OCR
Oxford Cambridge and RSA

GCE

Mathematics

Advanced GCE A2 7890 – 2

Advanced Subsidiary GCE AS 3890 – 2

OCR Report to Centres June 2014

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This report on the examination provides information on the performance of candidates which it is hoped will be useful to teachers in their preparation of candidates for future examinations. It is intended to be constructive and informative and to promote better understanding of the specification content, of the operation of the scheme of assessment and of the application of assessment criteria.

Reports should be read in conjunction with the published question papers and mark schemes for the examination.

OCR will not enter into any discussion or correspondence in connection with this report.

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OCR REPORT TO CENTRES

4721 Core Mathematics 1

General Comments

The vast majority of candidates were very well prepared for this paper. Many candidates achieved high marks and the proportion of candidates entered that have not been properly prepared continues to decrease. Candidates are now clearly comfortable with the answer booklet format. The use of additional sheets was rare this session, and the unnecessary use of graph paper is now almost non-existent. Many of those that did need to repeat a solution indicated so clearly, which was very helpful. A few still leave a choice of answers which should be discouraged.

Many candidates presented very clear and accurate solutions showing a good understanding of the mathematics needed for this module. As ever, differentiating powers of *x* and solving quadratics by an appropriate method continue to be approached very well, although their remains some reliance on the quadratic formula amongst weaker candidates. The use of clear sketches to help explain solutions was often very creditable. The best performing candidates were able to clearly explain all their reasoning in questions, such as 8 and 9, which required more than application of standard routines. Transformation of graphs, especially describing these, remains a challenge for many. Although the standard of arithmetic – particularly fraction arithmetic – was generally higher this session, a significant number of marks are still lost through careless calculation errors.

Comments on Individual Questions

1. This “completing the square” question was tackled well by the majority of candidates, many securing all four marks. In keeping with previous sessions, almost all earned the first two marks, seeing that p was 5 and q was 1, although $q = 5$ was seen relatively often among weaker candidates. Failure to multiply by five when working out the constant was the most common error amongst candidates who did not achieve full marks. Those who took out 5 as a factor of the full expression often made errors with the resulting fractions.

2. (i) Most candidates recognised the need to rationalise the denominator and did so efficiently. There were occasional numerical errors such as $\frac{6}{3} = 3$ and a small minority appeared not to know how to start, usually giving the answer as $6\sqrt{3}^{-1}$.

(ii) Again, most candidates were successful with this part. A relatively common error was to write $6\sqrt{27} = 6.3\sqrt{3} = 9\sqrt{3}$; with many of the weaker candidates it was unclear where incorrect answers had come from.

(iii) This was generally less successful than parts (i) and (ii), with only about three quarters of candidates earning both marks. $5\sqrt{3}$ was a commonly seen wrong answer.

3. This disguised quadratic was well approached by the vast majority of candidates, with just over half achieving all 5 marks. This continued an improving trend over the last few sessions, with fewer candidates going straight to the quadratic formula with no attempt to square root at the end, which has been a problem with similar questions in the past. The most common approach was to perform a substitution and then to factorise, although those who opted for a two-bracket approach using x^2 were also often successful. Fewer candidates than in previous sessions used the quadratic formula; those that did were usually successful. Completing the

square was rarely seen. Only a small number of candidates opted to square rather than square root, but the main loss of credit was due to lack of accuracy at the end. Although most candidates correctly dismissed any roots from $x^2 = -1$, some did not. More common was the absence of the negative square root

4.(i) This simple translation was recognised and the vast majority of candidates secured the mark.

(ii) This stretch proved much more demanding with only just over half of candidates obtaining the correct coordinate. Although (4, 10) was the modal incorrect answer, there were many other common errors involving halving/doubling either of, or both, the *x* or *y* values

(iii) Compared to previous sessions, candidates' attempts to describe a translation were better. The use of incorrect mathematical language, e.g. using "shift" or "move" and/or expressions like "in/along/on the *x* axis" were common. Several candidates erroneously stated " – 4 units in the negative *x* direction"; other errors included "translated by scale factor 4" and translation parallel to the *y* axis.

5 (i) This simple "double inequality" was well tackled by almost all candidates. Only the very weakest either tried to combine it into a single inequality and/or made arithmetical errors.

(ii) Just under half of candidates provided fully correct solutions to this quadratic inequality. Some failed to expand and rearrange at the start and thus earned no credit. Most were able to complete the first stage accurately, but the resulting quadratic proved more difficult to handle. Those who factorised were usually successful, whilst those who attempted to use the quadratic formula were often correct in performing the substitution but unable to find the square root of 289. Most of those who found the correct roots also chose the correct region, but there were a significant number who expressed this incorrectly, often as $-\frac{2}{3} \geq x \geq 5$.

6 (i) This differentiation was extremely well done, with around four in five candidates securing all the available marks; the ability to recognise and deal with a fractional negative term was much better than in some previous sessions. This term remained the main cause of error, although some errors were made with the first term. The inclusion of a constant when differentiating is now very rare indeed.

(ii) The need to differentiate again was apparent to most candidates, and again the standard of dealing with the fractional negative term was very high. Some candidates made arithmetical errors here and a few failed to simplify $\frac{6}{2}$.

7. (i) This was another area of improvement from previous sessions with very few candidates quoting or using an incorrect formula to find the mid-point. Thus the vast majority scored both marks.

(ii) Around two-thirds of candidates provided fully correct solutions to find the equation of the required line. For those who were not successful, errors occurred at all stages. Some failed to find the correct gradient whilst others omitted to find the negative reciprocal to give the gradient of the perpendicular. More commonly, candidates did not read the question carefully and found the equation of the line through the mid-point rather than the required point, or did not give their final answer in the correct form.

8. (i) Most candidates approached this very sensibly, differentiating and substituting in $x = 1$ to show the gradient is zero. Many, however, then failed to link this to question and state that this was why there was a stationary point. Some candidates equated their derivative to zero and solved the resulting quartic to show was a solution, again some omitted to explain the significance of this. In all cases, differentiation was generally accurate with some errors with the negative term.

(ii) Most candidates used the second derivative to determine that this was a minimum point, although there were a number of arithmetical errors at this stage. A few candidates found the second derivative to be 22 and then said "increasing", showing apparent confusion over the purpose and meaning of this method. Some candidates tried to find the gradient at either side of $x = 1$ but many of these chose $x = 0$ as the point to the left; as the function was undefined at this point, this invalidated this approach.

(iii) Less than half of candidates were successful on this part. Many realised the need to find the value of the function when $x = 1$ but then struggled to relate this to where the tangent would cut the axis. Q = – 2 was a common incorrect answer.

9 (i) The simplest, and most common, approach to show that the circle did not go above the x axis was to identify the centre and radius from the equation and state/show on a diagram that the circle just touched the axis at a single point. The majority of candidates showed clear solutions to this effect. Some, however, tested just a single point (usually $y = 1$) and showed this was not a point on the circle, which was of course insufficient.

(ii) Most candidates took the correct approach to this, substituting $x = 6$ and then solving the quadratic and finding the values of k that corresponded to the points on the circumference. A large number of candidates then stopped and failed to identify the correct range of values being any value between these. Those who carried on were usually correct, but it was fairly common to not give the answer as the strict inequalities required for the point to be inside the circle. There were some neat alternative solutions using Pythagoras' theorem to find the values of k from a good sketch.

(iii) There were a large number of fully correct solutions to the request to prove that the line and circle do not meet. Most performed the easier substitution for x, but $(2y)^2 = 2y^2$ was a fairly common error. Many were able to use the discriminant, or the quadratic formula, to explain their reasoning clearly. Only a few claimed that the line and circle did not meet because the quadratic could not be factorised. Some of the weaker candidates again resorted to testing a single point (sometimes the centre) or drawing a poor diagram.

10 (i) The sketching of this cubic graph was generally done well, with the majority recognising the need for a double root at $x = -2$. There were some errors such as the inversion of the positive root and the occasional negative cubic was seen. Those who sketched a quadratic were only able to score a mark if they correctly identified the intercept on the y axis.

(ii) Just over half of candidates scored full marks for this final unstructured question, with many others scoring highly. Indeed, most candidates structured their solutions very well and the vast majority of errors were arithmetical rather than conceptual. These included errors in the initial expansion and in the substitution of $x = -1$ into the derivative. A few candidates set their derivative to zero. Another fairly common error was to find the equation of the normal rather than that of the tangent as required.

4722 Core Mathematics 2

General Comments

In general candidates seemed well prepared for this examination and the vast majority were able to make an attempt at most, if not all, of the questions. There were some straightforward questions that allowed all candidates to demonstrate their knowledge, along with some more challenging ones where even the most able candidates had to carefully formulate appropriate strategies.

Candidates should ensure that they are proficient with the standard C1 techniques, such as manipulating surds and indices and using the quadratic formula, as these may be required as part of a C2 solution. This was particularly noticeable in Qu6, where candidates could state the correct binomial expansion but struggled to handle the indices correctly.

As always there were some questions where a number of candidates made multiple attempts and many are not heeding the advice given in previous reports, in that it is the final attempt that will be marked unless otherwise indicated. On a number of scripts it was the first attempt that was the best one; however, when this proved unsuccessful, candidates decided to try an alternative method instead. In these circumstances it is essential that candidates delete any attempts that they do not wish to be marked, including those on additional sheets of paper.

In general, candidates presented their solutions clearly and with adequate detail, though this was not the case on all scripts. This is particularly important when proving a given result when it is expected that each step will be shown clearly and with sufficient justification. Candidates should also ensure that they show details of the methods used rather than just writing down a calculator answer with no evidence of how it was obtained. Examiners cannot always award method marks if no explicit method is shown.

Candidates should check whether exact or approximate answers are required, and ensure that their final answer is commensurate with this. If an approximate answer is requested then they must ensure that the rounding is correctly done.

Comments on Individual Questions

1(i) This entire question proved to be a very straightforward start to the paper and most candidates gained all of the 6 marks available. In this part of the question the majority of candidates could quote the correct formula, though a few omitted the ½. Other errors included evaluating the expression in the incorrect calculator mode and incorrect rounding. These are all avoidable errors and candidates should be alert to them.

(ii) This was also very well done, with candidates able to either quote the correct formula or obtain it from the formula book. Candidates could identify which sides to use and the substitution was nearly always correct. A few candidates then spoiled their answer by inserting imaginary brackets to treat the whole expression as a coefficient of $\cos 65^\circ$. Once again, marks were sometimes lost through incorrect calculator mode or rounding errors.

(iii) The better candidates simply used triangle *ABC* and found the required length through efficient use of the Sine Rule. A surprising number decided instead to use a multi-step method working first of all within triangle *ABD* and then attempting the length of *BC*, possibly from assuming that part (ii) should be used. This was usually done correctly, but the extra steps did sometimes result in a loss of accuracy in the final answer.

2(i) Virtually all of the candidates were able to write down the required terms, gaining the two marks available. A few candidates mistakenly treated it as a recursive definition rather than an *n*th term definition.

(ii) Despite having written out the first three terms of the sequence in part (i), a number of candidates struggled to identify the correct values of a and d, with $a = 1$ being the most common error. The sight of the sigma sign resulted in other candidates attempting to use one of the summation formulae, but this was only occasionally done correctly. However, the majority of candidates could quote the relevant formula, substitute the correct values and obtain the required final answer to gain all of the marks available.

3(i) Most candidates could quote the relevant formula and then obtain the correct length of the arc, though a number spoiled their answer by then giving the decimal approximation. Some candidates used more cumbersome methods involving fractions of a circle, which were usually correct though rarely resulted in an exact answer. A significant minority found the length of the chord *AB* instead, either through not reading the question carefully or through a lack of understanding of the terminology.

(ii) Once again candidates could quote the relevant formulae and accurately substitute into them. It was common to see two separate exact values which then became a decimal once combined, possibly because of using a calculator. Candidates must appreciate that if an exact answer is requested then this should be the only final answer provided, and expect to get penalised if the decimal approximation is also given. The other common error was not ensuring that the calculator was in the correct mode when evaluating the area of the triangle.

4(i) A variety of methods were seen for this proof, some more efficient than others. Most candidates did get there in the end, but full credit was only given if the correct notation had been used throughout. Candidates must also ensure that each step is clearly and convincingly detailed when a proof has been requested.

(ii) This question was generally very well done, and many candidates gained full marks on this question. The most common error was to completely discount the solution resulting from $\tan^{-1}(-2)$ as it resulted in a negative angle rather than appreciating it would still generate other angles within the given range. It was also disappointing to see candidates with a correct method failing to gain full marks due to rounding errors. As in previous questions involving trigonometry, some candidates did not ensure their calculator was in the correct mode before proceeding. Angles given in radians could gain some credit, but candidates did not actually consider which measure they were using so the typical error was $\tan^{-1}(3) = 1.25$ and hence 189.25.

5 Most candidates were able to gain the first two marks for taking logarithms of both sides and using the power rule, though a number of candidates failed to use brackets. This lack of precision was penalised unless subsequent working clearly showed the correct intention. In order to make further progress candidates had to then expand the brackets and gather like terms which, only the better candidates realised the need to do. Even fewer managed the next step of making x the subject of the equation although some did manage to get a method mark for correctly combining two relevant logarithms. Recent examination sessions have shown candidates becoming more proficient in using logarithms to solve equations when a decimal answer is required, but it appears that algebraic manipulation of logarithms is still a challenge for many. Nevertheless, a pleasing number of fully correct solutions were still seen.

6(i) Most candidates were able to write down a correct binomial expansion, including coefficients. The correct brackets were invariably seen in the initial statement, and around half of the candidates then used these effectively to produce a fully correct solution. However a significant minority simply ignored the brackets resulting in incorrect coefficients as each index was only applied to the x^{-2} and not the 2 as well. An equally common error was an ability to deal with the indices involved, which is basic GCSE and C1 work. Whilst the first and last terms were often correct, the multiple index laws required for the middle three terms caused problems for many with confusion over whether the add or multiply the relevant indices.

