

# **Principal Learning**

## Engineering

Unit F563: Mathematical techniques and applications for engineers

OCR Level 3 Principal Learning

### Mark Scheme for January 2015

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Se	ction A	A Contraction of the second seco		
1		6(4x + 5) - 8x = 24x + 30 - 8x = 16x + 30 16x 30	[1] [1]	
2		$x^{2} - 10x + 24 = (x - 4)(x - 6)$ $(x - 4)$ $(x - 6)$	[1] [1]	
3		[(x + 5)/12] - (x - 4)/3 = (x + 5 - 4x + 16)/12 $= (-3x + 21)/12 = (7 - x)/4$ $(7 - x)$ $4$	[1] [1]	Accept –3x + 21 for one mark and 12 for one mark.
4		5(2x - 4) = 3(3x - 2) 10x - 20 = 9x - 6 x = 14	[1] [1]	
5	(a) (b)	$π$ radians = $π \times (360/2 π) = 180^{\circ}$ $60^{\circ} = 60 \times (2 π/360) = \frac{1}{3} π = 1.0472$ radians	[1]	Award one mark for correct numerical results with or without the unit. Accept $\frac{1}{3}\pi$ or answers between 1.04 and 1.05 radians or any equivalent.



Sec	tion A				
9		Given $y = \sqrt{x^3}$ $x^{3/2}$ $x^{0.5}$ gives dy/dx = 1.5 $\sqrt{x}$ So	Then $y = dy/dx = 1.5$ which		
		1.5	√x	[1] [1]	
10		Given $y = 2 e^{6x} + \sin x$			
		Then dy/dx = 12 $e^{6x}$ + cos x			
		12 e <sup>6x</sup>		[1]	
		COS X		[1]	
11		$\int \sin 3x  dx = -\frac{1}{3} \cos 3x + C$			
		– ⅓ cos 3x		[1]	
		+C		[1]	
12		3 $3$ $3\int 15 x^4 dx. = [(15 x^5)/5] = [3 x^5] = 729 - 3 = 726$			Do not accept error carried forward if incorrect integration is used.
		1 1 1 3 $[(15 x^5)/5]$		[1]	
		3 [3 x <sup>5</sup> ] or [3.3 <sup>5</sup> ] – [3.1 <sup>5</sup> ]		[1]	
		726		[1]	Accept for one mark with or without limits.
13		Normal distribution curve, Central tendency, Statistical distribution Even distribution, Symmetrical, Bell, Probability distribution Gaussian distribution etc.	١,	[1]	Award one mark for any other correct response.

Sec	ction A	N Contraction of the second seco		
14		Mean = $(30 + 27 + 26 + 28 + 29)/5 = 28$ Values arranged in order of magnitude: 26 27 <u>28</u> 29 30 Median point = 28	[1]	
15		When P(A or B) = P(A) + P(B) - P(A and B) P(A or B) = probability that event A or event B occurs P(A and B) = probability that event A and event B both occur.	[1] [1]	Allow reference to Venn diagram. Do not allow 'but not both' Do not allow 'but not together'
		Total	[30]	

Que	stion	Expected Answer	Mark	Rationale/Additional Guidance
Sec	tion E	3		
1	(a)	Given $A = \pi d^2/4$ . Substitute d = 10. Then $A = \pi d^2/4$ = 100 $\pi/4$ = 78.54	[1]	Accept (100 $\pi$ )/4 or 25 $\pi$ or answers between 78 and 79.
1	(b)	Given that $V = V_0(1 + 3aT)$		
		Open the brackets then V = $V_o$ + 3aT $V_o$	[1]	Accept any other correct method.
		Then $3aT V_o = V - V_o$	[1]	
		So T = $(V - V_o)/3a V_o$	[1]	
1	(c)	Given F = $m(V^2 - U^2)/2$		
		Divide both sides by m/2		Accept any other correct method.
		Then $2F/m = V^2 - U^2$	[1]	
		Then $U^2 = V^2 - (2F/m)$	[1]	
		Square root both sides		
		So U = $\pm \sqrt{[V^2 - (2F/m)]}$	[1]	
1	(d)	Given $y = x/(x + 1)$		
		Multiply both sides by $(x + 1)$ then $xy + y = x$	[1]	Accept any other correct method.
		So $x - xy = y$	[1]	
		Then $x(1 - y) = y$		
		So $x = y/(1 - y)$ or $x = -y/(y-1)$	[1]	
		Total	[10]	

