

Thursday 12 June 2014 – Afternoon

A2 GCE MATHEMATICS

4727/01 Further Pure Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

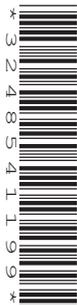
OCR supplied materials:

- Printed Answer Book 4727/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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- 1 (i) Find a vector equation of the line of intersection of the planes $2x + y - z = 4$ and $3x + 5y + 2z = 13$. [4]
 (ii) Find the exact distance of the point $(2, 5, -2)$ from the plane $2x + y - z = 4$. [2]

- 2 Use the substitution $u = y^2$ to find the general solution of the differential equation

$$\frac{dy}{dx} - 2y = \frac{e^x}{y}$$

for y in terms of x . [8]

- 3 (i) Solve the equation $z^6 = 1$, giving your answers in the form $re^{i\theta}$, and sketch an Argand diagram showing the positions of the roots. [4]
 (ii) Show that $(1 + i)^6 = -8i$. [3]
 (iii) Hence, or otherwise, solve the equation $z^6 + 8i = 0$, giving your answers in the form $re^{i\theta}$. [3]

- 4 The group G consists of the set $\{1, 3, 7, 9, 11, 13, 17, 19\}$ combined under multiplication modulo 20.

(i) Find the inverse of each element. [3]

(ii) Show that G is not cyclic. [3]

(iii) Find two isomorphic subgroups of order 4 and state an isomorphism between them. [5]

- 5 Solve the differential equation

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-x}$$

subject to the conditions $y = \frac{dy}{dx} = 0$ when $x = 0$. [10]

- 6 The line l has equations $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-7}{5}$. The plane Π has equation $4x - y - z = 8$.

(i) Show that l is parallel to Π but does not lie in Π . [3]

(ii) The point $A(1, -2, 7)$ is on l . Write down a vector equation of the line through A which is perpendicular to Π . Hence find the position vector of the point on Π which is closest to A . [4]

(iii) Hence write down a vector equation of the line in Π which is parallel to l and closest to it. [1]

- 7 (i) By expressing $\sin \theta$ in terms of $e^{i\theta}$ and $e^{-i\theta}$, show that

$$\sin^5 \theta \equiv \frac{1}{16}(\sin 5\theta - 5 \sin 3\theta + 10 \sin \theta). [4]$$

(ii) Hence solve the equation

$$\sin 5\theta + 4 \sin \theta = 5 \sin 3\theta$$

for $-\frac{1}{2}\pi \leq \theta \leq \frac{1}{2}\pi$. [4]

8 G consists of the set of matrices of the form $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$, where a and b are real and $a^2 + b^2 \neq 0$, combined under the operation of matrix multiplication.

(i) Prove that G is a group. You may assume that matrix multiplication is associative. [6]

(ii) Determine whether G is commutative. [2]

(iii) Find the order of $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. [3]

END OF QUESTION PAPER

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE



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