(ii) Nearly all of the candidates were able to make a good attempt at integrating their expression from part (i) and the majority gained two marks for correctly integrating at least three of their terms. This allowed for slips with the negative indices, which proved more problematical for some. A third mark was available for including + *c*, but this was omitted from a number of solutions.

7(i) The overwhelming majority of candidates gained both marks in this question, usually by using the remainder theorem. Some candidates decided to use algebraic division instead; a longer and often less successful tactic.

(ii) Virtually all of the candidates gained this mark, with the factor theorem being by far and away the most common method. It was not sufficient to do the division or state the factorisation, candidates also had to demonstrate that there was no remainder in order to gain the mark.

(iii) This part of the question was also very well done, with the majority of the candidates gaining all of the three marks available. Algebraic division tended to be the most common method, with most candidates coping admirably with the negative coefficient of x^3 and the function being written in order of ascending powers. Some candidates made the division easier by reversing the order of the terms. Others opted to use the negative of either the function or the factor, or both. Care then had to be taken when finally writing the function as the product of a linear factor and a quadratic factor as the reversal of signs was not always taken into consideration. Other methods included coefficient matching and inspection, both of which were usually successful. However, candidates using inspection do need to be careful as no method is shown which makes it unlikely that partial credit can be awarded in the case of an incorrect answer.

(iv) This was also very well done, with well over half of the candidates gaining full marks. Most remembered to include $x = 3$ as a root, and finding the other two roots was also very well done with a variety of methods being used. For candidates using the quadratic formula the method mark will only be awarded if the correct formula is seen, either the generic statement or after substitution, so it is essential that candidates show full details of their method.

8(a) The better candidates were able to gain three or four marks on this question, but many of the weaker ones struggled to gain any credit. The first mark was for a correct expression for the *k*th term; most candidates gained this but some used an index of *k* rather than $k - 1$. The other error was to initially work in terms of *a* and *r* and, by the time the numerical values were used, the expression was now incorrect. The next two marks were for attempting the value of *k*, either as an equation or an inequality. The most successful method was to first divide by 50 before introducing logarithms, but this was done by the minority. For those who introduced logarithms first, only the better candidates were able to correctly split the expression into two terms before using the power rule. The most common errors were for the power to be dropped too soon, for the expression to end up as the product of two terms or for the logarithms to only be partially applied to the expression. The final mark was for correctly concluding that $k = 28$, but this mark proved elusive for many. The most common errors were not appreciating the need to round the answer up to the next integer, especially as the rounded value was 27.0, and not realising the need for equality in the final answer at all.

(b) Candidates were able to make a good start to this question, but only the most able could make progress beyond the first five marks. The majority could attempt the two relevant equations and then eliminate one of the variables, usually *a*. Substituting the equation for the sum to infinity into the equation for the second term usually resulted in the correct quadratic, whereas the fraction involved in doing the substitution the other way around caused problems for some. Nevertheless, many candidates did obtain the correct quadratic which they could then attempt to solve. Candidates then had to select the correct common ratio and also provide some reasoning for this choice. No credit was available for picking $r = -0.5$ with no, or an incorrect, reason. To gain full marks, the reasoning for the selection of $r = -0.5$ had to be convincing and fully complete. It was not sufficient to reject $r = 1.5$ without also explaining why the other was being accepted.

9(i) Candidates were familiar with the trapezium rule and the majority were able to apply it accurately to the given situation. A surprising minority rounded the final answer to 10.3 rather than 10.2. This could be ignored provided that a correct, more accurate, answer had been seen previously. Slips in calculating the *y* values were condoned, as long as there was sufficient working to convey the correct intent.

(ii) The overwhelming majority of candidates gained both of the marks available, especially as full credit could still be awarded following an incorrect result to part (i).

(iii) This proved to be a suitably challenging end to the paper, with the most able candidates gaining full credit but the weaker ones struggling to make any kind of progress. Most candidates could attempt to change the subject of the equation but weak algebra skills meant that the result was not always the required equation. Most candidates were then able to make a reasonable attempt at the integration, with the most common error being for the constant term to be integrated with respect to *x* rather than *y*. There was a straightforward mark for identifying that the limits were 1 and 3, and candidates then had to use this to attempt the required area. Only the most able candidates were able to provide a convincing demonstration that the area of the region was indeed $^{14}/_{3}$, as given on the paper.

4723 Core Mathematics 3

General Comments

There were many excellent scripts produced for this unit and examiners were pleased to note that approximately 6% of the candidates recorded a mark of 70 or more out of 72. There were few scripts with low marks; approximately 95% of the candidates recorded a mark of 25 or more. The mean mark for the paper was 52 out of 72. There was no evidence that candidates did not have sufficient time to do themselves justice on this paper.

Most of the routine requests were generally answered very well. The differentiation techniques needed for Q.1 and Q.8(i), the numerical methods required for Q.3(i) and Q.6(ii), and the work on functions in Q.4 were well known and good sources of marks for the majority of the candidates. It is also encouraging to acknowledge the good work seen in both parts of Q.5 where candidates showed a sound grasp of exponential growth and decay. More challenging requests included Q.3(ii), Q.7, Q.8(ii) and Q.9. Candidates would have benefited here by giving some thought to the most effective way of approaching each request.

The comments below on individual questions refer to common errors and many of them are due to a general lack of confidence with basic algebra. This lack of confidence may explain why many candidates did not simplify $\frac{4x^2}{x}$ as soon as it appeared in Q.1. Uncertainty concerning roots was evident in Q.4(iii) where $\sqrt[3]{2x^3-6}$ often became $2x-6^{\frac{1}{3}}$ and in Q.7(i) where $\sqrt{3}(4x+1)^{-\frac{1}{2}}$ sometimes became $(12x+3)^{-\frac{1}{2}}$. The reluctance of candidates to insert necessary brackets has been referred to in previous examination sessions. On this occasion in Q.7(ii), solutions involving $\ln 4x+1$ were often seen; usually the candidates proceeded as if the phantom brackets did in fact exist and no harm was done. But, in Q.2, the insertion of brackets was also needed; it does not take much imagination to think of the various errors resulting from solutions commencing with statements such as " $5.1-2\sin^2\theta/\sin\theta \quad = \quad 2$ ".

2. Comments on Individual Questions:

Q. 1 This question was a suitable introduction to the paper for the majority of candidates, and 77% of them duly earned all five marks. Some provided very concise solutions taking only a few lines of working; the two applications of the product rule were handled without fuss. For many other candidates, solutions were more protracted with each attempt at the product rule needing some work at the side as functions *u* and *v* were defined and differentiated. Assembling the parts to form each derivative was prone to error and one that occurred frequently was a failure to include the derivative of $4x$ in the expression for the second derivative. Sound advice for candidates setting out solutions is to carry out obvious simplifications as the solution progresses. It was surprising that a significant number of candidates did not do this in this question. Having found the first derivative as $8x\ln x+\frac{4x^2}{x}$, they continued by correctly applying the product rule to the first term and then using the quotient rule to deal with the second term. A few candidates did not see the need to use the product rule at all and the first step in a few cases was the substitution of e^2.

Q. 2 All but a few candidates recognised $\operatorname{cosec}\theta$ as the reciprocal of $\sin\theta$ and most were able to express $\cos 2\theta$ in terms of $\sin\theta$, although in some cases only after some lengthy manipulations. Many candidates then had difficulties in reaching the equation $10\sin^2\theta + 2\sin\theta - 5 = 0$. Absence of necessary brackets and incorrect cancellation of $\sin\theta$ were frequently noted. Many candidates, having reached the correct equation, were then unsure how to proceed when factorisation proved impossible; some abandoned the solution at this stage and others embarked on incorrect approaches such as $2\sin\theta(5\sin\theta + 1) = 5$ leading to $\sin\theta = \frac{5}{2}$ and $\sin\theta = -\frac{1}{5}$. A few candidates opted for completion of the square as a means of solving the equation, sometimes successfully, but of course many did use the relevant formula correctly. The correct conclusion required two angles but some candidates provided only $37.9°$ whereas others provided four values within the required range.

Q. 3 Part **(i)** was answered very well with 83% of candidates earning all four marks. Arithmetical slips were the usual cause of candidates failing to earn all the marks.
Part **(ii)** was not answered so well and only 37% of candidates earned both marks. Most candidates recognised the need to multiply the previous result by 10 but, in many cases, followed by adding 1 rather than 2. A minority of candidates carried out a second full Simpson's Rule calculation and both marks were available for this approach. A few candidates appealed to ideas about stretch and translation of graphs in order to decide on their approach. Some recognised the need to add 2 but only as the result of applying Simpson's Rule to $\int_0^2 1\,dx$ and dutifully setting out $\frac{1}{6}(1 + 4\times1 + 2\times1 + 4\times1 + 1)$. There were a few attempts in both parts that involved an attempt at integration before values for *x* were substituted; no marks were available for an approach showing such a basic misunderstanding.

Q. 4 There were few problems with part **(i)** and 83% of candidates earned both marks. Few seemed to realise that the answer can be obtained by solving $f(x) = -50$ and the common approach was to find the inverse function. Many were guilty of careless notation, writing the inverse in a way to suggest $\frac{\sqrt[3]{x-4}}{2}$ when they clearly meant $\sqrt[3]{\frac{x-4}{2}}$; provided they proceeded to carry out the correct calculation, both marks were earned. Further carelessness was evident on some scripts where 50 was substituted.
Part **(ii)** was answered extremely well with almost all candidates showing sufficient detail and recording two marks.
There were more problems with part **(iii)**. Almost all candidates had the correct expression for $gf(x)$ but some chose not to carry out the obvious simplification, or simplified it to $\sqrt[3]{-20x^3 - 40}$ or to $(2x^3 + 4)^{\frac{1}{3}} - 10^{\frac{1}{3}}$. Common errors with the differentiation included a factor $6x$ instead of $6x^2$, an expression involving $(2x^3 - 6)^{-\frac{1}{3}}$ and a final answer not suitably simplified.

Q. 5 Both parts of this question were answered well with full marks earned by 87% of candidates in part **(a)** and by 79% of candidates in part **(b)**. In part **(a)**, the need for differentiation was almost always recognised and carried out accurately. There were a few numerical slips and not all candidates seemed clear about the request for 2 significant figures. Correct final answers given to 3 significant figures were accepted but not answers given to a greater level of accuracy, this being deemed inappropriate for a question placed in context.
Part **(b)** presented more problems and some candidates made the incorrect assumption that the mass would increase by 9.8 grams in each period of 6 years. Others made no progress because of an assumption that the formula from part **(a)** was still relevant. The usual method adopted was

to set up a formula of the form $42e^{kt}$ and proceed to establish the value of *k*. A lack of accuracy in the working marred some solutions. Some candidates displayed a clear understanding of exponential growth, knowing that the mass increases by the same proportion over equal time intervals, and were able to find the answer immediately from the calculation $42.0 \times (\frac{51.8}{42.0})^4$.

Q. 6 Part **(i)** was not answered well and only 23% of candidates earned two marks; candidates did need to show some care in order to earn the second mark. Many candidates drew a parabola with a minimum point, evidently believing that the curve $y = x^2 - 9$ was needed. Others realised that $y = 9 - x^2$ was required but the sketch did not show the maximum point anywhere near the *y*-axis. The curve given in the question is easily shown to cross the *x*-axis at the point $(2, 0)$ and candidates were expected to show the second curve crossing the *x*-axis to the right of this point. Not all candidates drew attention to the two points of intersection and not all gave the equation of the second curve in an acceptable form involving variables *x* and *y*.
The remaining parts of this question were answered very well. The necessary calculations in part **(a)** were shown and reference was made to the sign change. The iterative process in part **(b)** was carried out efficiently although, in a minority of cases, correct values were followed by an incorrect conclusion of 2.155 or 2.16.

Q. 7 There were more challenges for candidates in the two parts of this question. There were inaccuracies in carrying out the integration and in using the correct limits. A significant number of candidates ignored the need to deal with the region between *R* and the *x*-axis and produced answers that applied to a region between the curve and both axes.
Common errors in part **(i)** included initial steps such as $\int 3(4x+1)^{\frac{1}{2}}\,dx$ and $\int 3(4x+1)^{-\frac{1}{2}}\,dx$. Errors in the integration included $\int (\frac{3}{4x+1})^{\frac{1}{2}}\,dx$ leading to an expression involving $(\frac{3}{4x+1})^{\frac{3}{2}}$ and attempts leading to $\ln(4x+1)$. For candidates with the correct $\int \sqrt{3}(4x+1)^{-\frac{1}{2}}\,dx$, there was no certainty that the correct $\frac{1}{2}\sqrt{3}(4x+1)^{\frac{1}{2}}$ would follow. Incorrect limits such as $\frac{1}{9}\sqrt{3}$ and $\sqrt{3}$ or $\frac{1}{9}\sqrt{3}$ and 20 were sometimes used. An alternative approach was used by a number of candidates who treated *R* as a region between the curve and the *y*-axis. This needed care in expressing *x* in terms of *y*, in carrying out the integration accurately and in choosing the correct limits but many adopting this approach did reach the correct answer. They were among the 49% of candidates who did earn full marks for part **(i)**.
Some of the candidates treating *R* as a region between the curve and *y*-axis in part **(i)** continued in part **(ii)** to attempt to find the volume of revolution round the *y*-axis. Whether this was just careless reading of the question or whether they thought that the answer would be the same whichever axis they used was not clear. Generally in part **(ii)** candidates seemed to be on more familiar ground with the integration and most recognised that the result involved a natural logarithm although the integration often led to the incorrect $3\pi\ln(4x+1)$. Again, incorrect limits were applied in many cases. Many candidates did nothing about removing the volume of the unwanted cylinder; did they forget or did they not appreciate that this extra step was necessary? Of those making an attempt at this final step, many merely subtracted $\frac{20}{9}\sqrt{3}$ as they had in part **(i)** and others seemed to treat the shape to be removed as a cuboid or circle. 40% of the candidates managed to record all six marks on this part.

