

**Advanced Subsidiary GCE (H635)  
Advanced GCE (H645)**

**Further Mathematics B (MEI)**

**Formulae Booklet**

The information in this booklet is for the use of candidates following the Advanced Subsidiary in Further Mathematics B (MEI)(H635) or the Advanced GCE in Further Mathematics B (MEI) (H645) course.

The formulae booklet will be printed for distribution with the examination papers.

Copies of this booklet may be used for teaching.

This document consists of **16** pages.

**Instructions to Exams Officer/Invigilator**

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## A level Mathematics

### Arithmetic series

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n \{2a + (n - 1)d\}$$

### Geometric series

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \text{ for } |r| < 1$$

### Binomial series

$$(a + b)^n = a^n + {}^n C_1 a^{n-1} b + {}^n C_2 a^{n-2} b^2 + \dots + {}^n C_r a^{n-r} b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^n C_r = {}_n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

### Differentiation

f(x)	f'(x)
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient Rule } y = \frac{u}{v}, \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

### Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

**Integration**

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

**Small Angle Approximations**

$\sin \theta \approx \theta$ ,  $\cos \theta \approx 1 - \frac{1}{2}\theta^2$ ,  $\tan \theta \approx \theta$  where  $\theta$  is measured in radians

**Trigonometric identities**

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi)$$

**Numerical methods**

Trapezium rule:  $\int_a^b y dx \approx \frac{1}{2}h \{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$ , where  $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving  $f(x) = 0$ :  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

**Probability**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B | A) = P(B)P(A | B) \quad \text{or} \quad P(A | B) = \frac{P(A \cap B)}{P(B)}$$

**Sample Variance**

$$s^2 = \frac{1}{n-1} S_{xx} \quad \text{where} \quad S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

Standard deviation,  $s = \sqrt{\text{variance}}$

### The Binomial Distribution

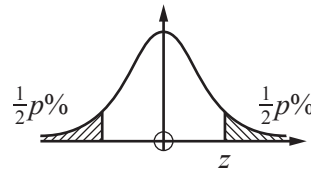
If  $X \sim B(n, p)$  then  $P(X = r) = {}^n C_r p^r q^{n-r}$  where  $q = 1 - p$   
 Mean of  $X$  is  $np$

### Hypothesis test for the mean of a Normal distribution

If  $X \sim N(\mu, \sigma^2)$  then  $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$  and  $\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$

### Percentage points of the normal distribution

$p$	10	5	2	1
$z$	1.645	1.960	2.326	2.576



### Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two and three dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

-

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

## Core Pure

### Complex Numbers

De Moivre's theorem:

$$\{r(\cos \theta + i \sin \theta)\}^n = r^n (\cos n\theta + i \sin n\theta)$$

Roots of unity:

The roots of  $z^n = 1$  are given by  $z = \exp\left(\frac{2\pi k}{n}i\right)$  for  $k = 0, 1, 2, \dots, n-1$

### Vectors and 3-D geometry

Cartesian equation of a plane is

$$n_1x + n_2y + n_3z + d = 0$$

Cartesian equation of a line in 3-D is

$$\frac{x-a_1}{d_1} = \frac{y-a_2}{d_2} = \frac{z-a_3}{d_3}$$

$$\text{Vector product } \mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$$

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix} = \begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix} = |\mathbf{a}||\mathbf{b}|\sin \theta \hat{\mathbf{n}} \text{ where } \mathbf{a}, \mathbf{b}, \hat{\mathbf{n}}, \text{ in that order, form a right-handed triple.}$$

Distance between skew lines is  $\frac{|\mathbf{d}_1 \times \mathbf{d}_2 \cdot (\mathbf{a}_1 - \mathbf{a}_2)|}{\|\mathbf{d}_1 \times \mathbf{d}_2\|}$  where  $\mathbf{a}_1$  is the position vector of a point on the first line and  $\mathbf{d}_1$  is parallel to the first line, similarly for the second line.

