Accredited

LEVEL 3 CERTIFICATE IN

Topic Exploration Pack

H866

QUANTITATIVE REASONING (MEI)

Statistical Experiments

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OCR
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Contents

This Topic Exploration Pack should accompany the OCR resource ‘Statistical Experiments’ learner activity and spreadsheet, which you can download from the OCR website.

This activity offers an opportunity for Maths skills development.

Introduction

There are four aspects of Statistical Experiments that need to be taught in Critical Maths:

Firstly, learners need to understand the importance of randomised controlled trials in science, medicine and social science, including blind and double blind trials.

Secondly, learners should understand that small samples are more variable than large ones and that for large random samples, the sum of observations are distributed Normally.

Finally, they need to learn that, when the probability of success in a single trial is $\frac{1}{2}$, then for a large number (n) of those trials

- the expected number of successes is $\frac{n}{2}$ (that is, half the trials)
- the standard deviation of the number of successes is $\frac{\sqrt{n}}{2}$.

Prior Knowledge

Students will need to be familiar with:

- The concept of **standard deviation** as a measure of spread.
- The **Normal distribution**.

Neither of these are included at GCSE and so both will need to be covered before considering Statistical Experiments (while studying the content of Introduction to Quantitative Reasoning which is an assumed knowledge before starting Critical Maths).

- The **variability of sample data**. The GCSE subject content includes, at both Foundation and Higher, the limitations of sampling and the fact that empirical, unbiased samples tend towards theoretical probability distributions with increasing sample size. This topic develops this idea.

Teaching Points/Misconceptions

Students are not required to calculate standard deviation in the examination. However, in order to understand what standard deviation is, it would be useful for them to know how it is calculated. Its calculation as the (square root of the) average of the squared differences from the mean is not difficult to introduce and will show students that it is a fairly natural and transparent way to measure spread.

Probability, beyond the basics of finding the probability of a simple outcome such as rolling a six on a fair die, can be a difficult area of mathematics and statistics to understand and to teach. It is certainly the case that people commonly make errors in interpreting evidence because they are unclear as to exactly what the numbers mean.

The mistake, most likely to confuse students in this topic is mixing up cases where order matters with those where it does not. So for example, if you flip 10 coins, students may believe that you are just as likely to get 1 head as to get 10 heads because: P(HTTTTTTTTT) = P(HHHHHHHHHH) = $\left(\frac{1}{2}\right)^{10} = \frac{1}{1024}$. This ignores the fact that there are 10 ways to get 1 head but only one way of getting ten heads.

One thing to avoid in this topic is reinforcing a belief in the Gambler’s Fallacy: that if you are flipping a **fair** coin and get six heads in a row, the next flip is more likely to be a tail ‘by the law of averages’. Or if a family has four boys, then the next baby is most likely to be a girl, again ‘by the law of averages’. The confusion here is between the probability of getting a head given that you already have six heads (which is $\frac{1}{2}$ with a fair coin) and the probability of getting seven heads in a row before you have started flipping the coins (which is $\frac{1}{128}$ with a fair coin). So, yes, it is unlikely to get seven heads in a row but, once you have got 6 heads, the probability of the next being a head is still $\frac{1}{2}$. Because we are teaching that when the probability of success in a Bernoulli trial is $\frac{1}{2}$, then for a large number (n) of those trials the expected number of successes is $\frac{n}{2}$, we are in danger of reinforcing the Gambler’s Fallacy (the so called ’law of averages’). To avoid this we need to emphasise the difference between the outcome of many trials and that of a single trial - we may know that about half the babies born are male but this does still not tell us whether a particular baby will be male or female.

Part A Randomised controlled trials

All students should have studied some science and/or social science at GCSE and so should be aware that the scientific method involves designing and carrying out experiments.

The video Against All Odds: Designing Experiments addresses experimental design, contrasting an observational study with an experiment and contrasting an experiment well-conducted with one where experimental design is ignored. You will need to stop the video to discuss with the group terms such as experimenter bias, confounding variables, random assignment, blind and double blind trials and sample size.