Q.8 Part **(i)** of this question involved a routine process and most candidates proceeded accordingly using the quotient rule. There were some errors including omitting the square in the denominator and getting the two parts of the numerator the wrong way round. Lack of precision with algebra led to further errors among which the commonest was the simplification of $2(x^2+5) - 2x(2x+4)$ to give $2x^2 + 10 - 4x^2 + 8x$. This particular mistake led to both stationary points having positive *y*-coordinates. Candidates making this elementary algebraic mistake

seemed totally unconcerned by the fact that the coordinates of their stationary points did not match the evidence of the curve shown in the question.
The requests in part **(ii)** were much more challenging and many candidates lacked the necessary skill and insight to answer them successfully. Most drew an acceptable curve of $y = \mathrm{g}(x)$ although not all gave sufficient attention to detail, particularly at either end of the *x*-axis. Perhaps mindful of the fact that modulus was involved, many candidates opted for $\mathrm{g}(x) \geq 0$ for the range of g; they did not seem to use the evidence provided by their graph. Only 33% of candidates earned three marks forpart **(ii)(a)**. Fewer candidates succeeded with the final part; those that did were usually able to write down the correct answer by consideration of their graph from the previous part. But most candidates were unable to make a sensible attempt or the request for two distinct real roots sent them into a $b^2 - 4ac > 0$ routine.

Q. 9 The requests in this question will have proved somewhat unfamiliar and it is pleasing to record that 15% of the candidates did rise to the challenges and record all twelve marks. Many candidates did not realise that some initial expansion and simplification were needed in part **(i)** and found *R* from $R^2 = 5^2 + 3^2$ with the value $30.96°$ for α following. For those candidates adopting the correct approach, there were some sign errors and the result of their initial simplification was often $\frac{11}{2}\cos\theta - \frac{5}{2}\sqrt{3}\sin\theta$. However, 49% of the candidates did reach the correct expression $7\sin(\theta + 51.8°)$.
Most candidates recognised that a stretch and a translation (although a few did refer to transform when presumably they meant translate) were needed in part **(ii)(a)** but the care needed to make sure that these were described accurately was not always present. In many cases, the stretch had scale factor 7 and the direction for the translation was incorrect. Presumably these candidates were assuming that the more usual request of the transformations needed to transform $y = \sin\theta$ to the more complicated curve was involved.

Success in part **(ii)(b)** needed the link between the left-hand side of the equation and the original expression to be noted. Some candidates did proceed easily to the correct final answer but many others did not see a need to use the obtuse angle $180° - \sin^{-1}\frac{3}{7}$ to find a positive value for β. Many others could make no relevant progress and attempts tended to consist of lengthy and involved trigonometric expansions.

4724 Core Mathematics 4

General Comments:

The paper proved accessible to a large majority of the candidates, yet there was enough to challenge the most able, and very high marks were rare. A significant minority of candidates demonstrated a fair degree of understanding of the Core 4 specification, but failed to do themselves justice in the examination because of poor GCSE level algebra and careless arithmetical slips.

Most candidates presented their work neatly and clearly, but in some cases work was very difficult to follow, and candidates should understand the importance of presenting a clear mathematical argument, especially when there is a "show that" request in the question. In these types of question, the onus is on the candidate to show sufficient working to convince the examiner that the result has been demonstrated satisfactorily, and that it hasn't simply been "back engineered".

The handing out of 16 page answer booklets to candidates who need extra space is unhelpful: often only one page is used – and frequently candidates choose to use a page in the middle of the answer booklet.

Comments on Individual Questions:

Question No. 1

Most candidates were able to identify the correct common denominator and write down the correct numerator in expanded form. Whilst many went on to earn the third mark, algebraic slips were quite common, $3 - x^2$ and $3 - x - x^2$ were commonly seen in the final answer. A surprisingly high number of candidates achieved the correct answer and then went on to "simplify" the result with incorrect cancelling out, thus losing the last mark.

Question No. 2

Some candidates didn't recognise the term "parallelogram" – the usual mistake was a sketch of a trapezium. Many more didn't seem to recognise the term "diagonal", and successfully used a scalar product to find the acute angle between the sides, or, rarely, one of the other acute angles in *OABC*. Those who drew a labelled diagram were usually successful. Nearly all candidates were able to demonstrate proficiency with scalar product, and understood "acute".

Question No. 3

This was very well done by nearly all candidates. A few candidates made a sign error with the first term, and some omitted "2" in part (i). Most gained at least the method mark in part (ii), although a few tried division instead of multiplication.

Question No. 4

Most candidates recognised at least one of the double angle substitutions, and many went on to spot the correct form of the integral and score full marks. A surprising number achieved the correct integrand, however, and then failed to progress. Incorrect splitting of the fraction such as $\int(\cos 2x + \cot 2x)\,dx$ and substitutions which went astray were fairly common. A small number of candidates didn't bother with double angles, but differentiated the numerator and spotted the logarithmic form, often going on to achieve full marks.

Question No 5

Most candidates knew what to do here. The majority used the coefficients of **i** and **j** to obtain a pair of correct values for *s* and *t*, before substituting in a third equation to demonstrate inconsistency. A minority of candidates went astray in the arithmetic and lost one or two of the accuracy marks.

In part (ii), the majority of candidates understood the relationship, but were unable to convey their understanding well enough to score, often associating the scale factor of – 2 with the wrong vector. Very few candidates checked whether the equations represented the same line or two distinct lines. A minority of candidates concluded that the lines were perpendicular.

Question No 6

Very many candidates showed mastery of implicit differentiation, and an overwhelming majority achieved the first 4 marks on this question. Many went on successfully to score full marks. However, weaker candidates set $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero and made no further progress. Surprisingly, solving $3y^2 - 12 = 0$ often led to $y = \pm 4$.

Question No 7

Part (i)
This proved accessible to most, with a good number of candidates achieving full marks. However, in many cases "2" went missing from the double angle, and
$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2\sin t + 2\cos t}{2\cos t} = 1 = 2\sin t$, in an attempt to achieve the given answer. Surprisingly, a large number of candidates simply omitted to find the co-ordinates of the turning point, or stopped at $t = \frac{\pi}{6}$.

Part (ii)
Those who attempted to find a polynomial equation often went astray in the substitution: $x^2 = (2\sin t)^2 = 2\sin^2 t$ was a common error, leading to $y = 1 - x^2 + x$. It was not always clear what substitution candidates were making: in cases where it went wrong a method mark was sometimes lost. Weaker candidates opted for an equation involving arcsin($\frac{x}{2}$); this nearly always resulted in zero in part (iii).

Part (iii)
Some candidates were able to deduce the range of values for *x*, but more often than not did not take the hint and relate this to their sketch. Only the best candidates produced a graph of the correct shape with endpoints in the correct quadrants, and only a handful identified a correct distinguishing feature for the third mark.

Question No 8

Part (i)
Most candidates took the expected route and showed the required result successfully using long division, although a proportion who adopted this approach made sign errors and fudged the rest. A variety of other approaches were also seen. Candidates are reminded that in this type of question, a convincing argument is required – it appeared that some strong candidates lost marks because the answer a andppeared obvious to them.

Part (ii)
Most candidates made some progress here. Integration by parts was generally used, and mostly successfully. Weak candidates failed to make the connection with part (i), but those who did make the connection generally went on to achieve at least the method marks. It was often in the manipulation following integration that marks were lost. A surprisingly common error was
$\int \frac{2t^3}{t+2}\,\mathrm{d}t = \int t^2\,\mathrm{d}t.$

Question No 9

Most recognised the correct form of partial fractions and successfully cleared the fractions. Although there were many fully correct solutions to this part of the questions, numerical slips such as 3*C* = 3 so *C* = 3 and $\frac{9}{4} = \frac{9}{4}A$ so *A* = 9 were surprisingly common. The integration was often well done, although $-(1-x)^{-1}$ was quite common, often leading to fudging of the subsequent arithmetic. As with 8(i), candidates are reminded of the need to show sufficient detail of the solution when working towards a given answer.

Question No 10

Part (i)
This defeated all but the best candidates. Many scored an easy B1 for $\frac{\mathrm{d}V}{\mathrm{d}t} = -0.01$(although a good number missed out on this because they wrote $\frac{\mathrm{dh}}{\mathrm{d}t} = -0.01$ instead). Thereafter very few made any progress: the need to eliminate *r* was often not appreciated. Those who did spot the relationship between *h* and *r* often went on to score full marks. Having obtained *V* in terms of *h* only, a few strong candidates derived the required result by considering $V(t) = 13.5\pi - 0.01t$ and differentiating.

Part (ii)
This proved surprisingly difficult for many. Those who did separate the variables either differentiated h^2 instead of integrating, or omitted the constant of integration and made no further progress. Those who did achieve a correct value for "*c*" often went on to spoil their answer, or simply left the equation in implicit form.

Part (iii)
Most realised that setting *h* = 0 was required here. Due to earlier errors, this sometimes led to a negative value for *t*; surprisingly this did not always set the alarm bells ringing. A good proportion of candidates who did everything right lost an easy mark because they failed to convert the answer to minutes. Some candidates who had made no progress earlier, were astute enough to realise that the correct answer could be obtained without the result from part (ii), as the rate of change of volume was constant.

4725 Further Pure Mathematics 1

General Comments:

Completely correct solutions to all questions were seen, with the majority of candidates scoring well on the first four questions, thus demonstrating a sound knowledge of a reasonable range of topics.

Candidates who presented their solutions neatly scored good marks, while those who were untidy usually made simple arithmetic or algebraic errors, such as missing brackets or loss of signs, lost a significant number of marks.

There was no evidence of candidates being under time pressure, with most making some attempt at the majority of the questions.

Comments on Individual Questions:

1 Almost all candidates showed a correct expansion method, with only minor arithmetic or algebraic errors occurring. Some found the correct value of the determinant, but then gave its reciprocal, as if they were finding the inverse matrix, which a few candidates actually did thus making the solution much longer.

2(i) The modulus and argument were both found correctly by most candidates. A common incorrect value for the modulus was $\sqrt{40}$, while a significant minority who gave either or both answers as decimals did not give sufficient significant figures.

2(ii) Most candidates multiplied by the conjugate of the denominator, but many then gave the value of (4 + i)(4 – i) as 15, rather than 17.

3(i) This was generally answered correctly. The two most common errors were finding **B** + 2**I** and subtracting this from 4**A**, while some thought **I** was $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

3(ii) Omission of the determinant was the most common reason for loss of marks.

3(iii) Many candidates did not know the result for the inverse of a product of two matrices and found the value of $\mathbf{A}^{-1}\mathbf{B}$. A significant number of candidates found $\mathbf{B}^{-1}$, then $\mathbf{AB}^{-1}$ and then attempted the inverse of this, usually making an arithmetic error somewhere or not being able to find the determinant of $\mathbf{AB}^{-1}$ correctly.

4(i) Most candidates wrote down the correct matrix, the most common incorrect answer seen being the columns of the matrix transposed.

4(ii) The transformation being a rotation was generally recognised, the difficulty was in giving a consistent direction and angle pair.

4(iii) The correct value was generally found and most gave a satisfactory explanation of its connection to the area of a transformed shape.

5(i) A surprisingly large number of candidates made errors in expanding $(u+2)^3$ or in simplifying a correct expression into a cubic equation with correct coefficients, often omitting the "= 0".

5(ii) This part proved to be quite testing. Many simply wrote down the value of "- *b*/*a*", or its reciprocal, from the new cubic equation. Others who combined the three fractions correctly, then expanded numerator and denominator but used the values of the symmetric functions from the new cubic equation, rather than the original equation. However, good candidates saw that a second substitution, $y = \frac{1}{u}$, could be used to find the required value.

6(i) Most candidates showed sufficient working to derive the given answer.

6(ii) A significant proportion of candidates did not set out the method of differences in a sufficiently detailed way, so that incorrect cancelling of terms occurred.

6(iii) Candidates either started the method of differences at the correct term, or subtracted the correct value from the sum to infinity.

7(i) Most understood that C_1 was a line, although the complete line thorough (2, 2) or a half starting at e.g. (-2, -2) was quite common. C_2 was frequently thought to be a circle, or a horizontal straight line. Those who recognised a vertical line usually identified its intersection with the *x*-axis correctly, a few giving the intersection at (10, 0).

7(ii) Errors in part (i) often meant that part (ii) could score few marks, while for those with a correct part (i), the shading was often incorrect. The two most common errors being to shade to the left of a vertical line or not to appreciate that the horizontal through (2, 2) was significant.

8(i) Many candidates did not realise that the sum to (n – 1), not n, needed to be subtracted and then just assumed that they could obtain the given answer. If they had been more careful, the mistake could have been identified, and possibly rectified.

8(ii) Having obtained an unsimplified expression for the required sum, many candidates expanded all terms, before trying to factorise and usually made an error in the process. Some did not use the answer from (i), which meant that the simplification and factorisation became more difficult.

9 The majority of candidates started by writing down some or all of the symmetric functions and then showed insufficient working to justify obtaining the given answer in part (i). Very few used the idea that a root satisfies the given equation and hence k can be found in terms of α.

Again, having obtained all 3 symmetric functions, many did not see the simplest way of finding the required result in parts (ii) and (iii), which often meant that algebraic errors were made. The incorrect result $(u + \text{i}v)(u - \text{i}v) = u^2 - v^2$ was seen very frequently.

10(i) Most candidates found the first 3 terms correctly and deduced a correct value for (ii) from their values.

10(iii) Some candidates tried to prove directly that u_{n+1} has a divisor of 3, with limited success. Those who showed that $u_{n+1} + u_n$ has a divisor of 3, then failed to explain clearly how this led to u_{n+1} having a divisor of 3.

4726 Further Pure Mathematics 2

General Comments:

The results of the cohort this year was very similar to that of last year with a mean only marginally higher.

We have commented before that at this level we would expect a higher standard of presentation; failure to write proper mathematics sometimes lead to a loss of marks if the essential detail of the answer is not shown. In particular this year:

Full working needs to be shown to achieve a given answer in a “show that...” question.

Care needs to be taken over sketches. If they do not illustrate the essentials needed then it is often because of a lack of care.

It was rare this series to find candidates answering a question in the space provided for another question.