Distance between point  $(x_1, y_1)$  and line  $ax + by + c = 0$  is  $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

Distance between point  $(x_1, y_1, z_1)$  and plane  $n_1x + n_2y + n_3z + d = 0$  is  $\frac{|n_1x_1 + n_2y_1 + n_3z_1 + d|}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$

### Hyperbolic functions

$$\cosh^2 x - \sinh^2 x = 1$$

$$\operatorname{arsinh} x = \ln[x + \sqrt{(x^2 + 1)}]$$

$$\operatorname{arcosh} x = \ln[x + \sqrt{(x^2 - 1)}], x \geq 1$$

$$\operatorname{artanh} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right), -1 < x < 1$$

### Calculus

$f(x)$	$f'(x)$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

$f(x)$	$\int f(x) dx$
$\frac{1}{\sqrt{a^2-x^2}}$	$\arcsin\left(\frac{x}{a}\right) \quad ( x  < a)$
$\frac{1}{a^2+x^2}$	$\frac{1}{a} \arctan\left(\frac{x}{a}\right)$
$\frac{1}{\sqrt{a^2+x^2}}$	$\operatorname{arsinh}\left(\frac{x}{a}\right) \text{ or } \ln(x + \sqrt{x^2+a^2})$
$\frac{1}{\sqrt{x^2-a^2}}$	$\operatorname{arcosh}\left(\frac{x}{a}\right) \text{ or } \ln(x + \sqrt{x^2-a^2}) \quad (x > a)$

The mean value of  $f(x)$  on the interval  $[a,b]$  is  $\frac{1}{b-a} \int_a^b f(x) dx$

Area of sector enclosed by polar curve is  $\frac{1}{2} \int r^2 d\theta$

### Series

$$\sum_{r=1}^n r^2 = \frac{1}{6} n(n+1)(2n+1) \quad \sum_{r=1}^n r^3 = \frac{1}{4} n^2(n+1)^2$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \dots + \frac{f^{(r)}(0)}{r!}x^r + \dots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^r}{r!} + \dots \text{ for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{r+1} \frac{x^r}{r} + \dots \quad (-1 < x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \dots \text{ for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^r \frac{x^{2r}}{(2r)!} + \dots \text{ for all } x$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

## Mechanics

### Motion in a circle

For motion in a circle,

tangential velocity is  $v = r\dot{\theta}$

radial acceleration is  $\frac{v^2}{r}$  or  $r\dot{\theta}^2$  towards the centre

tangential acceleration is  $r\ddot{\theta}$

## Further Pure with Technology

### Numerical solution of differential equations

For  $\frac{dy}{dx} = f(x, y)$ :

Euler's method:  $x_{n+1} = x_n + h$     $y_{n+1} = y_n + hf(x_n, y_n)$

Modified Euler method (A Runge-Kutta method of order 2):

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf(x_n + h, y_n + k_1)$$

$$x_{n+1} = x_n + h, y_{n+1} = y_n + \frac{1}{2}(k_1 + k_2)$$

Runge-Kutta method of order 4:

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

### Gradient of tangent to a polar curve

For a curve  $r = f(\theta)$ ,  $\frac{dy}{dx} = \frac{\frac{dr}{d\theta} \sin \theta + r \cos \theta}{\frac{dr}{d\theta} \cos \theta - r \sin \theta}$

## Extra Pure

### Multivariable calculus

$$\nabla g = \mathbf{grad} g = \begin{pmatrix} \frac{\partial g}{\partial x} \\ \frac{\partial g}{\partial y} \\ \frac{\partial g}{\partial z} \end{pmatrix}. \text{ If } g(x, y, z) \text{ can be written as } z = f(x, y) \text{ then } \mathbf{grad} g = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ -1 \end{pmatrix}$$



## Numerical methods

### Solution of equations

The Newton-Raphson iteration for solving  $f(x) = 0$ :  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

For the iteration  $x_{n+1} = g(x_n)$  the relaxed iteration is  $x_{n+1} = (1 - \lambda)x_n + \lambda g(x_n)$ .

