

**Monday 23 June 2014 – Morning**

**A2 GCE MATHEMATICS**

**4726/01** Further Pure Mathematics 2

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

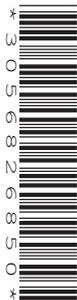
**OCR supplied materials:**

- Printed Answer Book 4726/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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1 Find  $\int_0^2 \frac{1}{\sqrt{4+x^2}} dx$ , giving your answer exactly in logarithmic form. [3]

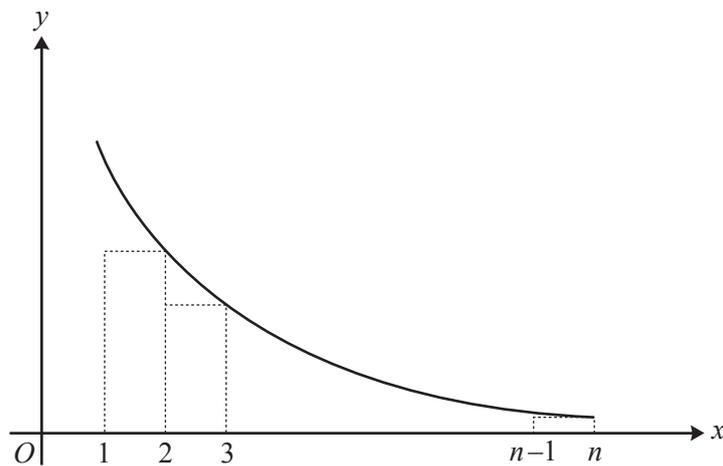
2 It is given that  $f(x) = \ln(1+x^2)$ .

(i) Using the standard Maclaurin expansion for  $\ln(1+x)$ , write down the first four terms in the expansion of  $f(x)$ , stating the set of values of  $x$  for which the expansion is valid. [3]

(ii) Hence find the exact value of

$$1 - \frac{1}{2}\left(\frac{1}{2}\right)^2 + \frac{1}{3}\left(\frac{1}{2}\right)^4 - \frac{1}{4}\left(\frac{1}{2}\right)^6 + \dots \quad [2]$$

3 The diagram shows the curve  $y = \frac{1}{x^3}$  for  $1 \leq x \leq n$  where  $n$  is an integer. A set of  $(n-1)$  rectangles of unit width is drawn under the curve.



(i) Write down the sum of the areas of the rectangles. [2]

(ii) Hence show that  $\sum_{r=1}^{\infty} \frac{1}{r^3} < \frac{3}{2}$ . [5]

4 The curves  $y = \cos^{-1}x$  and  $y = \tan^{-1}(\sqrt{2}x)$  intersect at a point  $A$ .

(i) Verify that the coordinates of  $A$  are  $\left(\frac{1}{\sqrt{2}}, \frac{1}{4}\pi\right)$ . [2]

(ii) Determine whether the tangents to the curves at  $A$  are perpendicular. [4]

- 5 A curve has equation  $y = \frac{x^2 - 8}{x - 3}$ .
- (i) Find the equations of the asymptotes of the curve. [3]
- (ii) Prove that there are no points on the curve for which  $4 < y < 8$ . [4]
- (iii) Sketch the curve. Indicate the asymptotes in your sketch. [2]
- 6 (i) Given that  $y = \cosh^{-1}x$ , show that  $y = \ln(x + \sqrt{x^2 - 1})$ . [4]
- (ii) Show that  $\frac{d}{dx}(\cosh^{-1}x) = \frac{1}{\sqrt{x^2 - 1}}$ . [2]
- (iii) Solve the equation  $\cosh x = 3$ , giving your answers in logarithmic form. [3]
- 7 It is given that, for non-negative integers  $n$ ,  $I_n = \int_0^{\frac{1}{2}\pi} \sin^n x \, dx$ .
- (i) Show that  $I_n = \frac{n-1}{n} I_{n-2}$  for  $n \geq 2$ . [3]
- (ii) Explain why  $I_{2n+1} < I_{2n-1}$ . [2]
- (iii) It is given that  $I_{2n+1} < I_{2n} < I_{2n-1}$ . Take  $n = 5$  to find an interval within which the value of  $\pi$  lies. [6]
- 8 A curve has polar equation  $r = a(1 + \cos \theta)$ , where  $a$  is a positive constant and  $0 \leq \theta < 2\pi$ .
- (i) Find the equation of the tangent at the pole. [2]
- (ii) Sketch the curve. [2]
- (iii) Find the area enclosed by the curve. [6]
- 9 The equation  $10x - 8 \ln x = 28$  has a root  $\alpha$  in the interval  $[3, 4]$ . The iteration  $x_{n+1} = g(x_n)$ , where  $g(x) = 2.8 + 0.8 \ln x$  and  $x_1 = 3.8$ , is to be used to find  $\alpha$ .
- (i) Find the value of  $\alpha$  correct to 5 decimal places. You should show the result of each step of the iteration to 6 decimal places. [4]
- (ii) Illustrate this iteration by means of a sketch. [2]
- (iii) The difference,  $\delta_r$ , between successive approximations is given by  $\delta_r = x_{r+1} - x_r$ . Find  $\delta_3$ . [2]
- (iv) Given that  $\delta_{n+1} \approx g'(\alpha)\delta_n$ , for all positive integers  $n$ , estimate the smallest value of  $n$  such that  $\delta_n < 10^{-6}\delta_1$ . [4]

**END OF QUESTION PAPER**

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