Comments on Individual Questions:

1. This was usually done well and provided a solid start to the paper. It was expected that candidates would give their final answer as a single logarithm.

2. Most candidates were also able to write down the series expansion. The range of values of *x* for which the series is valid was, however, poorly done and 3 marks was the norm.

3. Most could write down the sum of areas in part (i), some expressing it using the summation notation which was commendable.

   However, in part (ii) there was considerable confusion. Some started talking about an “upper bound” (which of course is nothing to do with the question). The most common error however was to see the extra 1 as an extra rectangle. It was necessary then to change the limits of the integral. The result of this differentiation cannot be evaluated but by this stage all that could be done was to fudge it. Those that used the summation notation with upper and lower values usually realised that what they had which was equal to ½ was not the summation of the question and the first term (which was 1) had to be added.

4. Unfortunately most candidates did not understand what ”verify” meant. Those that did simply substituted the value of *x* and the same value of *y* resulted which meant that the curves both passed through the same point. The most common procedure was to equate the two curves and end up with a quadratic in sin*x* or a quartic in cos*x.* Having got this far many then simply substituted the value for *x* to show that it satisfied the quadratic. This method led candidates to decide that there were in fact two roots and took a little time to reject the other one. All of this took more space than was given and a great expenditure on time for the 2 marks available.
   In part (ii) the differentiation of $y = \cos^{-1} x$ was done well but the differentiation of $y = \tan^{-1} \sqrt{2}x$ was not. A mark was on offer, however, for coming to a decision about whether they were perpendicular or not but it was necessary to see how that decision was made. In other words, simply saying “they are perpendicular” with no work done on the two

gradients was not sufficient. It was necessary to demonstrably use $m_1 m_2 = -1$ or refer to one being (or not) the negative reciprocal of the other.

5. The finding of the asymptotes was usually well done with only a very small minority dividing the top and bottom by *x* and saying that the asymptote was *y* = *x*. In part (ii) most candidates were able to rewrite the equation as a quadratic in *x*, but not very many produced convincing arguments from their inequality. Given that the answer was given it was usual to obtain $(y-4)(y-8) \leq 0$ or $(y-4)(y-8) \geq 0$ and jump to the result without any justification. For the graph in part (iii) we expected to see the two parts of the curve approaching the asymptotes. Sometimes the asymptotes were not straight lines and quite a number of curves veered away from them.

6. In part (i), most candidates set up a quadratic equation in e^y and solved it to find e^y and hence *y*. However, only a few candidates convincingly explained why the positive root needed to be taken, often referring in a vague way to the expression needing to be positive. The best explanations were those that talked about the shape of the graph of $y = \cosh^{-1} x$. Part (ii) was often answered well, using the derivative of the inverse (*x* = cosh *y*) tended to create fewer algebraic errors than direct differentiation. In part (iii), several candidates lost the second solution.

7. Setting up the reduction formula in part (i) was quite a challenge for some candidates, either because they tried to set $\frac{\mathrm{d}v}{\mathrm{d}x} = 1$, instead of sin*x*, or because they could not differentiate $u = \sin^{n-1} x$. The result was given, however, so this should not have prevented candidates from making progress with the other two parts of the question. Candidates often gave up on part (iii) having reached values for I_9 and I_{11}, at which point they had done most of the work but only earned 2 marks out of 6. The algebra in part (ii) was often not well done with candidates using expressions in *n* instead of 2*n* and not realising that the fraction $\frac{2n}{2n+1}$ had to be less than 1 for positive *n*.

8. Candidates should label the axes of the polar graph and mark any intersections with the axes. Part (iii) was often done well, although sometimes the a^2 disappeared part way through (or worse, the dimension of each term changed) and a few candidates could not deal with integrating $\cos\theta$.

9. The accuracy specified in the question needed to be strictly adhered to. Some candidates were clearly thrown by not being asked to use Newton-Raphson in part (i). In part (ii) a clear staircase diagram was expected. A number of sketches, however, were incorrect. Part (iv) gave candidates the relationship between consecutive differences using g'(α), some candidates chose to ignore this and others used this relationship to estimate the value of g'(α) rather than differentiating g. Some candidates then used successive multiplication to try to find the appropriate value of *n* and a few realised that they could jump straight to an approximate expression for the *n*th difference by multiplying a known difference by a suitable power of g'(α). A substantial number of candidates did not really make much progress with this part (although the fact that they were attempting it suggests that there was no issue with time on the paper).

4727 Further Pure Mathematics 3

General comments

Overall this paper was found to be slightly easier than recent ones for well-prepared candidates, although it still produced a good spread of marks. Most candidates were able to attempt all questions, and the time available appeared to be sufficient. Many of the questions allowed weaker candidates to demonstrate basic techniques, but also contained parts to stretch the most able.

There continues to be a problem for many candidates with questions where demonstration or proof is required. Candidates should have their attention drawn to assessment objective 2 (AO2) and the need, particularly at this level, to "construct rigorous mathematical arguments and proofs through use of precise statements, logical deduction and inference". This was particularly relevant in questions 3(ii), 4(ii), 7(i) and 8(i).

There were, once again, many candidates who had been well prepared for this exam with a sound knowledge of each topic area; there was also a significant minority who were unable to tackle questions on more than one topic.

Comments on Individual Questions:

1.Vector notation was generally used to a good standard this year.

(i) This question was generally answered well with most successful candidates using the vector product approach. Amongst candidates solving simultaneous equations there was some carelessness in arithmetic. There are still candidates who omit "r=" from the vector equation of a line; one suspects that they lack a secure conceptual understanding of the way vector equations operate.

(ii) A significant number of candidates appear unaware that the Mathematical Formulae booklet (MF1) contains a formula for the perpendicular distance of a point from a plane. Some of those who did not use this formula were able to derive the answer using geometrical reasoning, or an alternative (learnt) standard method. The most common mistake among those who used the given formula was to neglect to consider the equation of the plane with all terms on the left-hand side; this resulted in a wrong method (of adding 4 instead of subtracting it) and consequently scored zero.

2. The weakest candidates lacked the ability to change the variable, seemingly unable to either use the chain rule or implicit differentiation. However, very many were able to correctly gain full marks by application of standard processes with which they were obviously familiar.
Having substituted, a few candidates tried an invalid separation of variables, but most attempted to use an integrating factor. Some were unaware of the need, first, to put the equation into required standard form, leaving it as

$\frac{1}{2}\frac{\mathrm{d}u}{\mathrm{d}x} - 2u = e^x$, resulting in a non-integrable right-hand side (using "IF" of e^{-2x}).

Others neglected their negative sign with the 4 when calculating the IF. Then there were those who only multiplied the left-hand side by the IF. After integration, some omitted the arbitrary constant.
In this instance, the few who used a complementary function and particular integral found this to be a much more straightforward approach.
The final mark was occasionally lost by candidates who left the solution in the form "y^2 =" and did not precisely answer the question and solve "for *y* in terms of *x*."

3. (i) Although the majority of candidates could find the roots of unity, displaying them was not as well done. Candidates remain unwilling to give appropriate care and attention to Argand diagram sketches, resulting in lack of essential detail. Good answers were marked by clear labelling of axes and roots and an explicit reference within the Argand diagram to the polar coordinates of these roots. This could be (but didn't have to be) succinctly done by means of marked angles and a circle through all roots.

(ii) Candidates' solutions were split fairly evenly between the 3 approaches given in the markscheme. Arguments were sometimes insufficiently detailed; so those using the binomial expansion method occasionally neglected to show an unsimplified stage.
Candidates at this level are expected to structure their demonstrations and proof, so those who did not include " $(1+i)^6 = ...$" were not credited with the final accuracy mark.

(iii) This question was well answered by strong candidates, but many were unable to make any headway with it. Frequently, candidates had more joy through approaching the question independently of the preceding parts, simply using the alternative method given in the markscheme.

4. It was good to see better use of braces { } to define sets but few candidates knew formal mathematical notation for equivalence when stating an isomorphism. Centres may wish to draw candidates' attention to the list of Mathematical notation given in appendix B of the specification. (Once again poor notation was not, this year, penalised here.)
A few candidates had very poor basic understanding this topic; they were either unfamiliar with modulo multiplication, or unable to identify the order of an element or of a group. Most, however, were able to access the question well.

(i) This was standard work for most candidates.

(ii) In general, candidates understood how to show that G was not cyclic by considering the order of each element and of the group, but some failed to give sufficient detail to their demonstration, with many not showing clearly why elements such as 3 had order 4. Others neglected to say what they had shown in conclusion.

(iii) {1, 3, 7, 9} was generally found to be a subgroup of order 4 by most candidates but the second set of elements was far more varied and did not always constitute a group either due to the absence of inverse elements or due to lack of closure. Many failed to comprehend the request to "state an isomorphism between (the subgroups)", but those that did usually understood the need for equivalent elements to be of the same order. Whilst it was acceptable for candidates to give two alternative complete mappings, this needed to be precisely explained and those who gave "$3 \rightarrow 13 \text{ or } 17, \text{ and } 7 \rightarrow 13 \text{ or } 17$" were not awarded the final accuracy mark.

5. This question proved straightforward to many candidates. Marks were mainly lost due to calculation errors and again it is worth stressing the need for extra caution with calculations in a multistage problem involving so many marks. The place where most calculation errors crept in was in the finding of the arbitrary constants through solving two simple simultaneous equations. However, some candidates went wrong right at the start with their solutions to the quadratic, auxiliary equation. The most common method error occurred when candidates selected an unsuitable trial function for the particular integral.

6. (i) Most candidates scored at least 2 marks, but not all fully justified parallelism; strong answers did this either by referring to perpendicularity to the normal (where they had used dot product) or by saying that the line and plane had no intersection (where they had substituted a general point on the line into the equation of the plane). Some careless arithmetic also resulted in candidates losing the accuracy mark.

(ii) The initially requested equation of the line was often correctly given although some, erroneously, thought that the direction of the line was $\begin{pmatrix} 4 \\ -1 \\ -1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$. In calculating the required point it was common, sadly, to see candidates, using a correct method, deduce that $18t = 9$ but then go on to say that $t = 2$.

(iii) This usually followed through correctly from (ii). Again, however, the mark could be lost due to absence of "$\mathbf{r} =$".

7. (i) A significant number of candidates had not learned the basic result initially requested, and gave an expression for sin θ which omitted " i". This then led to incorrect conclusions being drawn at the end of their working as they looked to "show" the given result. It was also concerning how many candidates both here and in Qu. 3(ii) seemed completely unaware of the formula for a binomial expansion, leading them to carry out lengthy, often error ridden, expansions of brackets. Centres would do well to stress that, in proofs of this nature, the binomial expansion formula can be directly applied. The best answers were precise and concise, but showed each step of the proof, which is a standard one.

(ii) Few had problems in being able to rearrange the given equation to obtain $16 \sin^5 \theta = 6 \sin \theta$, or equivalent but many then ignored the $\sin \theta = 0$ solution or, when fourth rooting, omitted the negative root. It was pleasing to see a few strong candidates considering the fact that they were solving a quintic and so were on the lookout for 5 solutions for sin θ (two of which they observed to be imaginary). It is this ability to consider a problem analytically that forms the basis of the best approaches to the problem-solving aspect of the course.

8. As in question 4, a few candidates failed to access this question due to their lack of knowledge of matrix algebra, or of basic set vocabulary.

(i) The majority of candidates knew the basic axioms for a group, although there did seem some lack of awareness that elements not only have to have an inverse, but that the said inverse must be contained within the group. So the inverse was often found or stated but rarely was it explicitly said to be within the group. Closure was considered in most cases, but with extremely few considering the required condition on the determinant. Some candidates consider $a \circ a$ rather than $a \circ b$ when exploring closure.

(ii) This question and part (iii) were generally well answered. The most common errors made were: not using general matrices, using matrices not in the form of elements of the group or attempting to show associativity in place of commutativity.

(iii) Although most candidates tackling this question did all the calculations correctly, some then claimed that the order of this element was 3 or 5.

4728 Mechanics 1

General Comments:

In the main, candidates were well prepared for the examination. The only question which seemed unfamiliar was Q6ii, where the term "contact force" was not universally understood to be the resultant of combining a frictional force and a normal component of reaction.

There were several questions in the paper which required more careful thought than a candidate might appreciate. These are indicated below. One weakness which led to common errors was in the handling of positive and negative signs. This led to significant loss of marks in

- Q1, parts (ii) and (iii),
- Q4, parts (i) and (ii),
- Q5, part (iv),
- Q6, parts (iiia) and (iiib).

Candidates dissatisfied with an initial solution should not make it illegible when crossing out their answer. Each marker encounters scripts annotated "Please ignore crossing out". In some cases it is impossible to do so.

Every question includes marks for both method and accuracy. "Method" often involves substituting appropriate values in a standard formula, and candidates are advised to show the unevaluated expression, as careless evaluation can destroy the evidence of earlier correct (substitution) work. A related problem arises when simple equations are incorrectly rearranged to give an answer.

Comments on Individual Questions:

Question No. 1
Part (i) was almost always answered correctly, as was part (ii) save for a significant minority of candidates who had the wrong sign before the term involving *g.* One unusual feature was the high proportion of candidates who rearranged the standard *suvat* equation into a form which had *u* as its subject.
Part (iii) was nearly always answered by subtracting the magnitudes of the momentum on landing and on lift-off. A minority of candidates used the initial speed of $3.5\,\text{m}\,\text{s}^{-1}$ in their calculations.

Question No.2
The familiarity of the material gave rise to many correct answers in part (i). One error was using the value of 2.5cos73.7 as *F.* This gives 0.702 as the answer which is wrong correct to 3 significant figures; however there was no penalty for the mistake on this occasion.
Part (ii) was unfamiliar, and in consequence badly answered. There was little understanding that removal of the 2.4 N force from a system in equilibrium must create a resultant force of equal magnitude with the opposite sense. Indeed, calculations often ended when the resultant of the two remaining forces had been found

Question No.3
All three parts of this question were well answered by nearly all candidates. Part (ii) had an answer of exactly 5.175, which should be left as such, but the answer 5.18 was accepted. Inevitably some answers were based on *suvat* expressions, more commonly in (ii) where integration was needed than in (iii) which used differentiation.

