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LEVEL 3 CERTIFICATE IN

Topic Exploration Pack

H866

QUANTITATIVE REASONING (MEI)

Medical Testing

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MEI Innovators in Mathematics Education

OCR Oxford Cambridge and RSA

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Contents

This Topic Exploration Pack should accompany the OCR resource '*Medical Testing'* learner activity, which you can download from the OCR website.

This activity offers an opportunity for maths skills development.

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Introduction

Medical testing is an incredibly important process. Recent events have shown that the need for reliable medicines/vaccines is of paramount importance and one of the major issues of the current day. These medicines have to be tested on a large sample and certain confidence intervals have to be established in order to decide how reliable/successful a medicine will be on a certain patient.

The other side of this is the ability to detect diseases or drugs in the system. Essentially how reliable is a test that decides whether a patient has a disease or not. These tests have to be as close to 100% reliable as possible. It would be no good if the test consistently gave a negative result when it should be positive and vice versa.

The world of sport also needs a measure of the reliability of medical tests; this time in the context of detecting athletes who have used banned substances to enhance their sporting performance. The recent scandals surrounding the sport of cycling have highlighted the issues of how difficult it is to detect a drugs cheat and how accurate medical tests are.

It should be noted that no medical test will be 100% reliable or accurate. Scientists and pharmacists want to be as close to 100% as possible, however. The level to which they want it close to 100% is called the significance level and this is a difficult statistical concept that is not fully developed here but throughout the following activities the over-riding question is this: “How close to 100% is ‘okay’?”. For a test for a dangerous disease you might reasonably want this to be as close as possible, say 99.99%, but this may not be possible. When do we decide that a test is reliable?

This is not an easy question to answer but it should provide a background to the following mathematical explorations.

Prior Knowledge

- Students should be able to calculate a theoretical probability as:

$$P\ (Event) = \frac{No.\ of\ ways\ event\ can\ happen}{No.of\ \ possible\ outcomes}$$

- Students should be able to calculate a relative frequency; a measure of experimental probability:

$$Rel.Freq = \frac{Frequency\ of\ event}{Total\ frequency\ of\ all\ events}$$

- Students should understand the concepts of 'mutually exclusiveness' and 'independence' when using probability.

- Students should be able to draw a probability tree diagram and be able to calculate probabilities from it.

- Students should be able to use the 'OR' rule for mutually exclusive events:

Prob (A OR B) = Prob (A) + Prob (B)

- Students should be able to use the 'AND' rule for independent events:

Prob (*A AND B*) = *Prob* (*A*) × *Prob* (*B*)

Misconceptions

- The fundamental misconception when dealing with probability is the idea of 'randomness'. Unfortunately, in the last couple of years the word 'random' has entered the vernacular as a term being something bizarre happening. Learners could confuse a random event as being a 'rare' event rather than the actual meaning. There are a number of different definitions for the word 'random' but to keep it simple emphasise to students that a random process doesn't produce rare results but it produces results that are impossible to *predict*. Randomness is based on our inability to predict what will happen at a given moment of time. We may have probabilities to guide our predictions but essentially there is no way to be able to predict the outcome of a random event.

- Conditional probability is a very tricky concept for a student to understand. The best way to explain this is in the context of a sportsman taking part in a series of races each weekend. If, in the first race, the athlete wins then this could feasibly give them more confidence and hence the chances of them winning the next race increases. However, if the athlete loses the first race then they might lose confidence and the chances of them winning the next race could decrease. It is locked into the idea of dependent events. If one event affects the outcome of another then they are dependent events and the probability will depend on the outcome of the other.

Activity A – Performance enhancing drugs testing – Two-way tables

There is an annual cycle race, ‘Tour De Bidmas’, where 200 competitors take part each year. The previous year all the athletes were tested for performance enhancing drugs (PEDs) using a test and the results are shown in the table below. Some of the athletes got a positive result from the test but were later proven to be not guilty and had not taken any PED. Conversely, some athletes got a negative result and then were later proven to actually have taken them. Obviously, there is an unknown number who took a PED and got a negative result so were not proven guilty. It should be noted that if an athlete got a negative test result they were not investigated further unless there were extenuating circumstances.

	Positive Test	Negative Test	Total
Proven positive	40	5	45
Proven negative	10	145	155
	50	150	200

Students are to use this data to calculate conditional probabilities. The idea of calculating these probabilities from a two-way table is that the condition needs to be isolated and then the probability can be calculated. For example, to find the probability that an athlete is proven positive given they had a positive test we have to first isolate the condition; in this case they had a positive test:

	Positive Test	Negative Test	Total
Proven positive	40	5	45
Proven negative	10	145	155
	50	150	200

Now the condition has been isolated the probability can be calculated as there are 40 athletes who were positive *given* they had a positive test and there are 50 athletes who had a positive test. Hence the probability is $\frac{40}{50} = \frac{4}{5} = 80\%$

Another example is to find the probability that an athlete has had a positive test *given* they are proven positive. In this case the condition to be highlighted is:

	Positive Test	Negative Test	Total
Proven positive	40	5	45
Proven negative	10	145	155
	50	150	200

In this case the probability is $\frac{40}{45} = \frac{8}{9} \approx 89\%$.

