Thursday 12 June 2014 – Afternoon

A2 GCE MATHEMATICS (MEI)

4758/01 Differential Equations

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4758/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The displacement, x m, of a particle at time t s is given by the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = 30\cos 2t.$$

(i) Find the general solution. **[9]**

The particle is initially at the origin, travelling with velocity $10\,\mathrm{m\,s^{-1}}$.

(ii) Find the particular solution. **[4]**

(iii) Find the amplitude of the oscillations of the particle for large values of t. **[2]**

Consider now the differential equation

$$\frac{d^3x}{dt^3} + 2\frac{d^2x}{dt^2} + 2\frac{dx}{dt} = 0.$$

(iv) Show that 0 is a root of the auxiliary equation and write down the other roots**.** **[2]**

(v) Find the particular solution of this differential equation subject to the initial conditions $x = 0$, $\frac{dx}{dt} = 10$ and $\frac{d^2x}{dt^2} = 4$ when $t = 0$. **[5]**

(vi) Sketch the graph of this solution. **[2]**

2 The population, P, of a species at time t hours is to be modelled by a differential equation. The initial population is 100.

At first, the model $\frac{dP}{dt} - 0.25P = 0$ is used.

(i) Find P in terms of t and comment on the suitability of this model. **[4]**

To allow for certain environmental effects, the model is refined to

$$\frac{dP}{dt} - 0.25P = -18e^{-0.5t}.$$

(ii) Write down the complementary function for this differential equation. Find a particular integral and hence state the general solution. **[6]**

(iii) Find the solution subject to the given initial condition and comment on the suitability of this refined model. **[3]**

The following mathematical model for the population is now used.

$$\frac{dP}{dt} = 6 \times 10^{-4} P(400 - P)$$

(iv) Solve this differential equation subject to the given initial condition, expressing P in terms of t. **[8]**

(v) Show that the time T hours at which $P = 200$ is given by

$$T = \frac{25}{6}\ln 3.$$ **[1]**

(vi) What does this model predict for the population of the species in the long term? **[2]**

3 **(a)** The equation of a curve in the *x*-*y* plane satisfies the differential equation

$$(x+1)\frac{\mathrm{d}y}{\mathrm{d}x} - xy = \mathrm{e}^{2x}$$

for $x > -1$.

(i) Show that an integrating factor for this differential equation is $\mathrm{e}^{-x}(1+x)$ and hence find the general solution for y in terms of x. **[11]**

The curve passes through the point $(0, -2)$.

(ii) Find the equation of this curve. **[2]**

(b) The differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x^2 + y^2}$$

is to be solved approximately, first by using a tangent field and then by Euler's method.

(i) Show that the isocline for which $\frac{\mathrm{d}y}{\mathrm{d}x} = 4$ is a circle and state its centre and radius. **[2]**

(ii) Sketch the isoclines for the cases $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{4}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 1$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 4$. Use these isoclines to draw a tangent field. **[3]**

(iii) Sketch the solution curve through $(0, 1)$. **[2]**

Euler's method is now used, starting at $x = 0$, $y = 1$. The algorithm is given by $x_{r+1} = x_r + h$, $y_{r+1} = y_r + hy'_r$.

(iv) Use a step length of 0.05 to estimate y when $x = 0.15$. **[4]**

Question 4 begins on page 4

4 The simultaneous differential equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} + 2x = 4y + \mathrm{e}^{-2t}$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} + 3x = 5y + 2\mathrm{e}^{-2t}$$

are to be solved.

(i) Obtain a second order differential equation for x in terms of t. Hence find the general solution for x. **[12]**

(ii) Find the corresponding general solution for y. **[4]**

When $t = 0$, $y = -\frac{2}{3}$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = 0$.

(iii) Find the particular solutions for x and y. **[5]**

(iv) Find the set of values of t for which $y > x$. **[3]**

END OF QUESTION PAPER