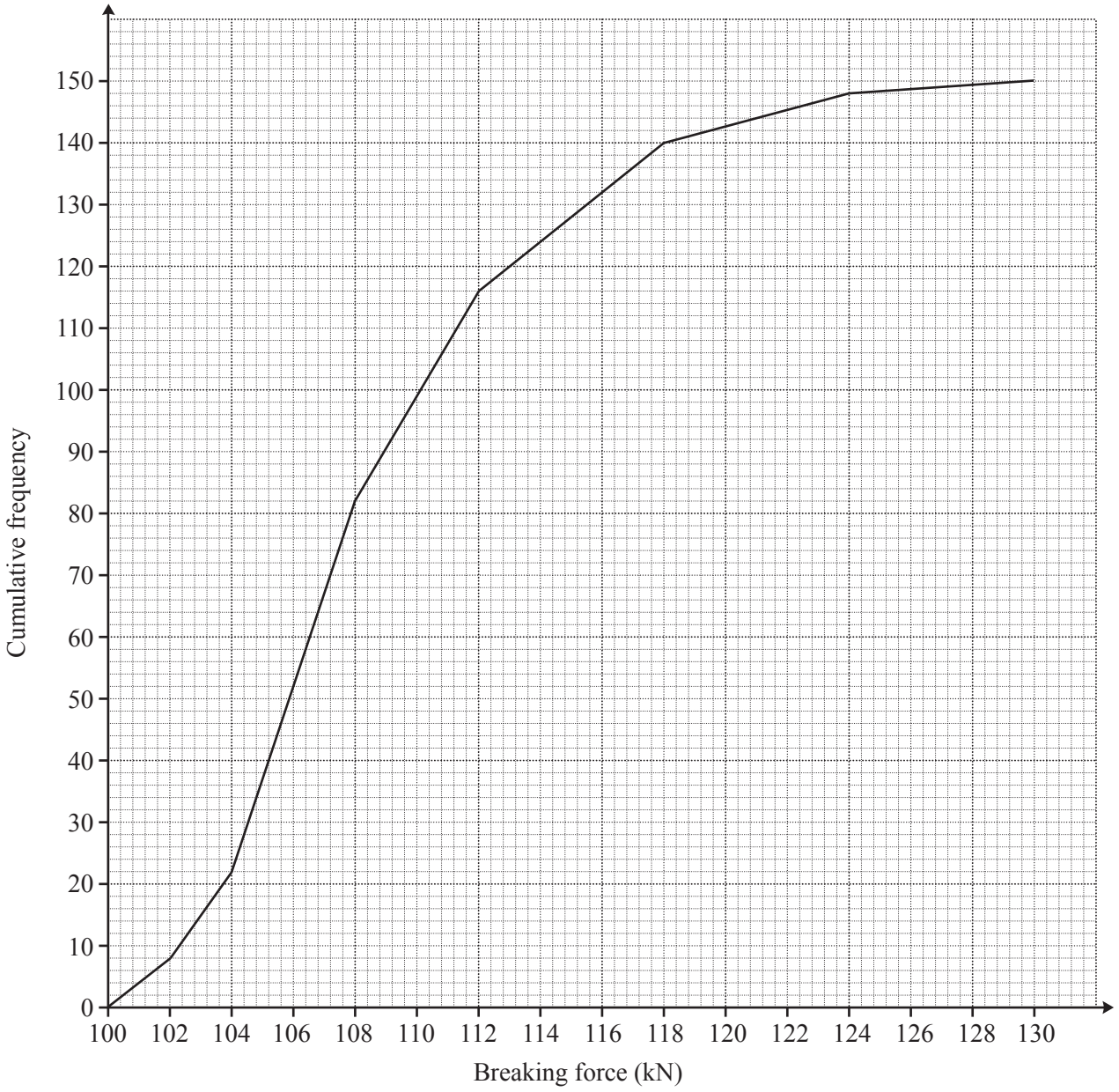

Fig. 1

(a) Copy the table below in your answer book and, by reading approximate values from Fig. 1, complete all the entries. The first two rows have been done for you.