Question No.4
For part (i), the explanation rested on the basic facts that the total "after" momentum has a sense which is in conflict with the "before" motion of *Q*. Convincing reasons for the change in the direction of motion of *Q* came from candidates who based their case on a form of proof by contradiction.
Part (ii) was often answered well, though some solutions were based on equations implying that *Q* would continue to move in its old direction after the collision. Candidates who offered only one solution used the case where the two particles effectively coalesce.

Question No.5
In recent examinations, a (t, v) graph has been presented to candidates. It was clear that a minority of candidates used methods inappropriate to a (t, x) diagram. Others wrongly used constant acceleration formulae, in a problem where changes of velocity are instantaneous. Only the best candidates were able to solve part (iv) fully, as only they realised that a speed of 4 m s^{-1} was consistent with $v = -4$.

Question No.6
Part (i) was invariably correct, and part (ii) was done satisfactorily by candidates who knew the term "contact force" mentioned in the specification. The valid solutions offered had only one common weakness, the specification of the direction of the force. An angle without a reference line or diagram is not specific. Many candidates gave their answer as a bearing, even though the contact force lies in a vertical plane. Too often, candidates' answers were "8 N" and "vertical".

In part (iii) the initial situation was extended by the introduction of an additional force, pulling towards "the left". Very many solutions implied that this force meant the particle was still on the verge of moving to "the right". Solutions based on this notion led to candidates finding $T = 0$ after having equations which erred only in the sign of the frictional force. A simpler common misunderstanding was that the horizontal effect of *T* was to eliminate friction.

Question No.7
Many good solutions were seen for three of the four parts of this question. It was pleasing that the correct forces were used in the Newton's Second Law equations in (iii) and (iv). Perhaps it was coincidence, but candidates who drew clear diagrams and included the forces and accelerations scored particularly well. A common error among candidates who left out a diagram was to have 0.6 as an acceleration.
It was part (ii) which presented the greatest challenge. The analysis of the motion of *P* rarely reflected its having two different accelerations, 0.9 m s^{-2} before *Q* reaches *B*, but 4.9 m s^{-2} subsequently.

4729 Mechanics 2

General Comments:

The standards achieved by candidates were generally high, and few poor scripts were seen. Each question posed some challenges, and these were successfully met in the majority of solutions seen. Generally, candidates who lost marks often did this through lack of care or not answering the precise question. As before, examiners advise candidates to be spatially aware, to use clearly labelled diagrams and to take care with basic geometry and trigonometry.

Comments on Individual Questions:

Question No.1

(i) This question was generally well answered by the majority of candidates.

(ii) There were two common approaches to the solution to this question. Candidates either found the time of flight first and then the horizontal distance, or used the range formula, although this is not a requirement of the specification. The most common error was for those found the time to the greatest height first but not doubling this time when finding the range.

Question No.2

(i) The majority of candidates appreciated the position of the centre of mass of the cylinder had to be vertically above the pivot point and proceeded accordingly to get a correct *r* value. The errors arose from the incorrect use of the height as 12 cm or 3 cm in a right-angled triangle, which included *r*.

(ii) The approach to solution of this was to compare the component of the weight down the slope with the maximum friction available. The majority used this approach. However a significant number of candidates used $F = \mu R$, and ended with an inequality for the cylinder NOT to slide down the plane.

Question No.3

(i) The process of finding the centre of mass of a combined object was well known and many fully correct answers were seen. However a significant number of candidates found difficulty in dealing with the triangle, either by using Pythagoras incorrectly to find the height, or finding the area of the triangle incorrectly, for which 30 cm^2 and 65 cm^2 were commonly seen.

(ii) The majority of candidates who did not arrive at the correct answer in (i) managed to subtract their value for x_G from 8 and used their answer correctly with tan so gaining 2/3 marks. A minority used 10 instead of 5 for the denominator in the trig equation and a small minority subtracted their value for x_G from 10.

Question No.4

(i) The best approach was to take moments about the point *P*, which many attempted successfully. However some just resolved the tension and forgetting to multiply by the distance *PQ*. Some candidates thought, incorrectly, that resolving would be sufficient and so totally ignoring any forces at *P*.

(ii) The most successful method employed was to find both horizontal and vertical components of the force exerted by the wall on the rod and combine these components. However a significant number of candidates only found the horizontal component, gaining credit for the vertical component in (iii).

(iii) The formula $F = mR$ was well known and most used their values correctly to find the coefficient of friction. However some used the value of *R*, the resultant, found in part (ii).

Question No.5

(i) As this was a given answer, candidates needed to have a convincing solution to arrive at the final answer and, on the whole, they did.

(ii) This proved to be the hardest question on the paper. The solution to this question was to consider energy and many efficient solutions were seen. However there were many examples seen of candidates wrongly assuming that either speed or acceleration was constant. Candidates should be reminded that the only constant in this type of situation is power.

Question No.6

(i) Most candidates know the standard approach to this type of request. Using conservation of momentum and Newton's experimental law to produce two simultaneous equations was usually successfully done, with only a minority of candidates using speed rather than velocity in these two equations. Unfortunately 5 marks were almost as common as full marks, due to candidates giving the velocities and not the speeds as their final answer.

(ii) Many fully correct answers or scores of 3/4 obtained with follow through. Some carelessness with the masses as *m* instead of 2*m* and 3*m* was seen. Also in some instances the *m* had a tendency to disappear at the subtraction stage; in some cases, but not all, it reappeared in the final answer.

(iii) This was the least well done part of this question. Errors in signs often leading to a negative mass, with the sign ignored at the final stage, were often seen. Confusion about what mass to use was significant. The mass was frequently taken to be *m*, in a few cases 5*m* was used or a mixture of 2*m* and 3*m* used in the same equation.

Question No.7

(i) This was well answered. The majority appreciating that the tensions would be equal.

(ii) Well answered by the majority. Only a small number of candidates found angular speed and then didn't convert to speed as requested.

(iii) This part was well done by those who realised that trigonometry was to be used. Some mistakenly thought that circular motion formulae needed to be used.

(iv) The main difficulty in this part was the failure to recognise that the radius of the circular motion was no longer 0.5 m. It was common also to see only one tension used although these candidates still had access to the majority of the marks.

Question No.8

(i) Many good solutions were seen to this question. Although candidates are getting better at describing the direction relative to a fixed direction, there is still room for improvement. A simple 'below the horizontal' accompanying the angle would have been sufficient. A few candidates lost marks because they were unable to rearrange
$8 = 12t\cos 20$ correctly to obtain the value of t. A more common error was to use
$v^2 = u^2 + 2as$ instead of $v = u + at$ to find the vertical component of velocity without justifying the sign taken when square rooting to find v.

This was well done in terms of the candidates knowing what was required, but in some cases the algebra wasn't always equal to the task. A small minority of candidates made the unfortunate assumption that the target was hit at the highest point of the trajectory.

4730 Mechanics 3

General Comments

This paper proved considerably more accessible to candidates than papers in recent sessions, and very many candidates gained well over half of the marks available. Even so, quite a number of scripts scoring below 20 marks were seen.

Perhaps the main difference in the standard of the paper was that the statics question (Q5) was done completely correctly by most candidates; in recent sessions many candidates have done badly on this type of question.

Candidates should be made aware that, when a value is given in a question, they should show full working to establish this value and not miss out any important stages in their working. It is also advisable for candidates to give answers to 3 significant figures (as indicated in the rubric on the front cover of the paper) and to use at least 4 significant figures in their working.

The presentation of the scripts was extremely good in many cases, and generally acceptable, though with a few exceptions. Most of the candidates who used additional paper did so since they genuinely needed to make an extra attempt at a question.

Comments on Individual Questions

1 (i) Almost all candidates found the greatest possible speed correctly, and realised that the impulse would act parallel to the particle's original direction of motion. The least possible speed caused some difficulty, with quite a number of candidates giving an answer of $-1.2\,\mathrm{m\,s^{-1}}$, others giving 0, and a small number finding the resultant speed when the impulse acted at right angles to the particle's original direction of motion. A small number of candidates worked with the impulse acting at a general angle to the original direction of motion, and these candidates were usually successful.

(ii) Most candidates tackled this part by drawing a triangle with sides of length 0.24, 0.6 and 0.75 (or else 0.8, 2 and 2.5) and then using the cosine rule, though not all put the sides in the right order, and not all found the right angle. However, a significant number of candidates answered the question by resolving momentum, either parallel and perpendicular to the particle's original direction of motion, or else parallel and perpendicular to the impulse. A small number of candidates using this second approach wrongly assumed that only one angle was required in their calculation. Almost all successful candidates clearly showed the required angle, often on their initial sketch.

2. (i) This question was done correctly by almost all candidates, though some made algebraic mistakes in moving from their original energy equation to a three term quadratic equation. A small number of candidates found the equilibrium position; while most then left this as their answer, a very small minority then went on to find the correct answer.

(ii) While this was generally done correctly, not all candidates remembered to take account of both the distance travelled of 1.3 m and the original length of 0.6 m in working out the extension of the string to be used. Just a few candidates either omitted the gravitational term completely or failed to make clear the direction of the acceleration.

3 (i) This part proved one of the most accessible on the paper, with most candidates working out the speed of *B* along the line of centres after the impact as $1.5\,\mathrm{m\,s^{-1}}$ and then correctly using Newton's experimental law. The most common errors were to omit the mass of *B* in the term for the momentum of *B* after impact, failing to use the fact that *A* was stationary after the impact and using the sum of speeds along the line of centres before impact, rather than the difference, in Newton's experimental law.

(ii) This question was done far better than similar questions have been done recently. Just a few candidates stopped after finding the angle the motion of *B* made with the line of centres after the impact, and a small number gave the answer as 165°.

4.(i) This question was done well, with most candidates successfully establishing the given answer. There were indications that some candidates had sensibly used the given answer to go back and correct slips in their working. However, quite a number did not give an explicit expression for $\frac{\mathrm{d}v}{\mathrm{d}x}$ which was asked for in the question. Most dealt competently with the arbitrary constant; those using limits for the integration found the answer rather more efficiently.

(ii) This part proved a little more challenging than part (i), and gave rise to quite a range of methods. Many started with the equation established in part (i), others started with $\frac{\mathrm{d}v}{\mathrm{d}t} = -4v^2$ and established that $\frac{1}{v} = 4t = \frac{1}{2}$. From here, some integrated again, others substituted $t = 0.5$, while others now used the expression for *v* in terms of *x* from part (i). Whichever of the two main methods was used, some candidates went wrong by omitting the arbitrary constant.

5. (i) Candidates found this statics question easier than those in recent years, though there were still a few who made no meaningful attempt at this question. Most candidates took moments about *A*, as hinted at in the question, with many of them correctly dispensing with cos60°, and even *L*, at an early stage to reach the answer very quickly. A few took moments about *B* for the whole body to find an equation involving the reactions at both *A* and *C*, but generally solved this correctly by using the result of resolving vertically for the whole body. Others first took moments about *B* for *AB* and *BC* separately, thus involving *T*; full marks were available for those who correctly established the given result. A small number of candidates gained only 1 mark for the force acting at *A*.

(ii) Most candidates were successful here, though some made errors in resolving forces.

(iii) This part was usually correct, though sometimes on follow through from part (ii). A small number of candidates got the directions of forces wrong, or else did not make the directions clear.

6. (i) Most candidates realised that *P* and *Q* would collide at the lowest point of the cylinder, with equal speeds. However, a number of candidates, having realised that the particles would collide at the lowest point, correctly worked out the speed of *P*, but made an error in the speed of *Q*, usually through having the mass of *Q* as *m* in the expression for KE and 5*m* in the expression for PE. Some follow through marks for the rest of the part were available to these candidates. A small number of candidates worked with the angle that a line joining O to the point of collision makes with the vertical; these candidates generally made no worthwhile progress. A small number of candidates were completely correct, except for not making clear the direction of motion of *Q* after the collision.

(ii) This part was done very well. The most common errors were to use the wrong height from the bottom of the cylinder to *D*, and to miss out the weight term when working out the normal reaction acting on *P* at *D*. A small number of candidates worked this out in terms of a general angle, putting that angle equal to 180°, usually correctly, at the end.

7. (i) Most candidates did this completely correctly, though not all candidates showed all the values needed to calculate the extension.

(ii) Most candidates successfully established SHM, usually using (x + 0.24) as the extension of the string, following on from the calculation in part (i). In a small number of cases there were errors or slips in the working, though most candidates ended up with the right value for 'ω^2', and were able to access the marks for establishing the period and amplitude. Some candidates lost a mark by failing to point out that an equation of the form $\ddot{x} = -\omega^2 x$ represents SHM. Some candidates worked with energy, which if done for a general point followed by differentiation with respect to x can establish SHM, though very few correct solutions of this form were seen. Most candidates who used energy used it to find the amplitude of the motion, and were then able to access the marks for showing that the period was 2.20 s.

(iii) This was done well by candidates who had been successful in part (ii). Some follow through marks were allowed for candidates who had calculated the amplitude wrongly. Not all candidates went on after finding the displacement at time 1.5 s to find the distance of *P* from *O*. It should be pointed out that candidates using $v^2 = w^2(a^2 - x^2)$ to find the speed of P needed to establish the direction of motion if they were to gain the final mark.

4731 Mechanics 4

General Comments

The work on this unit was generally of a high standard and many of the candidates were very competent and well prepared. These candidates demonstrated a sound understanding of all the principles of mechanics covered in this module. However, a small number of candidates struggled with the majority of the paper, seeming to be unprepared. Candidates seemed to be particularly confident when using calculus to find the centre of mass, applying conservation of energy, finding the force acting at an axis, and using energy to investigate stability of equilibrium. Topics which were found more challenging included relative velocity, using calculus to find the moment of inertia and applying the principle of conservation of angular momentum. The majority of candidates appeared to have sufficient time to complete the paper. The standards of presentation and communication were high, though some candidates failed to include necessary detail when establishing given answers.