Doing an example like this on the board will give students the tools to be able to answer the questions. You could use mini-whiteboards to get students to answer some questions using different numbers in the table to obtain a gauge on their understanding before giving them the worksheet. The wording in the questions is key here. Try to get students to separate each question into a condition and a probability, i.e. for the 5th question it could be separated like this:

Q5) What is the probability that an athlete has taken a PED given they had a positive test?

The blue text is the condition and helps isolate the column they need to use. The red text then tells them what probability they are looking for.

Questions

1) What is the probability that an athlete will have a positive test given they have taken a PED?

Answer: 40/45

2) What is the probability that an athlete will have a negative test given they have taken a PED?

Answer: 5/45

3) What is the probability that an athlete will have a positive test given they haven't taken a PED?

Answer: 10/155

4) What is the probability that an athlete will have a negative test given they haven't taken a PED?

Answer: 145/155

5) What is the probability that an athlete has taken a PED given they had a positive test?

Answer: 40/50

6) What is the probability that an athlete hasn't taken a PED given they had a positive test?

Answer: 10/50

7) What is the probability that an athlete has taken a PED given they had a negative test?

Answer: 5/150

8) What is the probability that an athlete hasn't taken a PED given they had a negative test?

Answer: 145/150

9) If there are 300 competitors in next year's event, how many would you expect to catch who are taking PEDs?

Answer:

Probability that someone has taken drugs is $\frac{45}{200}$

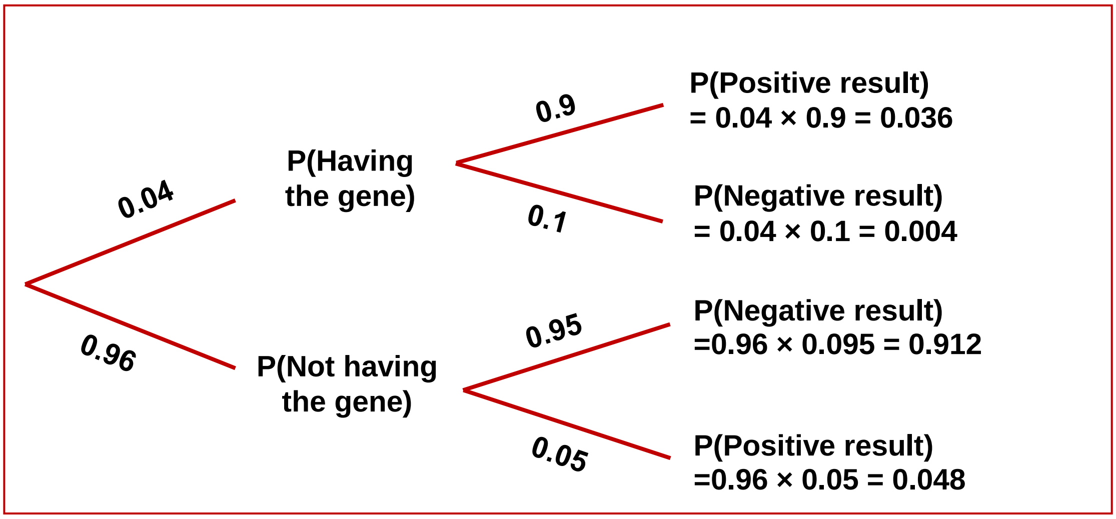
Hence expected number is $\frac{45}{200} \times 300 = 67.5$ athletes

Activity B – Diagnosing – Tree diagrams

It is known that 4% of the population have a particular gene that means that they are more likely to develop a certain medical condition. Scientists have developed a test that helps detect this gene. The outcomes of the test are either positive for the gene or negative for the gene. After numerous medical tests the scientists believe that if a person has the gene then there is a 90% chance the person will test positive. If the person doesn't have the gene then there is a 95% chance the person will test negative.