Breaking force, F kN	Mid-class value	Frequency
$100 < F \leqslant 102$	101	8
$102 < F \leqslant 104$	103	14
$104 < F \leqslant 108$		
$108 < F \leqslant 112$		
$112 < F \leqslant 118$		
$118 < F \leqslant 124$		
$124 < F \leqslant 130$		

[3]

(b) Draw a histogram of the data using the class intervals given in the table in part **(a)**. **[4]**

(c) Use the results of part **(a)** to calculate an estimate of

(i) the mean force required to break the cable, **[2]**

(ii) the standard deviation of the force required to break the cable. **[3]**

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2 **(a)** Fig. 2ai and Fig. 2aii show circuit diagrams each containing three resistors with resistances $R_1\,\Omega$, $R_2\,\Omega$ and $R_3\,\Omega$. The value of R_1 is the same in both circuits and the value of R_2 is the same in both circuits. In both circuits $R_3 = 3$.

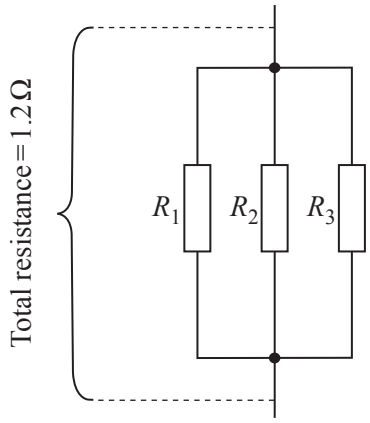

Fig. 2ai

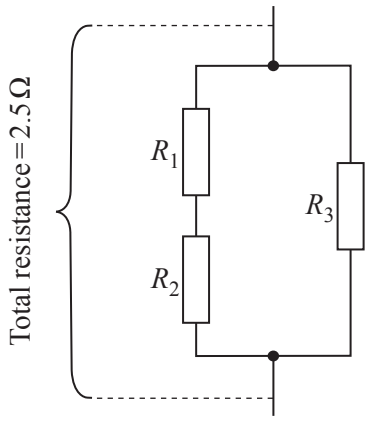

Fig. 2aii

The total resistance of the circuit shown in Fig. 2ai is $1.2\,\Omega$ and the total resistance of the circuit shown in Fig. 2aii is $2.5\,\Omega$.

Express the total resistance of each circuit in terms of R_1 and R_2 and hence calculate the values of R_1 and R_2 so that $R_1 < R_2$. **[6]**

(b) Fig. 2b shows a circuit containing four resistors R_1, R_2, R_3 and R_4 connected to a 10 V DC supply. $R_1 = 1\,\Omega$, $R_2 = 2\,\Omega$, $R_3 = 2\,\Omega$ and $R_4 = 12\,\Omega$.

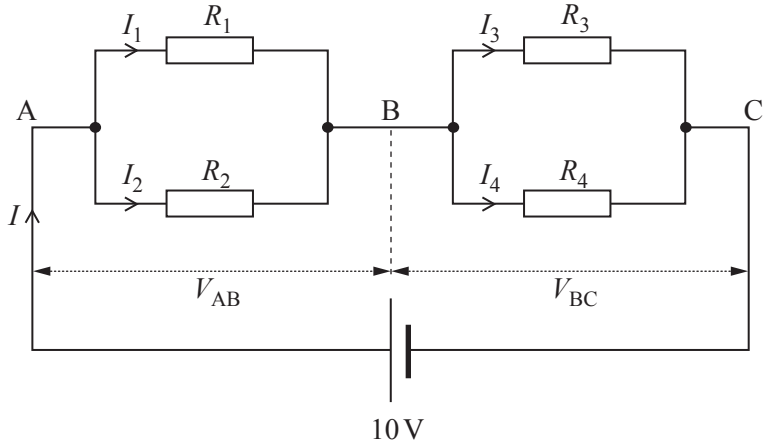

Fig. 2b

(i) Calculate the total current, I, flowing through the circuit. **[3]**

(ii) Calculate the values of the voltages V_{AB} and V_{BC} shown in the diagram. **[2]**

(iii) Calculate the current flowing through each resistor. **[3]**

3 A chemical engineering company produces three types of liquid insecticide (types X, Y, and Z). These are mixed from three basic ingredient chemicals, A, B and C. The amounts of chemical required to produce one litre of each type of insecticide are shown in Table 3a.

