OCR
RECOGNISING ACHIEVEMENT

Thursday 6 June 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4763/01 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4763/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 **(a)** A particle P of mass 1.5 kg is connected to a fixed point by a light inextensible string of length 3.2 m. The particle P is moving as a conical pendulum in a horizontal circle at a constant angular speed of 2.5 rad s^{-1}.

(i) Find the tension in the string. **[4]**

(ii) Find the angle that the string makes with the vertical. **[2]**

(b) A particle Q of mass m moves on a smooth horizontal surface, and is connected to a fixed point on the surface by a light elastic string of natural length d and stiffness k. With the string at its natural length, Q is set in motion with initial speed u perpendicular to the string. In the subsequent motion, the maximum length of the string is $2d$, and the string first returns to its natural length after time t.

You are given that $u = \sqrt{\frac{4kd^2}{3m}}$ and $t = Ak^{\alpha}d^{\beta}m^{\gamma}$, where A is a dimensionless constant.

(i) Show that the dimensions of k are MT^{-2}. **[1]**

(ii) Show that the equation $u = \sqrt{\frac{4kd^2}{3m}}$ is dimensionally consistent. **[2]**

(iii) Find α, β and γ. **[4]**

You are now given that Q has mass 5 kg, and the string has natural length 0.7 m and stiffness 60 N m^{-1}.

(iv) Find the initial speed u, and use conservation of energy to find the speed of Q at the instant when the length of the string is double its natural length. **[5]**

2 A particle P of mass 0.25 kg is connected to a fixed point O by a light inextensible string of length a metres, and is moving in a vertical circle with centre O and radius a metres. When P is vertically below O, its speed is $8.4\,\text{m}\,\text{s}^{-1}$. When OP makes an angle θ with the downward vertical, and the string is still taut, P has speed $v\,\text{m}\,\text{s}^{-1}$ and the tension in the string is T N, as shown in Fig. 2.

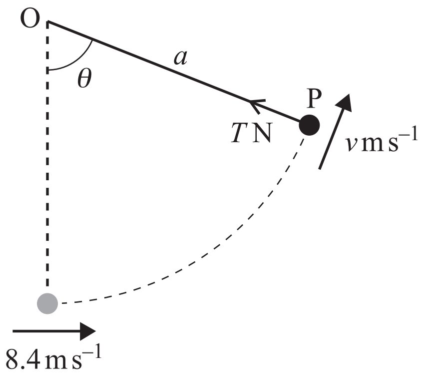

Fig. 2

(i) Find an expression for v^2 in terms of a and θ, and show that

$$T = \frac{17.64}{a} + 7.35\cos\theta - 4.9.$$ **[7]**

(ii) Given that $a = 0.9$, show that P moves in a complete circle. Find the maximum and minimum magnitudes of the tension in the string. **[4]**

(iii) Find the largest value of a for which P moves in a complete circle. **[3]**

(iv) Given that $a = 1.6$, find the speed of P at the instant when the string first becomes slack. **[4]**

3 A light spring, with modulus of elasticity 686 N, has one end attached to a fixed point A. The other end is attached to a particle P of mass 18 kg which hangs in equilibrium when it is 2.2 m vertically below A.

(i) Find the natural length of the spring AP. **[2]**

Another light spring has natural length 2.5 m and modulus of elasticity 145 N. One end of this spring is now attached to the particle P, and the other end is attached to a fixed point B which is 2.5 m vertically below P (so leaving the equilibrium position of P unchanged). While in its equilibrium position, P is set in motion with initial velocity $3.4\,\text{m}\,\text{s}^{-1}$ vertically downwards, as shown in Fig. 3. It now executes simple harmonic motion along part of the vertical line AB.

