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Friday 24 May 2013 – Morning

LEVEL 3 CERTIFICATE MATHEMATICS FOR ENGINEERING

H860/02 Paper 2

Candidates answer on the Answer Booklet.

OCR supplied materials:
- 12 page Answer Booklet (OCR12) (sent with general stationery)
- Insert (inserted)
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- The Insert will be found in the centre of this document.
- Write your name, centre number and candidate number in the spaces provided on the Answer Booklet. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a scientific or graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **40**.
- This document consists of **4** pages. Any blank pages are indicated.

Refer to the appendix of the insert for a list of symbols used in this paper together with their meaning.

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1 **(a)** Water is flowing at a constant rate of $100\,\text{kg}\,\text{s}^{-1}$ from a chute above an overshot waterwheel with a diameter of 4 m.

Calculate the power that is available due to the potential energy of the water flowing from the top to the bottom of the waterwheel. **[1]**

(b) A waterwheel, similar to that shown in Fig. 2 of the insert, has 18 buckets, a diameter of 4 m and an angular speed of 5 revolutions per minute. Water enters the top of the waterwheel at a rate of $100\,\text{kg}\,\text{s}^{-1}$. All of the water remains in the buckets until their lips reach an angle of $\frac{5}{9}\pi$ rad measured clockwise from the vertical centre line of the waterwheel. For all angles greater than $\frac{5}{9}\pi$ no water is retained.

(i) Calculate the mass of water in each bucket immediately after it has been completely filled. **[3]**

Formula (5) in the insert describes the power, P, generated by a waterwheel which has n buckets.

$$P \approx fgr\sum_{j=1}^{\frac{1}{2}n} \mathrm{X}\left(\frac{2\pi}{n}j\right)\sin\left(\frac{2\pi}{n}j\right)\Delta\theta$$

(ii) Define the function $\mathrm{X}(\theta)$ for $0 \leqslant \theta \leqslant \pi$. **[1]**

(iii) Use formula (5) to estimate the power generated by the waterwheel. **[5]**

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2 Formula (6) in the insert describes the power, P, generated by a waterwheel containing a large number of buckets.

$$P \approx fgr\int_0^{\pi} \mathrm{X}(\theta)\sin(\theta)\mathrm{d}\theta$$

where

θ is the angle, measured clockwise, between the vertical centre line of the waterwheel and the lip of the bucket (see Fig. 2 of the insert),

$\mathrm{X}(\theta)$ is a value between 1 and 0 that represents the proportion of water that remains in the bucket for angle θ.

Use this equation to answer the following.

A waterwheel has a diameter of 4 m. Water enters the top of the waterwheel at a rate of 100 kg s^{-1}. All of the water remains in the buckets until their lips reach an angle of $\theta = \theta_{\mathrm{A}}$.

(a) For all angles of $\theta \geqslant \theta_{\mathrm{A}}$ no water is retained. Show that the power generated by this waterwheel is

$$P \approx fgr(1 - \cos\theta_{\mathrm{A}}).$$ **[3]**

(b) The waterwheel is now fitted with modified buckets which still retain all their water for $\theta \leqslant \theta_{\mathrm{A}}$. For these modified buckets $\theta_{\mathrm{A}} = \frac{\pi}{2}$ and for $\theta \geqslant \theta_{\mathrm{A}}$, the proportion of water that remains is given by

$$\mathrm{X}(\theta) = 2\left(1 - \frac{\theta}{\pi}\right).$$

Calculate the output power of this waterwheel. **[5]**

(c) The efficiency, η, of the waterwheel in part **(a)**, before any other equipment losses are considered, is

$$\eta = \frac{\text{Output power}}{\text{Power of descending water}} = \frac{fgr(1 - \cos\theta_A)}{2rfg}.$$

Using trigonometric identities given in the List of Formulae (MF1) provided or otherwise, show that

$$\eta = \sin^2\left(\frac{\theta_{\mathrm{A}}}{2}\right).$$ **[3]**

3 Fig. 1 in the insert shows a waterwheel with associated equipment providing power to charge a battery bank and to operate electrical equipment in the henhouse. Assume that the waterwheel has a diameter of 4 m and the rate at which water flows from the chute is $100\,\mathrm{kg\,s^{-1}}$.

The output from the alternator is 14.4 V with a constant current of 120 A. Assume that the inverter has an efficiency of 90% and provides an output to power the equipment in the henhouse.

(a) Calculate the overall efficiency measured from the point at which water enters the waterwheel to the output of the alternator. **[2]**

(b) Assume that the henhouse is drawing 450 W from the output of the inverter.
Calculate the current available to charge the battery bank. **[2]**

(c) Assume that the battery bank is fully charged with a capacity, C_{20}, of 200 Ah and that the equipment in the henhouse is turned on and requires a total power of 2 kW.

(i) Calculate the current being drawn from the battery bank. **[2]**

(ii) Using Peukert's equation, as given in the insert with $n = 1.3$, calculate the time it will take to completely discharge the battery bank. **[2]**

4 For this question assume that the moment of inertia, J, for the waterwheel described in the insert is $2000\,\mathrm{kg\,m^2}$.

(a) For this part of the question any change in angular speed takes place with constant acceleration.
The waterwheel is rotating at a constant angular speed of 5 revolutions per minute with equal driving torque, τ_D, and load torque, τ_L, ie the net torque is zero.

(i) The net torque, τ_T, about the axle of the waterwheel now changes to a constant value of 50 N m.
Calculate the angular speed of the waterwheel after it has made one complete revolution with this new torque. **[3]**

(ii) Calculate the time it will take for the waterwheel to make the first revolution with the new torque. **[2]**

(b) Assume that the waterwheel is rotating at a constant speed of 5 revolutions per minute with a constant water flow. The water flow now suddenly stops but the waterwheel continues to turn for a short while. During this period you may assume that the driving torque created by the water that remains in some of the buckets for a short while is negligible. The load torque, τ_L, however, is still present and this is modelled by

$$\tau_L = 200(1 + \omega)\,\mathrm{N\,m}$$

where $\omega\,\mathrm{rad\,s^{-1}}$ is the angular speed of the waterwheel.

Calculate the time it will take for the waterwheel to come to a complete rest. **[6]**