Comments on Individual Questions

Question No. 1

Relative velocity remains a difficult and challenging topic for some candidates, however, there were many who answered this question correctly. The most succinct and efficient solutions were from those candidates who adopted a velocity triangle approach to find the bearing required for interception in part (i). Part (ii) was also well answered with about equal number of candidates using either the sine rule or cosine rule to calculate the magnitude of the required relative velocity. Almost all candidates who correctly calculated the velocity went on to find the time taken for interception.

Question No. 2

In part (i) many candidates correctly derived, by integration, the moment of inertia of a cone about its axis of symmetry and the presentation of these candidates' work was generally sound. However, a number of candidates seemed unprepared for a question of this nature and didn't progress any further than simply stating the mass per unit volume of the cone. A number of candidates had issues with deriving or stating the equation relating y and x or didn't realise that such a relationship was required. Those candidates that had the vertex of the cone at the origin were far more successful than those that candidates who had their origin at the centre of the base of the cone. Furthermore, in deriving the moment of inertia of the cone, a number of candidates assumed that the radius of the cone was equal to its height.

In part (ii) the constant deceleration and in part (iii) the time taken, were almost always found correctly.

Question 3

Most candidates found the exact value of the x coordinate of the centre of the mass of the lamina bounded between the two curves correctly. Two approaches were generally seen, the first was to find the centre of mass of the lamina by considering the difference between the two curves, and the second was to consider the centres of masses of the two laminae separately. The former approach was generally more successful. Most candidates were proficient in carrying out the integration by parts although sign errors were seen in the integration of the trigonometric terms.

Question 4

In part (i) most candidates correctly found the moment of inertia of the square lamina by applying either the perpendicular or parallel axis rule. Almost all candidates then went on to derive the correct form for the angular speed given in part (ii). In part (iii) many candidates went on to form the correct equations of motion for X and Y. The most common mistakes included sign errors, omission of the mass and either omission, or an incorrect, radius. However, the radial and transverse accelerations were almost always correct.

Question 5

In part (i) the moment of inertia of the pendulum was almost always found correctly. In part (ii) many candidates did not realise that the correct approach was to apply conservation of angular momentum and a significant number attempted to use an energy approach. Those that did realise that angular momentum was required usually went on to derive the given result correctly. Candidates found parts (iii) and (iv) the most demanding parts on the whole paper and only a few succeeded in getting both parts correct. While the majority of candidates realised that the correct approach was to apply the conservation of energy there were numerous errors seen. These included not calculating the correct position for the centre of mass of the pendulum, using m rather $6m$ for the mass of the pendulum, and using the mass rather than the moment of inertia in the calculation of the kinetic energy.

Question 6

In part (i) most candidates derived the given expression for the total potential energy correctly with only a small number having difficulty with calculating the extension in the string. Establishing the position and stability of equilibrium in part (ii) was very well done although some candidates did not show sufficient detail in establishing the given result. A small minority, while correctly showing the position of equilibrium, did not go on to show that this position was stable. In part (iii) the method of differentiating the energy equation to obtain the given equation of motion was quite well understood with the majority of candidates earning full marks for this part. Part (iv) was generally well done with the majority of candidates correctly applying the given substitution and going on to show that the motion of the system was approximately simple harmonic. The most common mistakes in this part were due to errors in the expansion of the trigonometric terms, failing to substitute for $\ddot{\theta}$, not applying the small angle approximation and not stating that the motion was (approximately) simple harmonic.

4732 Probability & Statistics 1

General Comments

This paper was found fairly accessible by most candidates. A few parts contained relatively non-standard requests (for example 4(ii), 6(ii) and 7) and some candidates could not handle the slightly different approaches that were needed. In particular, question 7 involved two-way tables which have rarely been set in this examination. For that reason the question was designed to be relatively simple, with all the probabilities capable of being read off from the table almost immediately. However, many candidates tried to use multiplication of probabilities and generally failed. Another unusual question was 6(ii), on Spearman's rank correlation coefficient. This was not the usual "turn the handle" calculation but required some thought. As a result fewer candidates than usual scored well on this question.

The questions that required an answer given in words were fairly well attempted, except for question 5(v) which was not well understood on the whole.

A few candidates lost marks by premature rounding or by giving their answer to fewer than three significant figures without having previously given an exact or a longer version of their answer. It is important to note that although an intermediate answer may be rounded to three significant figures, this rounded version should not be used in subsequent working. The safest approach is to use exact figures (in fraction form) or to keep intermediate answers correct to several more significant figures.

Few candidates appeared to run out of time.

In order to understand more thoroughly the kinds of answers which are acceptable in the examination context, centres should refer to the published mark scheme.

Use of statistical formulae and tables

The formula booklet, MF1, was useful in questions 4 (for binomial tables), 5 and 6. Candidates generally used the formula booklet more successfully than has sometimes been the case in the past. In question 5 very few candidates quoted their own (incorrect) formulae for r and/or b, rather than using the ones from MF1. In question 5, a small number of candidates thought that, eg, $S_{xy} = \Sigma xy$ or $\Sigma x^2 = (\Sigma x)^2$. Very few tried to use the less convenient versions, $r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2\,\Sigma(y-\bar{y})^2}}$ and $b = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$ from MF1. Almost all the candidates who did use these formulae misunderstood the Σ notation. For example they interpreted $\Sigma(x - \bar{x})^2$ as $(\Sigma x - \bar{x})^2$.

In question 6, Σd^2 was sometimes misinterpreted as $(\Sigma d)^2$ and the formula was sometimes misquoted as $\frac{6\times\Sigma d^2}{n(n^2-1)}$ or $\frac{1-6\times\Sigma d^2}{n(n^2-1)}$ or $1 - \frac{6\times\Sigma d^2}{n}$ or $1 - \frac{6\times\Sigma d^2}{n^2(n-1)}$.

In question 4, some candidates' use of the binomial tables showed that they understood the entries to be individual, rather than cumulative, probabilities. Also in this question, a few candidates tried to avoid using the table, which is quite understandable, but leads to a very long method in 4(i)(b).

Use of calculator functions

Increasingly nowadays, calculators can provide answers using statistical functions, binomial functions etc etc, without the need to quote a formula and substitute values into it. The problem here is that if candidates write down only an answer, they can only score either full marks or no marks, with no possibility of gaining any credit for partially correct working. In most cases, the use of such functions saves very little time and it is advisable to show working instead. However, if candidates wish to use these functions they should input all the relevant data twice in order to check their answer.

It should also be noted that there are sometimes questions in which a correct answer without working may not gain full marks.

Comments on Individual Questions

1.(i) Candidates used a variety of methods, many of them incorrect. In particular, the use of $\frac{n}{4}$ and $\frac{3n}{4}$ instead of $\frac{n+1}{4}$ and $\frac{3(n+1)}{4}$ in finding the quartiles was a common source of error. The latter method, although correct, requires interpolation, which some candidates failed to do correctly. The most successful candidates used the simplest method for the quartiles. This method takes the lower quartile to be the median of the lower half of the values (excluding the middle value if there is one). The IQR, using this method, is found by 7.75 – 6.7 = 1.05. Some candidates misread the key and gave answers which were 10 times the correct ones.

(ii) Most candidates ordered the digits correctly, but many failed to align them properly. These candidates seemed unaware that one of the points of a stem-and-leaf diagram is to illustrate the general “shape” of the distribution, which depends upon the leaves being of the correct lengths, i.e. the digits being aligned correctly. In some cases, misalignment was caused by crossing out incorrect work and replacing it with correct digits, but in the wrong place. A few candidates, faced with the dilemma of how to align properly after crossing out, started a new diagram. This was acceptable, so long as the leaves for B were on the left of the stem. Wise candidates firstly drew a rough diagram on the left hand side of the answer space, and then gave their final version in the place expected. Many candidates gave the key incorrectly, not appreciating that for the digits in the left hand half of the diagram, 2 | 5 means 5.2, not 2.5. Others simply omitted the key. The digits 4, 2, 2 were often seen in the second row instead if the first.

(iii) To be sure of gaining both marks in questions of this type, candidates should follow the following guidelines: 1 Always refer to the context. 2. Give answers that refer to the groups as a whole, rather than to individual values. 3. Give one answer about size and one about spread. Many candidates fell down on one or more of these criteria. While it is not absolutely impossible to gain the marks without adhering to these guidelines, it is extremely difficult to give a convincing answer that does not do so.

2 (a) Many candidates calculated the mean rather than spotting that its value is obvious by symmetry. Otherwise, this part was answered well on the whole. A few candidates omitted “ $-\mu^2$”, and some gave “ $-\mu$”. Others made the cardinal error of dividing by 3. Some spurious attempts at *npq* were seen.

Candidate who used $\Sigma(x-\mu)^2p$ were generally successful, because the arithmetic in this particular question is simple. However, centres should note that in most question of this type this method is cumbersome and is rarely completed successfully by candidates, whereas the use of $\Sigma x^2p - \mu^2$ tends to produce the correct answer very easily. A few candidates began with a correct method, but at some point divided by 3, resulting in the loss of all the marks.

(b)(i) This part was also answered well. In questions like this one, where the answer is given, the main problem is ensuring that a complete solution is given. For example, the solution 2 + 3 + 4 + 5 = 14, hence $k = \frac{1}{14}$, was deemed to be insufficient. The intermediate statement $14k = 1$ was required. A few candidates used a verification method, which was accepted.

(b)(ii) A few candidates gave probabilities of $\frac{1}{14}$, $\frac{2}{14}$, $\frac{3}{14}$, $\frac{4}{14}$, effectively using the formula kx, rather than $k(x + 1)$. Others had all four probabilities equal to $\frac{1}{14}$.

3.(i) Despite this question being in principle straightforward, candidates found many false paths down which to travel. Some simply made arithmetical errors. Others divided by 5 or 6 or by Σx, instead of Σf. Some used Σx instead of Σfx, and Σx^2 instead of Σfx^2. Some found the mean correctly, but used $\sqrt{(\Sigma fx^2 - \bar{x}^2)}$ for the standard deviation. A few found $\Sigma(fx)^2$ or $(\Sigma fx)^2$. The last class caused difficulty for some candidates. The instruction was to treat "5 or more" as "5 or 6", This led some candidates to find two values of fx for this class, one for 5 and one for 6. Some of these halved the class frequency, giving a plausible method. Others did not. A few failed to take the square root at the end. Candidates who attempted to find the standard deviation using $\Sigma(x - \bar{x}^2)f$ became lost in the arithmetic.
Probably the safest method, both for achieving correct answers and for enabling examiners to understand candidates' working, is to complete a table showing the values of x, f, fx and fx^2, and the totals for the last three columns.

(ii) Some candidates gave values taken from the second row of the table instead of the top row. Others tried to interpolate, giving answers such as 2.3 and 3.6.

4. Many candidates used incorrect versions of $\frac{2}{3}$, such as 0.6, 0.66 and 0.666. Some did so consistently throughout the question. Others varied their version of $\frac{2}{3}$ in different parts. Some of those using 0.66 had to resort to the column for 0.65 in the binomial table. When fractions are so easily handled by calculators, it is difficult to understand why candidates shy away from using $\frac{2}{3}$. A few actually appeared to believe that $\frac{2}{3} = 0.6$, but others seemed to be (consciously or unconsciously) using a rounded decimal.
A few candidates treated the situation as geometric rather than binomial.

(i) (a) Some candidates omitted the binomial coefficient. Others just read a single value (0.6228) from the table.

(i)(b) Only candidates using $\frac{2}{3}$ and using the tables succeeded in this part. A few found 1 – their answer to part (i)(a) or 1 – 0.6228 (from the tables).

(i)(c) Those who used np and npq generally gained both marks. Those who tried to use Σxp and $\Sigma x^2p - \mu^2$ generally gained neither mark.

(ii) Despite the broad hint, few candidates seemed to understand what was required, and the simple method using B(27, $\frac{2}{3}$) was seen infrequently. Some attempted to list some triples which add up to 18, but soon gave up. Many of these failed to find the probabilities of their triples, which could have gained them a mark. A few thought that just 6, 6, 6 was enough. Others counted triples, and those who were persistent arrived at a probability of $\frac{55}{729}$. Unfortunately this method ignores the fact that the triples are not equally likely, and so gained no marks.

5(i) This part was well answered by most candidates. A few made the errors mentioned above. A significant number made an arithmetical slip leading to S_{xx} = –66 instead of –65. Over rounding was frequently seen in this question, i.e. '0.96'.

(ii) Many candidates failed to mention the context. Others stated that the value of *r* shows "negative correlation" between price and distance, omitting to include "strong" or an equivalent. A few candidates simply discussed the nearness of points to a line of regression. Some candidates thought that a high negative value of *r* means that there is very little correlation.

(iii) Most candidates answered this part correctly, although some did not appreciate the difference between "affect" and "effect". They were not penalised for this non-mathematical error.

(iv) Like part (i) this was generally well answered. Some candidates started from scratch, not appreciating that they had already found the values of S_{xx} and S_{xy} in part (i). A few of these, having found the *S* values correctly in part (i), tried to use the formula for *b* from MF1 ($b = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$) which they clearly did not understand. Some used incorrect values such as *Sxy* = 241. Others made errors in the sign or the decimal point or rounding. Subtracting a negative to find '*a*' caused problems for some candidates.

(v) Many candidates answered correctly. A few gave inadequate statements, such as that *y* is dependent on *x*. Others gave incorrect statements such as that *y* is independent or that *x* depends on *y*. Some wrote "because *r* is close to – 1". A large number stated that this would be a better method because no rearrangement would be necessary.

6. (i) Almost all candidates gave the correct answer. A very few gave 1, 2, 3, 4, 5, 6. An even smaller number gave 2, 1, 4, 3, 6, 5 or 0, 1, 2, 3, 4, 5.

(ii) A good number of candidates understood this rather non-standard question and gained full marks. However, the errors mentioned above were seen. But the most common error was to take the ranks from part (i) and use them in this part, giving $r_s = \frac{1}{7}$. Some considered only the two teams G and H. A few tried to work backwards from an *r*-value of –1 (or even 1) and find d^2. Then they added 2 to this and proceeded to use the formula. *n* = 6 was used by a few candidates.