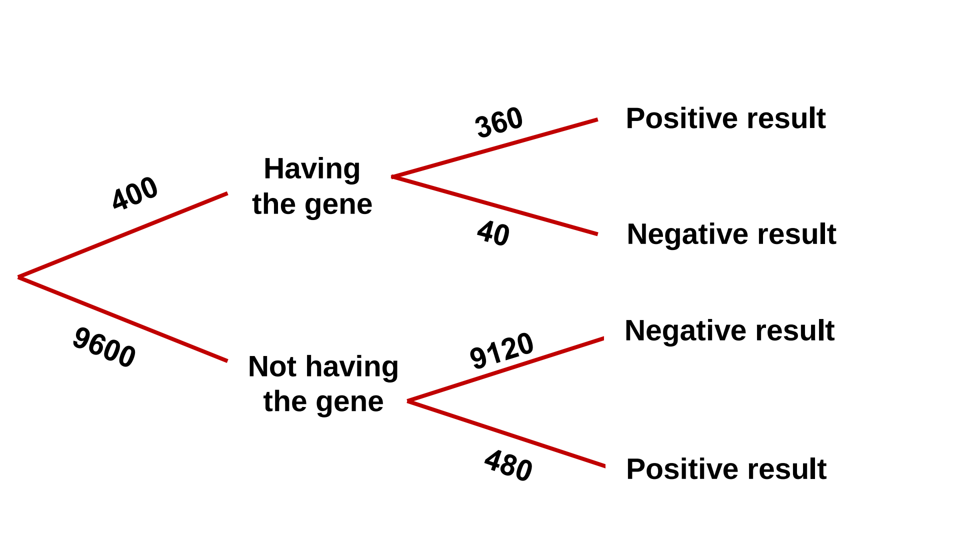
When students have to display this information in a tree diagram the difficult bit for them is to decide which 'event' occurs first. Do they choose the testing +ve and –ve as the first branch or the fact that a resident has the gene or not as their first branch. To explain this use the fact that a person will have the gene *before* they have the test. Therefore the test is dependent on whether the gene is present or not. The first branch on a tree diagram should be whether a person has the gene or not. Remind students that a way to check if they are correct or not is to add up the probabilities at the end to see if they add to 1. If they don't then they've made a mistake somewhere!

1) Display this information in a probability tree diagram by calculating the probabilities along the branches and finding the probabilities at the end.



2) In a particular town of 10 000 people all of the residents are tested. How many would you expect to test positive given they have the particular gene?

NB. For questions 2 and 3 learners may find it easier to draw the tree diagram again or update the one above with the amount of people in each branch (see below).



Answer:

Probability of testing positive given they have the gene is 0.036. Therefore number we would expect to detect is 10 000 x 0.036 = 360 residents.

3) How many would you expect to have the gene but test negative?

Answer:

Probability of testing negative given they have the gene is 0.004. Therefore number we would expect to not detect is 10 000 x 0.004 = 40