Insecticide type	Amount of chemical required (litres)		
	Chemical A	Chemical B	Chemical C
X	0.5	0.4	0.1
Y	0.2	0.4	0.4
Z	0.7	0.3	0

Table 3a

The number of litres of chemical available each week and their cost per litre are shown in Table 3b.

	Weekly availability (litres)	Cost per litre (£)
Chemical A	2500	0.50
Chemical B	2000	0.50
Chemical C	1000	0.50

Table 3b

The selling price and the production costs of each type of insecticide are shown in Table 3c.

Insecticide type	Selling price per litre (£)	*Production cost per litre (£)
X	1.40	0.20
Y	1.20	0.20
Z	1.30	0.20

Table 3c

(*The production costs are in addition to the ingredient costs.)

The variables x, y and z are the number of litres of insecticide types X, Y and Z respectively which are produced each week.

(a) Assuming that all the insecticide produced is sold show that the weekly profit is given by

$$0.7x + 0.5y + 0.6z.$$ **[2]**

(b) Formulate three linear constraints in terms of x, y and z that have to be satisfied simultaneously which will place limits on the maximum weekly production of each type of insecticide. **[3]**

(c) It has been decided that exactly 1000 litres of type Z insecticide is to be produced each week. The profit is to be maximised.

(i) Formulate the problem as a linear program which includes an objective function to be maximised and a set of constraints, all involving variables x and y only. **[3]**

(ii) Represent the constraints formulated in part **(c) (i)** on a graph.
On your graph shade the feasible region. **[4]**

(iii) Determine how many litres of each type of insecticide should be produced each week in order to maximise the profit. **[2]**

[Questions 4 and 5 are printed overleaf.]

4 A quantity, N, is said to be subject to exponential decay if its decay rate is proportional to its magnitude. This relationship is described by the differential equation

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\lambda N$$

where λ is the decay constant,
t is time.

(a) Given $N = N_0$ when $t = 0$, solve the differential equation and show that

$$N = N_0 \mathrm{e}^{-\lambda t}.$$ **[3]**

(b) The half-life, $t_{\frac{1}{2}}$, of a quantity that is subject to exponential decay is the time required for the quantity to be reduced to half of its initial value.
Show that

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}.$$ **[2]**

(c) When an organism dies the amount of carbon-14 within it is subject to exponential decay. It is known that the half-life of carbon-14 is 5730 years.

A particular sample of animal bone, discovered at an archaeological site, is examined and found to have only 35% of its original quantity of carbon-14.
Estimate the age of the bone. **[4]**

5 A strong-motion accelerometer has recorded a predominant east-to-west motion of the ground during a particular seismic event. Treating east-to-west movement as positive and west-to-east movement as negative, the acceleration, $a\,\mathrm{m\,s^{-2}}$, of the ground is modelled by the equation

$$a = g\mathrm{e}^{-\frac{1}{2}t}\sin t$$

where t is time, in seconds, and $g = 9.8\,\mathrm{m\,s^{-2}}$.

(a) Calculate the maximum acceleration. **[4]**

(b) **(i)** Show by integration that

$$\int \mathrm{e}^{-\frac{1}{2}t}\sin t\,\mathrm{d}t = -\frac{4}{5}\mathrm{e}^{-\frac{1}{2}t}\left(\cos t + \frac{\sin t}{2}\right) + C.$$ **[5]**

(ii) Given that the ground is at rest when $t = 0$, calculate the velocity of the ground when $t = 1$. **[2]**