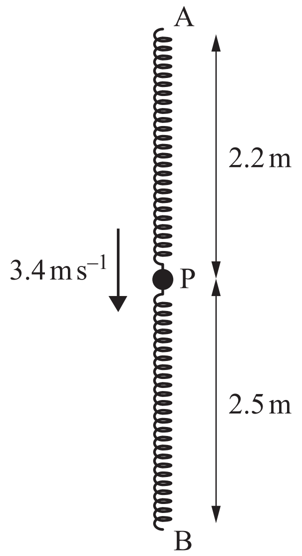

Fig. 3

At time t seconds after it is set in motion, P is x metres below its equilibrium position.

(ii) Show that the tension in the spring AP is $(176.4 + 392x)$N, and write down an expression for the thrust in the spring BP. **[3]**

(iii) Show that $\dfrac{d^2x}{dt^2} = -25x$. **[3]**

(iv) Find the period and the amplitude of the motion. **[3]**

(v) Find the magnitude and direction of the velocity of P when $t = 2.4$. **[3]**

(vi) Find the total distance travelled by P during the first 2.4 seconds of its motion. **[4]**

4 **(a)** A uniform solid of revolution S is formed by rotating the region enclosed between the x-axis and the curve $y = x\sqrt{4-x}$ for $0 \leqslant x \leqslant 4$ through 2π radians about the x-axis, as shown in Fig. 4.1. O is the origin and the end A of the solid is at the point $(4, 0)$.

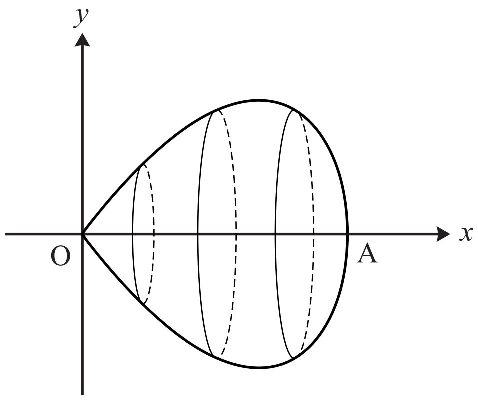

Fig. 4.1

(i) Find the x-coordinate of the centre of mass of the solid S. **[6]**

The solid S has weight W, and it is freely hinged to a fixed point at O. A horizontal force, of magnitude W acting in the vertical plane containing OA, is applied at the point A, as shown in Fig. 4.2. S is in equilibrium.

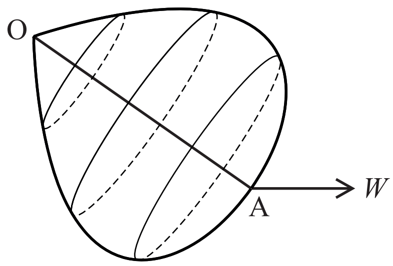

Fig. 4.2

(ii) Find the angle that OA makes with the vertical. **[3]**

[Question 4(b) is printed overleaf]

(b) Fig. 4.3 shows the region bounded by the x-axis, the y-axis, the line $y = 8$ and the curve $y = (x - 2)^3$ for $0 \leqslant y \leqslant 8$.

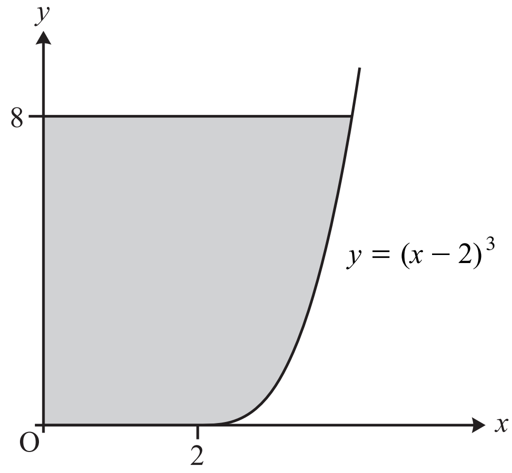

Fig. 4.3

Find the coordinates of the centre of mass of a uniform lamina occupying this region. **[9]**

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