Friday 6 June 2014 – Afternoon

A2 GCE MATHEMATICS (MEI)

4772/01 Decision Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4772/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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1 Keith is wondering whether or not to insure the value of his house against destruction. His friend Georgia has told him that it is a waste of money. Georgia argues that the insurance company sets its premiums (how much it charges for insurance) to take account of the probability of destruction, plus an extra fee for its profit. Georgia argues that house-owners are, on average, simply paying fees to the insurance company.

Keith's house is valued at £400 000. The annual premium for insuring its value against destruction is £100. Past statistics show that the probability of destruction in any one year is 0.0002.

(i) Draw a decision tree to model Keith's decision and the possible outcomes. **[6]**

(ii) Compute Keith's EMV and give the course of action which corresponds to that EMV. **[2]**

(iii) What would be the insurance premium if there were no fee for the insurance company? **[1]**

For the remainder of the question the insurance premium is still £100.

Suppose that, instead of EMV, Keith uses the utility function $utility = (money)^{0.5}$.

(iv) Compute Keith's utility and give his corresponding course of action. **[3]**

Keith suspects that it may be the case that he lives in an area in which the probability of destruction in a given year, p, is not 0.0002.

(v) Draw a decision tree, using the EMV criterion, to model Keith's decision in terms of p, the probability of destruction in the area in which Keith lives. **[1]**

(vi) Find the value of p which would make it worthwhile for Keith to insure his house using the EMV criterion. **[2]**

(vii) Explain why Keith may wish to insure even if p is less than the value which you found in part **(vi)**. **[1]**

2 **(a)** A national Sunday newspaper runs a "You are the umpire" series, in which questions are posed about whether a batsman in cricket is given "out", and why, or "not out". One Sunday the readers were told that a ball had either hit the bat and then the pad, or had missed the bat and hit the pad; the umpire could not be sure which. The ball had then flown directly to a fielder, who had caught it.

The LBW (leg before wicket) rule is complicated. The readers were told that this batsman should be given out (LBW) if the ball had not hit the bat. On the other hand, if the ball had hit the bat, then he should be given out (caught). Readers were asked what the decision should be.

The answer given in the newspaper was that this batsman should be given not out because the umpire could not be sure that the batsman was out (LBW), and could not be sure that he was out (caught).

(i) Rachel thinks that the answer given in the newspaper article is not sensible. Give a verbal argument why Rachel might think that the batsman should be given out. **[3]**

Rachel tries to formalise her argument. She defines four simple propositions.

- o: "The batsman is given out."
- lb: "The batsman is given out (LBW)."
- c: "The batsman is given out (caught)."
- b: "The ball hit the bat."

(ii) An implication of the batsman not being out (LBW) is that the ball has hit the bat. Write this down in terms of Rachel's propositions. **[1]**

(iii) Similarly, write down the implication of the batsman not being out (caught). **[1]**

(iv) Using your answers to parts **(ii)** and **(iii)** write down the implication of a batsman being not out, in terms of b and $\sim \mathrm{b}$.
[You may assume that if $\mathrm{w} \Rightarrow \mathrm{y}$ and $\mathrm{x} \Rightarrow \mathrm{z}$, then $(\mathrm{w} \wedge \mathrm{x}) \Rightarrow (\mathrm{y} \wedge \mathrm{z})$.] **[1]**

(v) By writing down the contrapositive of your implication from part **(iv)**, produce an implication which supports Rachel's argument. **[2]**

(b) A classroom rule has been broken by either Anja, Bobby, Catherine or Dimitria, or by a subset of those four. The teacher knows that Dimitria could not have done it on her own.

Let a be the proposition "Anja is guilty", and similarly for b, c and d.

(i) Express the teacher's knowledge as a compound proposition. **[1]**

Evidence emerges that Bobby and Catherine were elsewhere at the time, so they cannot be guilty. This can be expressed as the compound proposition $\sim (b \vee c)$.