### Numerical integration

To estimate  $\int_a^b f(x) dx$ :

The midpoint rule:

$$M_n = h(y_{\frac{1}{2}} + y_{\frac{3}{2}} + \dots + y_{n-\frac{3}{2}} + y_{n-\frac{1}{2}}) \quad \text{where } h = \frac{b-a}{n}$$

The trapezium rule:

$$T_n = \frac{1}{2}h \{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\} \quad \text{where } h = \frac{b-a}{n}$$

Simpson's rule

$$S_{2n} = \frac{1}{3}h \{(y_0 + y_{2n}) + 4(y_1 + y_3 + \dots + y_{2n-1}) + 2(y_2 + y_4 + \dots + y_{2n-2})\}$$

$$\text{where } h = \frac{b-a}{2n}$$

These are related as follows:

$$T_{2n} = \frac{1}{2}(M_n + T_n)$$

$$S_{2n} = \frac{1}{3}(2M_n + T_n) = \frac{1}{3}(4T_{2n} - T_n)$$

### Interpolation

Newton's forward difference interpolation formula:

$$f(x) = f(x_0) + \frac{(x-x_0)}{h} \Delta f(x_0) + \frac{(x-x_0)(x-x_1)}{2!h^2} \Delta^2 f(x_0) + \dots$$

Lagrange's polynomial:

$$P_n(x) = \sum L_r(x) f(x_r) \quad \text{where } L_r(x) = \prod_{\substack{i=0 \\ i \neq r}}^n \frac{x-x_i}{x_r-x_i}$$

## Statistics

### Discrete distributions

$X$  is a random variable taking values  $x_i$  in a discrete distribution with  $P(X = x_i) = p_i$

Expectation:  $\mu = E(X) = \sum x_i p_i$

Variance:  $\sigma^2 = \text{Var}(X) = \sum (x_i - \mu)^2 p_i = \sum x_i^2 p_i - \mu^2$

	Probability	$E(X)$	$\text{Var}(X)$
Uniform distribution over $1, 2, \dots, n$	$P(X = r) = \frac{1}{n}$	$\frac{n+1}{2}$	$\frac{1}{12}(n^2 - 1)$
Geometric distribution	$P(X = r) = q^{r-1}p$ $q = 1 - p$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Poisson distribution	$P(X = r) = e^{-\lambda} \frac{\lambda^r}{r!}$		

### Correlation and regression

For a sample of  $n$  pairs of observations  $(x_i, y_i)$

$$S_{xx} = \sum x_i^2 - \frac{(\sum x_i)^2}{n}, S_{yy} = \sum y_i^2 - \frac{(\sum y_i)^2}{n}, S_{xy} = \sum x_i y_i - \frac{\sum x_i \sum y_i}{n}$$

product moment correlation coefficient:  $r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sqrt{\left(\sum x_i^2 - \frac{(\sum x_i)^2}{n}\right) \left(\sum y_i^2 - \frac{(\sum y_i)^2}{n}\right)}}$

least squares regression line of  $y$  on  $x$  is  $y - \bar{y} = b(x - \bar{x})$  where  $b = \frac{S_{xy}}{S_{xx}} = \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}$

least squares regression line of  $x$  on  $y$  is  $x - \bar{x} = b'(y - \bar{y})$  where  $b' = \frac{S_{xy}}{S_{yy}} = \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sum y_i^2 - \frac{(\sum y_i)^2}{n}}$

Spearman's coefficient of rank correlation:

$$r_s = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)}$$

### Confidence intervals

To calculate a confidence interval for a mean or difference in mean in different circumstances, use the given distribution to calculate the critical value,  $k$ .

To estimate...	Confidence interval	Distribution
a mean	$\bar{x} \pm k \frac{\sigma}{\sqrt{n}}$	N(0, 1)
a mean	$\bar{x} \pm k \frac{s}{\sqrt{n}}$	$t_{n-1}$
difference in mean of paired populations	treat differences as a single distribution	

### Hypothesis tests

Description	Test statistic	Distribution
Pearson's product moment correlation test	$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$ $= \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sqrt{\left[\sum x_i^2 - \frac{(\sum x_i)^2}{n}\right] \left[\sum y_i^2 - \frac{(\sum y_i)^2}{n}\right]}}$	
Spearman's rank correlation test	$r_s = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)}$	
$\chi^2$ test	$\sum \frac{(f_o - f_e)^2}{f_e}$	$\chi_v^2$
Normal test for a mean	$\frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$	N(0, 1)
$t$ -test for a mean	$\frac{\bar{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)}$	$t_{n-1}$
Wilcoxon single sample test	A statistic $T$ is calculated from the ranked data	