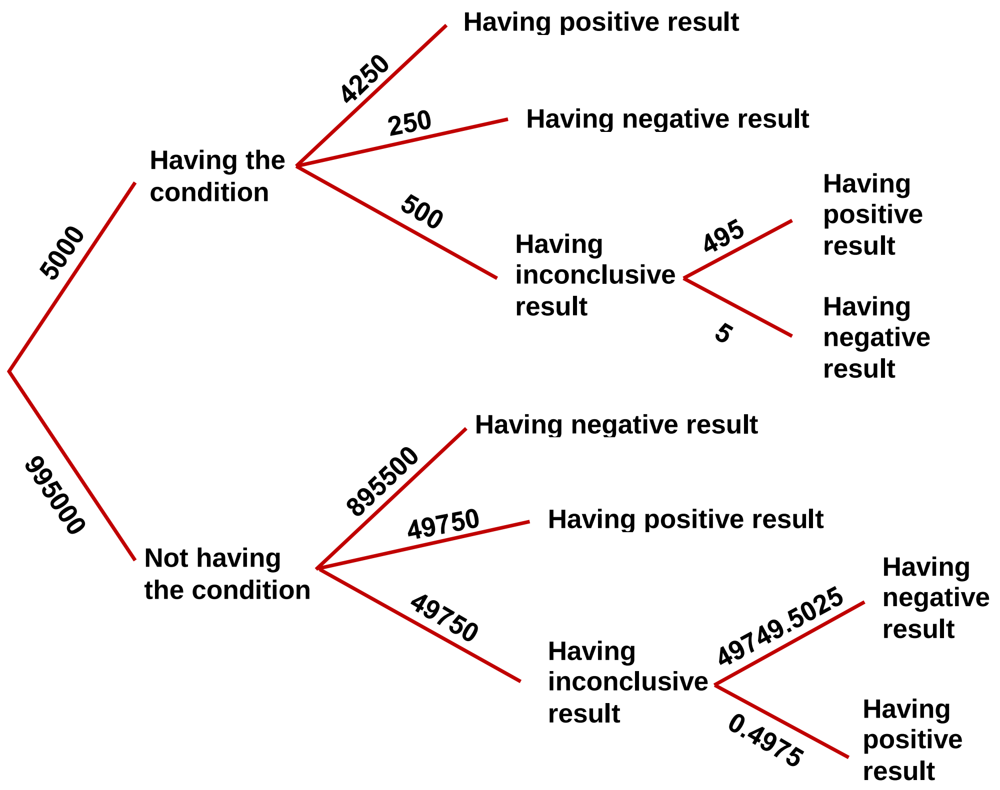
Activity C – Advanced medical testing

Pharmaceutical companies manufacture a test that checks whether patients have a particular medical condition. This test has been trialled extensively among one million people. The outcomes indicated that 0.5% of the population have this particular medical condition. After testing, the outcomes are positive, negative or unsure. It is found that if the patient has the condition then there is an 85% chance that they test positive, 5% chance they test negative and a 10% chance the results are inconclusive. If the patient doesn't have the condition then there is a 90% chance they test negative, 5% chance they test positive and a 5% chance the tests are inconclusive.

If the results are inconclusive then the patient has to have a thorough examination. If they do not have the condition then the probability that a thorough examination determines they have the condition is 0.001%. If the patient does have the condition then the probability that a thorough examination detects this is 99%.

1) Draw a frequency tree diagram to display this information.

Answer:



2) How many people that do have the condition do not get diagnosed?

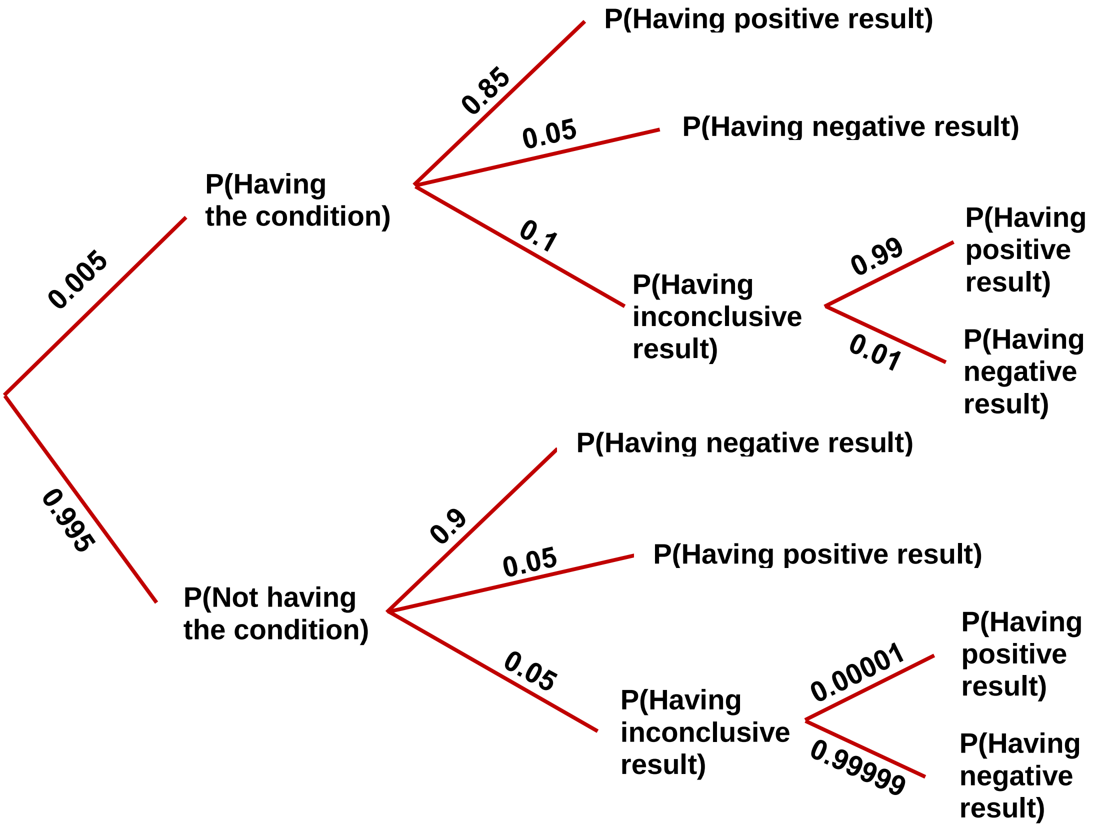
Answer:
People with negative result given they have the condition + People with negative result given they have the condition and inconclusive result = 250 + 5 = 255.

3) How many people that do not have the condition get a positive test result?

Answer:
People with positive result given they do not have the condition + People with positive result given they do not have the condition and inconclusive result = 49750 + 0.4975 = 49750.4975

4) Draw a probability tree diagram to display the original information.

Answer:



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