Q	uestio	n Expected Answer	Mark	Rationale/Additional Guidance		
2	(a)	Given 0.5 $I_1$ + 0.6 $I_2$ = 7 eqn. 1		Accept error carried forward from $I_1$ or $I_2$ .		
		1.2 I <sub>1</sub> + 0.4 I <sub>2</sub> = 9 eqn. 2				
		Multiply eqn. 1 by 4 then $2 I_1 + 2.4 I_2 = 28$ eqn. 3	[1]			
		Multiply eqn. 2 by 6 then 7.2 $I_1$ + 2.4 $I_2$ = 54 eqn.4	[1]	Accept any other correct method.		
		Subtract eqn. 3 from eqn. 4	[1]	Allow up to three marks for finding $I_1 \mbox{ or } I_2$		
		So 5.2 I <sub>1</sub> = 26	[1]	then		
		Then $I_1 = 26/5.2 = 5$		up to two marks for finding $I_2$ or $I_1$ .		
		Substitute $I_1$ = 5 into eqn. 1				
		Then 2.5 + 0.6 $I_2 = 7$	[1]			
		So 0.6 I <sub>2</sub> = 7 – 2.5 = 4.5	[4]			
		So $I_2 = 4.5/0.6 = 7.5$	[1]			
2	(b)	Given formula $\Theta = \tilde{\omega}t + 0.5\alpha t^2$ Where $\Theta = 5$ , $\tilde{\omega} = 4$ and $\alpha = 0.6$		Do not accept error carried forward if incorrect quadratic equation formula is used.		
		So $5 = 4t + 0.3t^2$ Then $0.3t^2 + 4t = 5 = 0$	[1]			
		Solution of quadratic equation by formulae	1.1			
		$t = [-b \pm \sqrt{(b^2 - 4ac)}]/2a$	[1]			
		where $a = 0.3$ , $b = 4$ and $c = -5$				
		so t = $[-4 \pm \sqrt{(4^2 - [4 \times 0.3 \times -5])}]/(2 \times 0.3)$	[1]			
		t = – 14.48 or 1.151 s	[1]			
		(Take t as 1.151 s)	[1]	Accept answers between 1.1 and 1.2 with or without the unit.		
		Total	[10]			

Q	uestio	n	Expected Answer	Rationale/Additional Guidance	
3	(a)	(i)	Given i = I sin 314.2 t. then $2\pi f = 314.2$ so f = $314.2/2\pi = 50$	[1]	
3	(a)	(ii)	Given i = I sin 314.2 t. and I = 20 A and t = 0.002 s then i = 20 sin (314.2 x 0.002) so i = 20 sin (0.6284) = 11.76 A	[1] [1]	Accept answers between 11 and 12 with or without the unit. Remember the angle is in radians.
3	(a)	(iii)	Given i = I sin 314.2 t. so I = i / sin 314.2t I = 4/(sin 314.2 x 0.008) = 6.8 A	[1] [1] [1]	Do not accept error carried forward if incorrect formula is used. Accept answers between 6.5 and 7 with or without the unit. Remember the angle is in radians.
3	(b)	(i)	Cosine rule: $\cos B = (a^2 + c^2 - b^2)/2ac$ Where side $a = 3$ m, side $b = 5$ m and side $c = 6$ m. $\cos B = (3^2 + 6^2 - 5^2)/(2 \times 3 \times 6) = 20/36$	[1]	Do not accept error carried forward if incorrect formula is used.
			$B = \cos^{-1} 20/36 = 56.25^{\circ}$	[1]	the unit.
3	(b)	(ii)	Area = $\sqrt{s(s - a)(s - b)(s - c)}$ $s = \frac{1}{2} (a + b + c) = \frac{1}{2} (3 + 5 + 6) = 7$ Area = $\sqrt{7(7 - 3)(7 - 5)(7 - 6)} = 7.48 \text{ m}^2$	[1] [1]	Do not accept error carried forward if incorrect formula is used. Accept any alternative correct method. Accept answers between 7.2 and 7.6 with or without the unit.
			Total	[10]	