7. (i) Although many candidates had no problem with this part, a disappointing number appeared to have trouble coping with simple ratio and proportion. A common incorrect response was $\frac{5}{8} \times 45 = 28.125$, rounded to 28. Another common response was $n = 45 \times \frac{8}{5} = 72$.

(ii) This is a simple question requiring the addition of three numbers (or subtraction of one number from the total), followed by division by the total. However, presumably because of unfamiliarity with two-way tables, only a minority of candidates used this approach.

Some candidates tried to use multiplication of probabilities, i.e. P(female)×P(adult) + P(female)×P(child) + P(male)×P(child). These candidates generally failed to realise that the second probability in each product must be a conditional probability. A correct version of this method is possible, but this method is unnecessarily long and was rarely carried out correctly.

Many candidates added the frequencies (or the probabilities) for females plus children, thus including the female children twice. Others added females plus children plus female children, thus including the female children three times.

Perhaps these mistakes were prompted by a misunderstanding of the phrase "or both" in the question. A few correctly added females plus children and subtracted female children.

(iii)(a) Many candidates attempted P(female child) × P(adult male). Most of these made one or both of two errors: omission of "2 ×" and/or failure to reduce the second denominator to 24. Thus $\frac{4}{25}\times\frac{6}{25}$ and $\frac{4}{25}\times\frac{6}{24}$ and 2 × $\frac{4}{25}\times\frac{6}{25}$ were frequently seen.
A correct method using combinations was seen, but some candidates added ${}^{10}C_1$ and ${}^{6}C_1$ in the numerator instead of multiplying. Others had a denominator of 25 instead of ${}^{25}C_2$.

(iii)(b) Mistakes made in part (i) were carried over into this part. Some candidates found P(female child) × P(adult male), but not P(adult female) × P(male child). Some very common errors involved such working as P(female) × P(child) = $\frac{14}{25}\times\frac{15}{25}$.

8. (i) This part was found difficult by many candidates. Most used a combinations approach, but were unable to deal correctly with including two people while excluding another. Most failed to realise that the other two people must be chosen from 5 people, rather than 8 or 6 or 4. These candidates often used a correct denominator (${}^{8}C_4$) but few used the correct numerator (${}^{5}C_2$).
Some candidates used a probability approach and many of these got as far as $\frac{1}{8}\times\frac{1}{7}\times\frac{5}{6}\times\frac{4}{5}$, but not many went on to multiply by the correct figure of 12 (from ${}^{4}C_2$ × 2 or 4! ÷ 2).

The above methods consider the four *places* to be filled. A method that considers the three *people*, rather than places, was attempted by a few candidates. But most of these only gave something like $\frac{1}{8}\times\frac{1}{7}\times\frac{5}{6}$ and very few arrived at the correct product ($\frac{4}{8}\times\frac{3}{7}\times\frac{4}{6}$).

(ii) Here the key point is to "stick" Kathy and David together making them one element, and then to fix Harpreet, leaving 6 elements to arrange. Thus the key part of the working is either 6! or 5! × 6 (which is them multiplied by 2, to swap Kathy and David around). Many candidates used 5! rather than 6!, (e.g. 5! × 2). Others used 7! or 8!.

(iii) Here again, 6! or 5! × 6 is a crucial part of the working, but many candidates used 5! instead of 6!. Common errors were 5! × 3!, 5! × 2 × 2 and 5! × 4 × 5! Rather better, but not fully correct, was 6! × 2. Also seen were 6! × 5! and 6! + 4!.

9. (i) This question was well answered. A few candidates gave $0.7^5 \times 0.3$.

(ii) Candidates who used the "straightforward" method [i.e. P(X = 2, 3 or 4)] were generally successful, although a few added an extra term. Those who tried a more sophisticated method [i.e. P(X < 5 – P(X < 2) or P(X > 1) – P(X > 4)] generally made errors such as $(1 - 0.7^5) - 0.3$ or $(1 - 0.7^4) - 0.7$ or $0.3 - 0.7^4$. A very common error was P(X < 5) × P(X = 1).

(iii) Many candidates attempted the standard method for this type of question, i.e. $1 - 0.7^r$, but some had r = 5 or 7 instead of 6. Others correctly found $1 - 0.7^6$, but then multiplied by 0.3. Those who attempted the "long" method often omitted a term or added an extra term. Some added terms to their answer(s) to part(s) (i) and/or (ii), but many of these effectively omitted a term or counted a term twice.

(iv) Disappointingly few candidates made use of their answer to part (iii). Some started from scratch, and most of these were unsuccessfully, although a few correctly found $(0.7)^6 \times (0.7)^6 \times (1 - 0.7^6)$ or $(1 - 0.7^{18}) - (1 - 0.7^{12})$. Many misunderstood the question completely and gave $0.7^2 \times 0.3$.

4733 Probability & Statistics 2

General Comments:

This paper was found to be generally accessible. Many excellent scripts were seen, and a large number of candidates were able to get correct numerical answers to most of the questions. Conclusions to hypothesis tests were generally well stated, although it is emphasised that no quantity of evidence can ever demonstrate that the null hypothesis is true. If the conclusion of a test is "do not reject H_0" ("accept H_0" is equally permitted, though perhaps less good practice), it is wrong to say "there is significant evidence that the mean time is 80 minutes"; the correct statement requires a double negative, for example "there is insufficient evidence that the mean time is not 80 minutes".

As usual there are quite a few candidates who seem unable to engage with the questions that require demonstration of understanding, instead attempting to give verbal answers using familiar phrases that they have seen before. Questions and mark schemes are not sympathetic to this approach. Weak candidates often answer last year's question rather than this year's.

There are persistent misunderstandings as to when the normal distribution can be assumed. Some think that all continuous distributions are normal; some think that any distribution that is not binomial or Poisson must be normal; some think that the central limit theorem applies only to distributions that are normal in the first place. In any case, candidates are advised to read each question carefully to identify whether a particular distribution is stated in the question.

Two topics that continue to be weak every year are the Central Limit Theorem (question 7(iii)), and the concepts and notation for probability density functions (question 5(v)).

This year there were more candidates who obtained most answers by putting numbers straight into their calculator. There were quite a lot of wrong answers unsupported by any working, and these can get no credit even if they are nearly right. In particular, candidates would be well advised always to show any standardisation steps (such as $(40.5 – 35)/\sqrt{10.5}$ in question 1). The wider availability of calculators that will, for instance, produce test statistics directly has an impact on examination papers; questions have to be set and marked so that candidates without such calculators are not disadvantaged.

Centres are reminded that knowledge of the specification content of Modules C1 to C4 and S1 is assumed, and that candidates may be required to demonstrate such knowledge in answering questions in unit S2. Synoptic assessment is a significant part of the assessment of S2.

Comments on Individual Questions:

Question No.

1 This was for most a straightforward start to the examination, and it was pleasing to see so many correct uses of the continuity correction.

2(i) Most candidates knew conditions for approximating the binomial distribution with either the normal or the Poisson, though answers were often poorly expressed; if conditions involving *np* and *nq* are used, the numerical values of these should be stated (here 147 and 3), while the use of *npq* (2.94) is wrong. Weaker candidates attempted to give answers about *modelling assumptions* for the Poisson and normal distributions ("events do not occur independently", etc), instead of the conditions for *approximations* to the distribution to be valid.

2(ii) A pleasingly large number answered this successfully. The most common mistake was to calculate 1 – P(< 4) rather than 1 – P(≤ 4). Candidates who attempted to use the exact binomial distribution scored no marks; the question required "a suitable approximation". Some attempted to use a normal approximation even though in the previous part it had been stated to be invalid.

3(i), (ii) Placing a question about finding normal parameters in the context of a frequency distribution did not worry most candidates, though some inevitably attempted to use a factor of √100 in the standard deviation. Signs were generally well handled, and the correct answers were often seen. Only a few used 0.17 or 0.25 in their equations, but continuity corrections in this context continue to be seen quite often. If the distribution is stated to be normal then a continuity correction must not be used.

3(iii) Many candidates realised that the probabilities were based on only a sample rather than on the whole population. However, there were also many who attempted to use a familiar answer to a different question, namely the routine answer to S1 questions about why calculations of sample mean and variance were not exact: "you don't know the exact data values, only the ranges". Others said "it's only approximately a normal", even though it was clearly stated in the question that the distribution *was* normal.

4(i) Most gave a correct statement such as "snakes must be independent of one another", though some showed confusion by saying things like "snakes and land must be independent of one another". It was pleasing that few gave regurgitated answers such as "randomly, singly, independently and at constant rate", which as usual did not receive credit. Nor did meaningless statements such as "snakes must occur in a fixed period of time and space".

4(ii) Almost always correct.

4(iii) Very often correct. Only a handful attempted to multiply the two probabilities, or to present them as separate answers. A few thought that λ was 4/0.77 instead of 4×0.77.

5(i) A large number of candidates could integrate $\sin(\pi x)$ correctly, even if they found that some subsequent adjustment of the negative sign or the factor of π was necessary, and most could demonstrate that the area under the curve was 1. Almost all, however, omitted the other condition for a function to be a PDF: it must be non-negative for all values of x.

5(ii) Most knew the correct shape of the graph, though some drew either not enough of a sine wave or too much. Some candidates tried to use integration in order to find E(X), some realising later that they could and should simply have written the answer down. A common mistake was to give the y-coordinate of the maximum, $\pi/2$, instead of the x-coordinate, ½.

5(iii) The common mistake here was to equate the integral between 0 and q to 0.75, rather than the integral between q and 1. Otherwise many correct answers were seen.

5(iv) Most candidates scored the first mark; some lost the second one by omitting either the limits of the integral, or the value of $[E(X)]^2$.

5(v) As usual this type of question revealed severe misunderstandings as to the nature of a PDF. Many candidates clearly think that X and x represent completely different things, a misunderstanding that can perhaps be summed up by a typical (wrong) expansion of the mis-statement in the question: "Whether or not X happens depends on how close to ½ the value of x is". Examiners were looking for evidence that candidates understood that X takes numerical values, and/or that x merely represents a possible value that X can take. Centres might find it advisable to focus on the two facts that (i) X stands for a number, not an event; and (ii) x represents merely a number that X could be.

6(i) Most realised that the sample had to be a random one. Those who used the word "representative" sometimes indicated, wrongly, that it had to be representative of the city rather than of the company. Some even negated the point of the test by stating that the proportion from group *Z* had to be the same in the company as in the city.

6(ii) Many candidates do not appreciate the difference between "select numbers randomly" and "select using random numbers". The word "randomly" in the former context is not specific and gives no indication of how the selection is to be done, whereas "random numbers" represents a specific mathematical concept, knowledge of which is required by the specification.

6(iii) This question was often well done, perhaps because finding the relevant probability for a left-hand tail is easier than for a right-hand tail. Few made the error of finding P(< 2) or P(= 2) as opposed to the correct P(≤ 2). Those who used the critical region method generally did so correctly but this method always needs validation by displaying the relevant probability (here 0.0196).
Too many stated the acceptance of H_0 meant that there is significant evidence that the proportion from group *Z* is 40%. This is wrong. The correct statement is that there is not sufficient evidence that the proportion from group *Z* is less than 40%.

6(iv) This question could be successfully answered either in hypothetical terms ("it is more likely that the null hypothesis is rejected") or in terms of the actual sample ("the null hypothesis is now rejected"). Some thought that the issue was whether causality could be proved – but once again that is answering last year's question and not this one.

7(i) It was perhaps indicative of candidates' over-rigid ways of answering questions that many omitted the $n/(n - 1)$ factor for the variance in this part, yet went on and used it in the more familiar context of part (ii). More predictable was that many attempted to use a $\sqrt{n}$ factor in the standard deviation in this part, where it is wrong. However, the correct answer was often seen.

7(ii) Many answered this question very well, although relatively few achieved all 7 marks. Those who omitted the $\sqrt{50}$ factor here lost 4 marks, as did those who stated their hypotheses in terms of 81 and not 80 (a serious mistake emphasised in all recent Reports to Centres). Those who used a critical region often centred it on 81 rather than 80; were these S3 candidates who had confused the method with confidence intervals? The same comments about the need to state the conclusion properly apply as in question 6(iii).

7(iii) As usual a question that tests understanding of the Central Limit Theorem was poorly answered. "No, yes" was more common than the correct "yes, no" (+ reason). Many said that you didn't have to assume a normal distribution in part (i) as *n* was large; clearly they had not realised that in part (i) we are talking about probabilities for a single observation. These candidates often gave "yes" as their answer to (b), presumably on no better grounds than expecting the two answers to be different. Another common wrong answer to (a) was "no as we know it is normal"; Examiners find it hard to account for the misconception here.

8(i) A pleasing number got the correct answer of 0.0273, or 2.73%, although some insisted on converting that to a more familiar value such as 5%. Some could not decide whether the answer was 2.73%, 12.57% or somewhere between.

8(ii) The correct answer of 1.61 (1.609…) was seen pleasingly often, although many used P(W = 0) = 0.8, rather than P(*W* > 0). The use of tabulated values is not a sufficiently accurate method.

4734 Probability & Statistics 3

General Comments:

There were 360 candidates, similar to recent years. The candidates were usually well-prepared. There were four significance tests, the candidates answering the numerical parts well, but statements of hypotheses were not as good. Hypotheses must use standard symbols or the word 'population'. Conclusions have improved in recent years. Most candidates now use the standard structure e.g. "[test statistic] > [critical value], so reject H_0. There is sufficient evidence that ...". There were fewer over-assertive conclusions. Questions involving algebra and calculus were usually answered well.

Comments on Individual Questions:

Question No.

1(i) Most candidates answered this part correctly. Var (Z) = ½(16) + 2 and ¼(16) – 2 were the common wrong answers.

1(ii) This question was not answered well. This is disappointing, because a similar question was set in 2011. If centres used this question for revision, most candidates missed the point. The fact that E(Z) = Var(Z) does not necessarily make the distribution Poisson. Similarly, an incorrect Var(Z) in part (i) usually led to the incorrect answer "No, because E(Z) ≠ Var(Z)". Correct responses, pointing out that it was possible for Z to take negative or fractional values, which are not possible for a Poisson distribution, were rare.