(ii) Construct a truth table to show the truth values of the compound proposition given by the conjunction of the two compound propositions, one from part **(i)** and one given above. **[4]**

(iii) What does your truth table tell you about who is guilty? **[3]**

3 Three products, A, B and C are to be made.

Three supplements are included in each product. Product A has 10 g per kg of supplement X, 5 g per kg of supplement Y and 5 g per kg of supplement Z.

Product B has 5 g per kg of supplement X, 5 g per kg of supplement Y and 3 g per kg of supplement Z.

Product C has 12 g per kg of supplement X, 7 g per kg of supplement Y and 5 g per kg of supplement Z.

There are 12 kg of supplement X available, 12 kg of supplement Y, and 9 kg of supplement Z.

Product A will sell at £7 per kg and costs £3 per kg to produce. Product B will sell at £5 per kg and costs £2 per kg to produce. Product C will sell at £4 per kg and costs £3 per kg to produce.

The profit is to be maximised.

(i) Explain how the initial feasible tableau shown in Fig. 3 models this problem. **[6]**

P	a	b	c	s1	s2	s3	RHS
1	–4	–3	–1	0	0	0	0
0	10	5	12	1	0	0	12000
0	5	5	7	0	1	0	12000
0	5	3	5	0	0	1	9000

Fig. 3

(ii) Use the simplex algorithm to solve this problem, and interpret the solution. **[8]**

(iii) In the solution, one of the basic variables appears at a value of 0. Explain what this means. **[1]**

There is a contractual requirement to provide at least 500 kg of product A.

(iv) Show how to incorporate this constraint into the initial tableau ready for an application of the two-stage simplex method.

Briefly describe how the method works. You are **not** required to perform the iterations. **[5]**

4 John is designing a hot water system for his new house. The vertices on the network in Fig. 4 represent positions for 6 hot water taps. The arcs represent possible connections between taps. The weights represent the lengths in metres of pipes that would be needed to connect the taps.

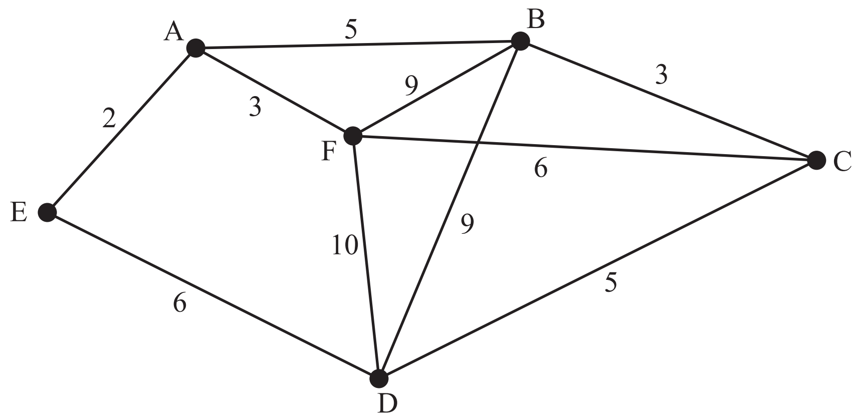

Fig. 4

(a) John wants to position the hot water cylinder so that the maximum distance from it to a hot tap is as small as possible.

(i) Use Dijkstra's algorithm repeatedly to find the complete network of shortest distances for the network of hot taps and pipes. **[7]**

(ii) Complete the matrix of shortest distances given in your answer book. **[2]**

(iii) Use your matrix to find the best vertex for John to choose for the hot water cylinder, explaining how you found it. **[3]**

(iv) Using your answer to part **(ii)**, show that there is no point on the arc AB which represents a better position for the hot water cylinder than vertex A or vertex B. **[1]**

(b) Alongside each hot tap there is a cold tap. Fig. 4 also represents the network for the cold taps. John wants to connect the cold taps with one run of pipe with no branches.

(i) Starting at vertex A in Fig. 4, apply the nearest neighbour algorithm to find a Hamilton cycle. Now delete the longest arc in that cycle to find a possible way for John to connect his cold taps. Give your connections and their total length. **[3]**

(ii) Repeat part **(b) (i)** for the other starting vertices, where possible, and give the best connector that you have found. **[4]**

END OF QUESTION PAPER

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