Qu	estio	n	Expected Answer	Mark	Rationale/Additional Guidance
4	4 (a)		Given $\sin^2 x - \cos^2 x = \frac{1}{4}$ But $\sin^2 x + \cos^2 x = 1$ so $\sin^2 x = 1 - \cos^2 x$ Then $(1 - \cos^2 x) - \cos^2 x = \frac{1}{4}$ So $1 - 2\cos^2 x = \frac{1}{4}$ $1 - \frac{1}{4} = 2\cos^2 x$ $\frac{3}{4} = 2\cos^2 x$ So $\cos^2 x = \frac{3}{8}$ $\cos x = \sqrt{\frac{3}{8}} = 0.6124$ Angle $x = \cos^{-1} 0.6124 = 52.24^{\circ}$ Since the cosine may be positive or negative there will be one		Award marks for any other correct response.
			Since the cosine may be positive or negative there will be one solution in each quadrant, ie. 52.24°, 127.76°, 232.24° and 307.76°	[1]	Accept answers between: 52 and 52.5, 127.5 and 128, 232 and 232.5. 307.5 and 308.
4	(b)	(i)	Dimension AC = $3 \cos 30^\circ$ = 2.598 cm.	[1]	Accept answers between 2.5 and 2.7. Award one mark for correct numerical result with or without the unit.
4	(b)	(ii)	Dimension BC = $3 \sin 30^\circ = 1.5 \text{ cm}.$	[1]	Award one mark for correct numerical result with or without the unit.
4	(b)	(iii)	Angle between the lines DA and DC = $\tan^{-1} 2.598/4 = 33^{\circ}$	[1]	Accept answers between 32.5 and 33.5. Award one mark for correct numerical result with or without the unit.

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Qı	uestio	n	Expected Answer	Mark	Rationale/Additional Guidance	
4	(b)	(iv)	Dimension DC = $\sqrt{(4^2 + 2.598^2)} = 4.77$ cm.	[1]	Accept answers between 4.5 and 5. Award one mark for correct numerical result with or without the unit.	
4	4 (b) (v)		Angle between the lines DC and DB = $\tan^{-1} 1.5/4.77 = 17.46^{\circ}$	[1]	Accept answers between 17.3 and 17.6. Award one mark for correct numerical result with or without the unit.	
			Total	[10]		

Question		Expected	Answer			Mark	Rationale/Additional Guidance	
5	(a)		x y	-2 4	-1.5 2.25	-1 0 1 0	[1]	
	(b)	(i)	х у х у -х		-1.5 2.25 1.5 2.25 y y -1		[2]	Graph – award one mark for correct left hand side and one mark for right hand side. Allow error carried forward from part (a).
			Fig. 3					

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Qı	uestio	on	Expected Answer	Mark	Rationale/Additional Guidance	
5	(b)	(ii)	At $x = -1.5$ , gradient will be $= -3$	[1]	Check that a tangent has been drawn at $x = -1.5$ Accept answers between $-2.8$ and $-3.2$ .	
5	(c)	(i)	Given $s = 2t^3 + 4t^2 + 6t$ . Velocity = ds/dt = $6t^2 + 8t + 6$ At initial velocity t = 0 Then velocity = 0 + 0 + 6 = 6 ms <sup>-1</sup>	[1] [1] [1]	Award one mark for correct numerical result with or without the unit.	
5	(c)	(ii)	Given $s = 2t^3 + 4t^2 + 6t$ . Velocity = ds/dt = $6t^2 + 8t + 6$ When t = 5 s Then velocity = $150 + 40 + 6 = 196 \text{ ms}^{-1}$	[1]	Award one mark for correct numerical result with or without the unit	
5 (c) (iii)		(iii)	Given $s = 2t^3 + 4t^2 + 6t$ . Velocity = ds/dt = $6t^2 + 8t + 6$ Acceleration = $d^2s/dt^2 = 12t + 8$ When t = 3 s Then acceleration = $36 + 8 = 44 \text{ ms}^{-2}$	[1] [1]	Award one mark for correct numerical result with or without the unit	
			Total	[10]		