2 Over half the candidates scored full marks. The test statistic was almost always correct, but there were occasional errors in the critical value.

3 About one-third of the candidates scored full marks. Errors included inadequate hypotheses, failure to use √5 in the standard error and incorrect critical values. A few candidates tried to compare a probability to 0.05, rather than use a test statistic/critical value argument. They were almost always unsuccessful.

4 More candidates scored 3 out of 7 than any other mark. This was due to an incorrect variance. 114, 242, 392, 464 and 976 were all widely seen. However, about one-third of candidates did score full marks.

5(i) Most candidates scored full marks on this question. The most common error was to use an incorrect z-value, often 3.09.

5(ii) Most candidates earned both marks. Sociological answers were more common than statistical ones.

5(iii) Most candidates earned full marks. Common errors were to use 0.05 instead of 0.025, to use an incorrect z-value and failure to give final answer as an integer.

6(i) Almost always correctly answered. $\int \sin x = \cos x$ was the most common error.

6(ii) The same error was also seen in this part. However, most candidates scored full marks.

7(i) A surprisingly large number of candidates failed to score this mark. "H_0 : there is no association between ... and ..." or "H_0: ... and ... are independent" should be used.

7(ii) Most candidates scored full marks, but many said that one of the observed values was < 5 so combination of rows or columns was necessary.

7(iii) Almost all candidates scored full marks.

7(iv) Many candidates did not score both marks. An incorrect critical value was the usual reason, but some candidates said "10.51 > 9.488, reject H_0" and nothing else.

8(i) Most candidates scored this mark.

8(ii) Over half the candidates scored 9 out of 9. A few lost marks for not calculating the pooled variance. Others had an incorrect critical value.

9(i) Most candidates scored full marks, but some scored only one mark for $A = 10x - x^2$. They then omitted $A = -(x^2 - 10x)$, jumping straight to the given answer, which was not acceptable.

9(ii) Most candidates answered this part correctly.

9(iii) The most common mark was 3 out of 5, for $F_X(x)=\frac{1}{2}x$, satisfactory explanation of the change of limits and $P(A \le a) = P(25 - (X - 5)^2 \le a)$. Most obtained $P((X - 5)^2 \ge 25 - a)$ but did not go on to say that this was equal to $P[X \ge 5+\sqrt{(25 - a)}$ or $X \le 5 - \sqrt{(25 - a)}]$ and to explain that the first inequality was impossible, or an equivalent statement.

9(iv) Almost all the candidates knew that they had to differentiate the answer given in part (iii). Some did not do this correctly, obtaining ½, -½ or -¼ instead of ¼.

4735 Probability & Statistics 4

Comments on Individual Questions:

1(i) Almost all candidates scored at least 6 out of 7. Those losing a mark lost it for inadequate hypotheses or conclusions. It is pleasing that most candidates now conclude by saying 'TS>CV, do not reject H_0 so there is insufficient evidence that the second paper was easier.'

1(ii) Most candidates scored this mark, but many did not mention 'differences' in their answer and so failed to score.

2(i) Almost all candidates scored full marks.

2(ii) Most candidates scored full marks. A few lost the final mark by saying m = 0.0238. Some weaker candidates could not solve the equation.

3(i) Almost all candidates scored this mark.

3(ii) Most scored full marks. Almost all candidates knew that they needed to differentiate and substitute t = 1 into their derivative.

3(iii) Most scored full marks. Some weaker candidates could not set up $\frac{t}{a}(1-\frac{bt}{a})^{-1}$.

3(iv) Most candidates recognised that they had been dealing with a Geometric distribution and almost all of these candidates knew the parameter was $\frac{1}{a}$.

4(i) Most scored full marks. Those who multiplied out $(2-x)e^{xt}$ were more likely to lose marks than those who did not.

4(ii) Almost all candidates knew that they had to take the square root of the given answer to part (i).

4(iii) Most scored full marks. Some did not know that $E(Y^2)$ is twice the coefficient of t^2.

4(iv) Almost all candidates knew that they had to double their answer to part (iii)

5(i),(ii) Almost all candidates scored full marks in these parts.

5(iii) Most candidates scored full marks. Some weaker candidates did not realise that $E(Y)=E(X)$ and made errors in finding $E(Y)$.

5(iv),(v) Almost all candidates scored full marks in these parts.

6(i) Almost all candidates answered this part correctly.

6(ii) The most common mark was 5 out of 6. The usual errors were to omit the continuity correction and to fail to give the final answer as an integer.

7(i),(ii) Almost always answered correctly.

7(iii) Most scored the mark for the expectation. The variance proved to be more difficult. Some did not use $\left(\frac{3}{n}\right)^2$.

7(iv) As in part (iii), the coefficients $\frac{1}{9}$ and $\frac{25}{9}$ caused problems for some.

4736 Decision Mathematics 1

General Comments:

Almost all candidates had enough time to attempt every question including checking and making alterations, however there were several instances where candidates had not read the questions properly and spent time answering what they had assumed was being asked rather than what actually was being asked.

Poor presentation and messy handwriting also created problems, some candidates wrote their digits so badly that they then misread their own work. Centres need to be aware that work done in pencil can sometimes partially show through, so candidates who draft out an answer in pencil and then go over it in ink may end up with an unreadable answer on the scanned script.

Candidates need to use technical vocabulary correctly, the terms arc and node were sometimes confused and several candidates insist on trying to use 'vertice' instead of 'vertex' (or 'vertexes' instead of 'vertices'). However there was evidence that centres had responded to some of the issues raised in previous reports, for example candidates appreciating that 'positive' and 'non-negative' are not the same thing.

Comments on Individual Questions:

Question No. 1
Generally answered well, although too many candidates forgot to write the number of deliveries needed. Various forms of presentation were used, all of which were accepted provided the order in which the boxes were 'packed' could be identified. Some candidates lost (or gained) a box in part (iii) and many focussed on the weight limit without taking account of the box limit given in the stem to this part.

Question No. 2
Most candidates drew a correct graph in part (i)(a) and identified it as semi-Eulerian, but their reason was often not specific enough – we needed to know that there are 'exactly two odd vertices'. There were very few fully correct responses to part (i)(b), the question asked candidates to consider the sum of the vertex orders (which was $2 \times 5 = 10$) and many candidates realised (from part (i)(a)) that one way to achieve this was using vertex orders 2, 2, 3, 3; some candidates explained why the vertex order could only be 1, 2 or 3 but they often missed the apparent possibility of 1, 3, 3, 3 (although those who spotted it usually then explained that this would make a graph that was not simple). The point that was missed by most candidates in part (i)(b) was that there is only one graph with vertex orders 2, 2, 3, 3 – being the complete graph K_4 with one arc removed (so the two vertices of order 2 are not directly connected). Candidates were clearly aware that a given vertex order can create more than one graph since they did not assume that there was only one way to make 1, 2, 2, 2, 3 in part (ii). Most candidates were able to find at least three of the five graphs for part (ii) and several found all five.

Question No. 3
Candidates could usually follow the instructions correctly in parts (i) and (ii), a few slipped up with rounding errors (if not using surds) or through continuing into an extra pass beyond the stopping point. (eg C = 3 in part (ii)). Most candidates understood when lines 70 and 110 are used but they did not always say the reason why they are needed (to force the algorithm to stop when it has failed). In part (iv), a few candidates assumed that the algorithm had order n^2 instead of square root, as given in the question, and some made arithmetic slips, but most were able to find the time as 1.4 seconds.

Question No. 4

Several candidates made mistakes in carrying out Dijkstra's algorithm, in particular having a temporary label of 10 at G. Candidates should only show temporary labels when they are smaller than the current best value. The instruction to 'not cross out temporary labels' was included so that candidates could gain credit for correct temporary values - although examiners try to recognise correct work, if the values cannot be read then they ca
nnot be given credit. Many candidates dropped the final mark in part (i) through not recording the values in metres. There were many good answers to parts (ii), (iii) and (iv). Some candidates got into a muddle over the units and ended up gaining or losing powers of 10.

Question No. 5

Part (i) was often answered well, although some candidates did not close the route (cycle) in part (i)(a) or chose an indirect (although shorter) way to close it. The use of Prim's algorithm in part (ii) should have been straightforward, some candidates found a correct tree but had used Kruskal's algorithm, others used Prim's algorithm but listed the order in which the vertices had been chosen, rather than the arcs (as asked in the question), and some wasted time by drawing out a distance matrix and then carried out the matrix version of Prim's algorithm. Some candidates had constructed a nearest neighbour route in part (i), rather than a spanning tree, but this usually corrected itself in part (ii) because these candidates did not need to reconnect G. Those candidates who had plausible upper and lower bounds were often able to find a suitable route in part (iv), although there were some arithmetic errors with the length of the route (in metres). Parts (v) and (vi) caused difficulties for several candidates, in part (v) some candidates did not divide their distance from part (iv) by 100 or counted vertices more than once (which would not have been consistent with the context) and in part (vi) candidates often counted the vertices up to and including H, rather than just those before H was reached. The need to be aware of the practical context and the network solution at the same time seemed to confuse some candidates, or possibly candidates had not read the information in the stems carefully enough.

Question No. 6

The definitions of the variables in part (i) were often incomplete, lacking units, or else referred to the amount of water. Correct and precise definitions of a, b and c helped candidates to interpret the solution in part (vii). Many candidates were able to construct appropriate inequality constraints for parts (ii) and (iii), although some candidates included the slack variables in their response to part (iii). The iteration of the tableau was carried out well, with fewer arithmetic errors than on some past papers, although some candidates chose an inappropriate pivot and some did not express the row operations in an appropriate form. A few candidates then ignored the given (final) tableau and continued to work with their own tableau, and some used the given tableau but could not read off the values correctly (for example stating that $a = 0$, $b = 0$, $c = 14.3$). Many candidates were able to interpret the tableau well and understood the significance of their solution.

4737 Decision Mathematics 2

General Comments:

Several excellent scripts but a few candidates were let down by messy handwriting or poor diagrams. Centres need to be aware that work done in pencil can sometimes partially show through so candidates who draft out an answer in pencil and then go over it in ink may end up with an unreadable answer on the scanned script. The vast majority of candidates were able to make an attempt every question.

Comments on Individual Questions:

Question No. 1
Many good responses were seen, although some candidates rushed into an answer without reading the question carefully. In part (i), the question required candidates to find the shortest possible alternating path and to write it down, as well as producing an incomplete matching. Part (iii) required candidates to identify the two complete matchings and to explain why there could not be more than two.

Question No. 2
This question was generally answered well. In part (i), most candidates were able to add appropriately weighted arcs leading from a supersource and to a supersink, although the arcs were not always shown as being directed. Many candidates were successful on part (ii) provided they read the vertex sets carefully. In part (iii) most candidates stated that the maximum flow was 25 but not all candidates gave both a flow of 25 and a cut of 25, there is still a common misconception amongst candidates that the value of a cut can be calculated from the flows rather than the capacities. Some good responses to part (iv) although to achieve both marks candidates needed to state the maximum useful increase to the capacity of CE and the value of the resulting maximum flow.

Question No. 3
Most candidates were able to achieve good marks on this question, there were, inevitably, some arithmetic errors – although fewer than in previous sessions. Some candidates dropped a mark in part (ii) for not stating that the total cost was £170. Many successful attempts at part (iii) using the method indicated, some candidates started the Hungarian algorithm again, which wasted some time but did usually lead to the three correct solutions.

Question No. 4
Most candidates could give a correct statement of how the entries in the table had been calculated. To show that it gave a zero-sum game candidates needed to state that for every cell the sum of Ross's score and Collwen's score was originally 8 (and so now it is 0), equivalently they could give two independent ways of calculating the entries in the table. Parts (ii) and (iii) were usually answered well, and there were many good attempts at part (iv), although candidates did not always support their statements with appropriate numerical comparisons (or sometimes did not support the numerical comparisons with any evidence of which columns were being compared and what conclusion could be reached). Part (v) was usually successful and most candidates could use the values in the table to draw an appropriate graph, although some graphs were far too small. When a graph is required and no axes or scales are given, candidates should aim to fill at least half the width and at least half the height of the given grid with useful plots. In particular, when graphing the expected pay-off against probability in game theory questions, the probability axis (from 0 to 1) should fill at least half the width of the grid (rather than just one square. However several candidates then chose the wrong intersection - the lowest line for each value of p form the upper boundary of the feasible region, the optimum is then found by bringing a horizontal 'profit' line up across the graph until it hits the highest point

that is within the feasible region - in this case the required point occurred when $-2p = 5p - 2$, giving $p = 2/7$.

Question No. 5

Most candidates were able to have a good try at the activity network, although often D ended up following C as well as A and B (a pair of dummies were needed at B). The mark scheme allowed such networks to be followed through in (ii) and (iii), so there was usually only a small penalty provided further mistakes did not occur. Some candidates gave the biggest value when two arcs merged on the backward pass, when it should be the smallest value so that there is enough time to complete all the activities from that point to the end. Because activity networks were being followed through in (iii), candidates needed to state the float on C as given on their network. Some candidates gave a clear and concise explanation for part (iv) but far too many of the responses were confused or only dealt with specific cases, despite this there were several good answers to part (v) – provided the letters for the activities could be read on the schedule. In critical path analysis questions a very small minority of candidates use activity on node instead of activity on arc. Centres should note that this specification only accepts activity on arc.

Question No. 6

Parts (i), (ii) and (iii) were answered well, apart from the omission of vertex (4; 0) from the route in part (ii) or the diagram in part (iii) – despite the structure being given in the stem to part (v). In part (iv), candidates needed to use the context of this question (flows) to interpret both the maximin value and the route from part (ii), many good candidates only described one of these, however they were then often able to say how this was equivalent to flow augmentation, with the values on the arcs showing the excess capacities, until no further augmentation was possible. The cut in part (v)(b) suffered from the same common misconception as the cut in question 2(iii), but otherwise the candidates who reached the end of the paper usually scored some marks in part (v).

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