This could be followed up with:

- The Gold Standard: What are randomised controlled trials and why are they important?
- Monkey See Monkey Take which is a clip about monkey behaviour where some possible confounding variables are controlled for, this video is in .mov format and requires Quicktime to play which can be downloaded for free here. Suggested questions about the clip can be found at Lesson activities.
- Power Bracelets, which a double blind test shows are not all they seem to be.

Part B Large samples and the Normal distribution

In **Activity 1**, students discover that, for large samples, the sum of observations approximates the Normal distribution.

Activity 1a starts with a skewed distribution of fish with mean 24.3 (resource adapted from CIMT). The weights of the catches of 30 fish should be distributed fairly Normally with a mean of about 729 (24.3 × 30). The accompanying Excel worksheet 'Activity 1a Fish and dice graphs' allows comparison of the original distribution with that of the catch weights. You will need to decide how students are to generate random numbers between 1 and 57 - you could use Random Number Tables, pieces of paper, calculators or spreadsheets.

Activity 1b is similar but uses dice. The original distribution here is rectangular but the sums of 30 rolls should be distributed Normally with a mean of about 105 (3.5 × 30).

The dice experiment makes the point that sums of observations are Normal more dramatically than the fish experiment but are a more artificial context using a discrete random variable rather than a continuous one. Catching fish is a realistic and useful context.

Part C Outcome of trials when P(success) = 0.5

Activity 2a is a visual demonstration of the convergence of the proportion of successes to $\frac{1}{2}$, and the number of successes to $\frac{n}{2}$, in Bernoulli trials when $p = \frac{1}{2}$.

It requires each member of the class to be given a coin. Each person takes it in turns to throw their coin and the result is recorded on the accompanying Excel file 'Activity 2ab Number of heads graph' (worksheet Coins 2a). So, for example, if the first person throws a head, enter H in cell B5; if the next person throws a tail, put T in the cell B6 and so on. The graph will not complete immediately - you will need to copy cell D5 down the column. This should be done one cell at a time to start with, checking that students understand what is being plotted.

NB. To reuse the spreadsheet, delete all the cells in column B and column D except for D5.

To follow you could pose the following question:

At a small Hospital A, there are about 3 births a week.
At a larger Hospital B, there are about 30 births a week.
Which of the following statements (about babies born in one week) is true?

1. The probability of at least $\frac{2}{3}$ of the babies being girls is **greater** at Hospital A then at Hospital B.
2. The probability of at least $\frac{2}{3}$ of the babies being girls is **smaller** at Hospital A and Hospital B.
3. The probability of at least $\frac{2}{3}$ of the babies being girls is **the same** for Hospital A and Hospital B.

This is a good opportunity for discussion and to check understanding. If anyone is unsure, work out the probability of at least $\frac{2}{3}$ being girls at Hospital A $= \frac{1}{2}$.

Then let them throw 30 coins a few times until they realise how unusual it is to get 20 or more heads. This is because, for a large number of coins (or births) the proportion of heads (or girls) converges to $\frac{1}{3}$ and so it becomes increasingly unlikely that as many as $\frac{2}{3}$ will be heads (or girls).

Activity 2b The only way of demonstrating that the standard deviation converges to $\frac{\sqrt{n}}{2}$ for Bernoulli trials with $p=\frac{1}{2}$ is to do an experiment. The theoretical explanation (that the distributions are binomial and so the variance for a given n is $npq\left(=\frac{n}{4} \text{ when } p=q=\frac{1}{2}\right)$ and the standard deviation is therefore $\frac{\sqrt{n}}{2}$ is beyond the scope of this qualification.

Give each pair 10 coins and ask them to complete the table. How many trials you ask them to do will depend on how many you have in your class and how much time you want them to spend throwing coins and entering data? Aim for at least 30 trials for each number of coins.

Firstly, ask about their proportion of heads – are they close to $\frac{1}{2}$?

Excel worksheet Coins 2bi simply plots their data. You could ask if they can see a pattern – what would they predict if they threw 12 coins?

Excel worksheet Coins 2bii compares their data with the theoretical standard deviation (σ).

Their results should be close to the curve $\sigma = \frac{\sqrt{n}}{2}$.

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