LEVEL 3 CERTIFICATE IN
Topic Exploration Pack

H866

QUANTITATIVE REASONING (MEI)

Fermi Estimation

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Contents

This Topic Exploration Pack should accompany the OCR resource '*Fermi estimation'* learner activities 1 and 2, which you can download from the OCR website.

This activity offers an opportunity for maths skills development.

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Fermi Estimation

Introduction

The concept of Fermi estimation is incredibly important. Whilst this would not have been covered explicitly at Key Stages 3 and 4 most learners would have had some experience in using this technique; whether they knew the formal name for it or not. Indeed, in this era of mobile phone technology, where most people have almost constant access to a calculator on their mobile phone, it makes the process of Fermi estimation more applicable and relevant to learners' lives than most other mathematical concepts.

The aim of Fermi estimation is to provide a realistic estimate to a numerical problem by making a series of approximations and then using the four operations (+, -, ×, ÷) to reach an estimate. This is by no means an accurate method of obtaining an answer but by following the process a 'good' estimate can be found with little working out, whereas an accurate answer may need far more detailed calculations or may not even be obtainable at all. It teaches learners the idea of making 'plausible estimates' and it provides a way of checking to see if their accurate answer is correct. The process is demonstrated by an example.

Say we were asked to find out the number of piano tuners in the city of Norwich. First of all, an accurate answer can be obtained (by looking in business directories etc.) but we wish to get a 'plausible' estimate by making a series of approximations. The approximations we need to make are:

- What is the population of Norwich?
- How many houses in Norwich?
- How many houses have pianos?
- How many times does a piano have to be tuned?
- How many pianos can be tuned per day?
- How many days does a piano tuner work?

Now for someone not aware of the size of the city of Norwich, this could be found from an internet engine search (not guaranteed to be correct!) and then round it to an 'easier' number. The exact population of Norwich is 214 000 so we approximate this to 200 000, to make the calculations easier.

Now from this initial estimation we can make the other 'plausible' estimates. We make the assumption that there are 4 people living in each house ('plausible') and hence there will be 200 000 ÷ 4 = 50 000 houses in Norwich.

Now how many pianos are in these houses? Let's make an assumption there is one piano every 50 houses, this means that there are 50 000 ÷ 50 = 1000 pianos in Norwich.

Now we have to estimate how many times a year they need to be tuned; let's say twice a year. That means that there are 1000 × 2 = 2000 piano tunings a year.

How many days does a tuner work? Let's assume they have the standard 4 weeks holiday and work for 48 weeks a year, not including weekends, and hence work for 48 × 5 = 240 days a year. If a tuner can do 2 tunings a day then there are 2000 ÷ 2000 = 1000 days of work available to the piano tuners.

Therefore as each piano tuner works for 240 days then the number of piano tuners is 1000 ÷ 240 = 4.16, so approximately 4 piano tuners.

The assumptions made were:

- 200 000 people live in Norwich.
- 4 people per household.
- 1 piano every 50 houses.
- 2 piano tunings a year.
- It takes $\frac{1}{2}$ a day to tune a piano.
- A piano tuner works for 48 weeks of the year.

These assumptions are 'back of an envelope' estimations of what happens in reality; but they are plausible estimates; they are not completely off the mark. You could argue that an estimate of 4 people per house is too much but if it is only 2 people per house, the size of the eventual answer doesn't change much. In fact it is this aspect of Fermi estimation which makes it so appealing. If we over-estimate or under-estimate the assumptions, they will balance themselves out in the overall answer. [Note the actual answer is 7 if you look at a map provided by yell.com (see activity 1 solutions).]

Misconceptions

The main misconceptions that learners may have are:

- When to use division and multiplication. Usually a check on the suitability of their answer will help them overcome this. For example if we had done 600 x 240 in the final calculation at the end we would have 144 000 piano tuners in a city of 120 000 people! This is clearly incorrect and hence division should be done instead of multiplication.

- Learners may choose numbers that are difficult to perform arithmetic with. Although calculators are fine to use, it is useful for learners to choose 'nice' numbers that have lots of divisors so that the calculations can be performed in the head or by hand rather than solely using the calculator. In the previous example 200 000 was chosen as a population instead of 210 000 because 200 000 is easily divided by 4 and 5 whereas 210 000 is not, although it can be divided, less usefully, by 3 and 7. Also, it is often easier to do mental calculations with numbers written to one significant figure, rather than two significant figures.