Qı	lestic	on	Expected Answer	Rationale/Additional Guidance		
6	(a)		$\pi/6$ Integral = $\int \cos 3x.  dx$ 0		Do not accept error carried forward if incorrect integration takes place.	
			$\pi/6$ = [(sin 3x)/3.] 0 = (sin $\pi/2$ )/3 - (sin 0)/3 = $\frac{1}{3}$	[1] [2] [1]	Award one mark for ( sin $\pi/2$ )/3 and one mark for – (sin 0)/3	
6	(b)		Given $y = (1 - x)^2$ Then integral = $\int (1 - x)^2 dx$ = $\int (1 - 2x + x^2) dx$ = $x - x^2 + \frac{1}{3}x^3 + C$	[1] [1]	Award one mark for correct integration with or without constant C. Accept for two marks $y = [-(1 - x)^3]/3 + C$	
6	(c)		Given $y = (x^3 - 2x)/3x$ Then integral = $\int \{(x^3 - 2x)/3x\} dx$ = $\int \{(x^3/3x) - (2x/3x)\} dx$ = $\int \{(x^2/3) - (2/3)\} dx$ = $(x^3/9) - (2x/3) + C$	[1] [1] [2]	Award one mark for correct integration with or without constant C. Award one mark for $(x^{3}/9)$ and one mark for – $(2x/3)$	
			Total	[10]		

Question		n	Expected Answer			Ν	Mark	Rationale/Additional Guidance
7	(a)	(i)						
			Time (minutes)	Tally	Frequency		[2]	Consider frequency column: award two marks for all correct
			21 – 30		9			frequencies . Award one mark for two correct frequencies.
			31 – 40	11111 11111	10			
			41 – 50	1111	4			
			51 – 60	11111 11	7			

Question Expected Answer						Rationale/Additional Guidance
Qu 7	(a)	<u>1</u> (ii)	Expected Answer frequency   12   11   10   9   8   7		Mark	Rationale/Additional Guidance         Allow error carried forward.         Award one mark for each correct bar.
			12 11			Allow error carried forward.
			10		[4]	
			8			Award one mark for each correct
			7			bar.
			6			
			5			
			4			
			2			
			1			
			0			
			20.5	30.5 40.5 5	0.5 60.5	

Question		n	Expected Answer	Mark	Rationale/Additional Guidance
7	(b)		First quartile = 23.5 Third quartile = 31.5 Interquartile range = $31.5 - 23.5$ = 8	[1] [1] [1] [1]	First quartile – accept answers between 23 and 24. Third quartile – accept answers between 31 and 32.
					Interquartile range – accept answers between 7 and 9. Allow error carried forward.
			Total	[10]	

Question		n	Expected Answer		Rationale/Additional Guidance
8	(a)	(i)	A <b>trial</b> is any process which, when repeated generates a set of results or observations or a single repetition of an experiment.		Award one mark for each correct explanation.
		(ii)	An <b>outcome</b> is the result of carrying out a trial.	[3]	Accept any form of words that gives an adequate explanation.
		(iii)	An <b>event</b> is a set which consists of one or more of the possible outcomes of a trial or a subset of sample space.		
	(b)		The first three positions can be filled in 3! ways and the next in 5! ways. Clients with eight-character codes = $3! \times 5!$		
			= (3 x 2 x 1) x (5 x 4 x 3 x 2 x1) = 720	[1] [1]	
	(c)	(i)	Probability of three fives = $(1/6)^3 = 1/216$ or 0.00463	[1] [1] [1] [1] [1]	Accept answers between 0.0045
		(ii)	The first dice may be a five with the second and third not or the second a dive and the first and third not or the third a five with the first and second not. Probability of one, and only one, five $= 3[(1/6] \times (5/6) \times (5/6)] = 25/72$ or 0.347		and 0.0047. Accept answers between 0.34 and 0.35.
		(iii)	Probability of obtaining at least one five = $1 - $ the probability of obtaining no fives		
			$= 1 - (5/6)^3 = 91/216$ or 0.421		
					Accept answers between Accept answers between 0.42 and 0.425.
			Total	[10]	

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