- A lack of life experience may hinder the learners approach to Fermi estimation. It would be unreasonable for learners to be able to make the assumptions in the previous example as they wouldn't, in general, know about the life and workings of an average piano tuner. So, taking into account a learner's life experience is essential when deciding what problems they should attempt. Alternatively, encourage learners to use internet search engines to get some rough estimates. We live in a world of near 24 hour internet access and there is no reason why this cannot be utilised to provide some initial estimates.

Activity 1 – Piano Tuners

In this introductory activity learners are required to work out the number of piano tuners in their nearest city. They can then find the exact amount using the internet and compare this with their estimate. Note calculators are allowed and are encouraged, as this is what would happen in real-life; the difficulty for your learners will be deciding when to use multiplication or division. Their answers may be different, as the assumptions they make could be different. Use the city of Norwich example to demonstrate to the learners the steps required, or you may want to provide an example for the town nearest to you.

Explain that the Italian scientist Enrico Fermi (responsible for the naming of the sub-atomic particle the Fermion amongst other scientific achievements) developed this method to find the number of piano tuners in Chicago.

Example Solution

Make plausible estimates for the following questions *(you may use an internet search engine to find estimates only!)*

1. What is your nearest city?

Answer: Norwich

2. What is the population of your nearest city?

Answer: 214 000 people

3. Round the population to a suitable degree of accuracy.

Answer: 200 000 people

4. How many people are there per household on average? How many houses?

Answer: 4; 200 000 ÷ 4 = 50 000 houses

5. How many houses are there per piano? How many pianos in total?

Answer: 50; pianos 50 000 ÷ 50 = 1000 pianos

6. How many times does a piano need tuning per year?

Answer: Twice

7. How many tunings are there per year?

Answer: 2 x 1000 = 2000

8. How many days does a tuner work?

Answer: 48 x 5 = 240 days

9. How many tunings can a tuner do per day?

Answer: 2

10. How many tunings can a single tuner do each year?

Answer: 240 x 2 = 480

11. Divide your answer to 7) by your answer to 10) to find the number of piano tuners in your nearest city.

Answer: 2000 ÷ 480 = 4.167

12. Check the exact answer by performing an internet search and looking at Google maps or equivalent. How close were you to the correct answer? What estimate do you think could be improved to get a more accurate answer?

Actual number 7 – see figure below. The answer was quite accurate – a bit smaller than reality, but this estimate only took into account private homes. There would be more pianos and hence piano tuners if we had included school, pubs, music halls etc.
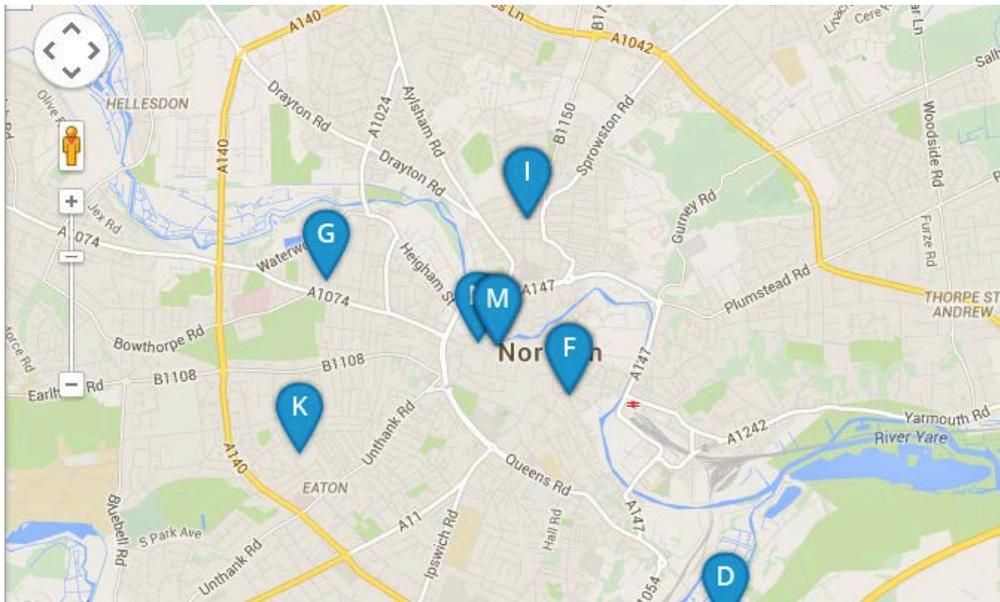
Map Data: Google Maps via Yell.com
Map data ©Google 2015

Activity 2 – Daily Life

This is a more detailed Fermi estimation following on from Activity 1 and less guidance is given in terms of questions they need to ask.

Figures and statistics are often quoted by newspapers/TV and radio etc. to create a certain impression or a 'shock' effect. Learners are going to investigate the following claims and decide whether they are accurate or not by using Fermi estimation techniques.

"There are more than 100 billion hours spent playing computer games/mobile phone apps in the UK."

Learners will split the population into different age groups to perform a more detailed estimation to account for the fact that different age groups behave significantly differently to each other when considering mobile phone app usage. There is a table on the student sheet to help guide the initial part of the problem and then some questions to help steer them in the correct direction. Note there is no one correct way of doing these calculations, so encourage methods that are different to the one mentioned here; use the solution as a guide/example only. In the extension task, learners should be encouraged to perform the calculations in the way that best suits them.

Example Solution

First we have to calculate the number of hours played per day per person. This will vary from age group to age group. Using the table:

Age Group	Hours Per day	Days played per year	Total hours per year
Less than 10	1	4 × 52 = 208	1 × 208 = 208
Between 10 and 16	5	6 × 52 = 312	5 × 312 = 1560
Between 16 and 18	6	7 × 52 = 364	6 × 364 = 2184
Between 18 and 30	3	5 × 52 = 260	3 × 260 = 780
Over 30	0.5	4 × 52 = 208	0.5 × 208 = 104

Now the total population of the UK is 60 000 000, of which there are approximately: 60% over 30; 20% between 18 and 30; 5% between 16 and 18; 5% between 10 and 16; and 10% less than 10. Therefore working out the number of hours for each age group gives:

(600 000 000 × 0.6 × 104) + (60 000 000 × 0.2 × 780) + (60 000 000 × 0.05 × 2184) + (60 000 000 × 0.05 × 1560) + (60 000 000 × 0.01 × 208)

= 3 744 000 000 + 9 360 000 000 + 6 552 000 000 + 4 680 000 000 + 1 248 000 000

= 2.5584×10^{10} hours.

In comparison to 100 billion hours = 1×10^{11} hours; this is smaller than the claim suggested. There are two possible reasons for this:

1. The estimates made were under-estimates. It could be that, for example, the number of hours for the over 30 group has been under-estimated.
2. The 100 billion hours quoted was an over-estimate and exaggerated.

The nature of Fermi estimation means that there could be massive discrepancies between the answer and quoted answer but it doesn't mean the calculations are necessarily incorrect; it could be the claim is incorrect. However, in this case as we are out by a factor of 10, then perhaps an under-estimation is more likely. This could be explored as an extension task:

"Can you find estimates so that the number of hours per day is as close to 1 billion as possible?"

This question could be given to early finishers.

Activity 3 – 'You Reckon'

In this activity the package 'You Reckon' has to be accessed from the website http://www.bowlandmaths.org.uk/projects/you_reckon.html. The Bowland Maths website has a number of very interesting projects that can take 3-5 lessons for the learners to complete but the 'You Reckon' module has a number of shorter as well as longer activities. Click on the 'Run online' button at the top of the screen and a list of activities come up with full teaching notes. They are excellent and engaging activities for practising Fermi estimation and making plausible estimates in interesting examples. A couple of examples include:

Stop Thief (Section C: Lesson Blocks)

A bank is claiming that a robber ran off with £5000 in £1 coins.
Could the robber lift this much money?

Pavarotti (Section E: One Liners)

The Chancellor of the Exchequer wants to give pensioners an extra £10 a year. How much would this cost the government per year?
There are full teacher's notes and printed worksheets on the website.

Extension Activity – The Drake Equation

This is a research activity to help consolidate learning on Fermi estimation in the interesting context of extra-terrestrial civilisations.

The Drake equation gives an approximate number of intelligent civilisations in the nearby galaxy. Learners' are expected to research this equation and create a presentation on it. This is an independent research task and can be used as an introductory homework task or as a consolidation task at the end of the topic. The presentations must include:

- The Drake equation
- History of the Drake equation
- The first estimate of Drake's equation
- A current estimate for the answer to Drake's equation
- An explanation of Fermi's paradox

Learners should use the internet and other research materials in order to complete this project and presentations can include PowerPoints, hand-outs, discussions, debates etc. The presentations could form the basis of a debate about whether there are intelligent species on other planets.

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