### Continuous distributions

$X$  is a continuous random variable with probability density function (pdf)  $f(x)$

Expectation:  $\mu = E(X) = \int xf(x) dx$

Variance:  $\sigma^2 = \text{Var}(X) = \int (x - \mu)^2 f(x) dx = \int x^2 f(x) dx - \mu^2$

Cumulative distribution function  $F(x) = P(X \leq x) = \int_{-\infty}^x f(t) dt$

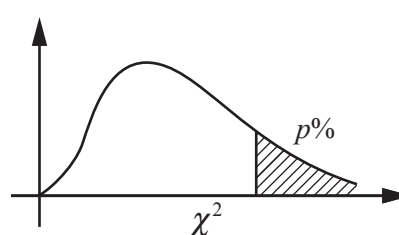
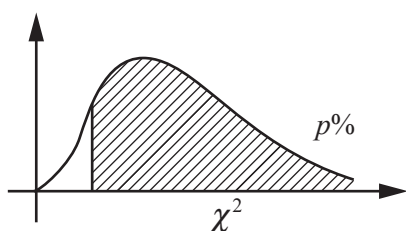
	$E(X)$	$\text{Var}(X)$
Continuous uniform distribution over $[a, b]$	$\frac{a+b}{2}$	$\frac{1}{12}(b-a)^2$

Critical values for the product moment correlation coefficient,  $r$ 

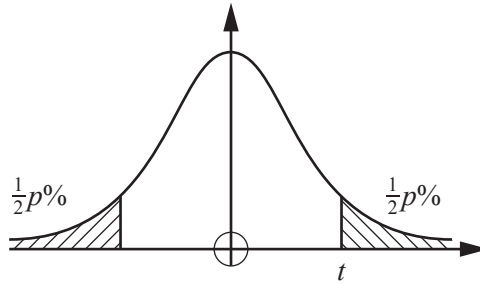
$n$	1-Tail Test				2-Tail Test			
	5%	2½%	1%	½%	10%	5%	2%	1%
1	-	-	-	-	-	-	-	-
2	-	-	-	-	-	-	-	-
3	0.9877	0.9969	0.9995	0.9999	0.3009	0.3550	0.4158	0.4556
4	0.9000	0.9500	0.9800	0.9900	0.2960	0.3494	0.4093	0.4487
5	0.8054	0.8783	0.9343	0.9587	0.2913	0.3440	0.4032	0.4421
6	0.7293	0.8114	0.8822	0.9172	0.2869	0.3388	0.3972	0.4357
7	0.6694	0.7545	0.8329	0.8745	0.2826	0.3338	0.3916	0.4296
8	0.6215	0.7067	0.7887	0.8343	0.2785	0.3291	0.3862	0.4238
9	0.5822	0.6664	0.7498	0.7977	0.2746	0.3246	0.3810	0.4182
10	0.5494	0.6319	0.7155	0.7646	0.2709	0.3202	0.3760	0.4128
11	0.5214	0.6021	0.6851	0.7348	0.2673	0.3160	0.3712	0.4076
12	0.4973	0.5760	0.6581	0.7079	0.2638	0.3120	0.3665	0.4026
13	0.4762	0.5529	0.6339	0.6835	0.2605	0.3081	0.3621	0.3978
14	0.4575	0.5324	0.6120	0.6614	0.2573	0.3044	0.3578	0.3932
15	0.4409	0.5140	0.5923	0.6411	0.2542	0.3008	0.3536	0.3887
16	0.4259	0.4973	0.5742	0.6226	0.2512	0.2973	0.3496	0.3843
17	0.4124	0.4821	0.5577	0.6055	0.2483	0.2940	0.3457	0.3801
18	0.4000	0.4683	0.5425	0.5897	0.2455	0.2907	0.3420	0.3761
19	0.3887	0.4555	0.5285	0.5751	0.2429	0.2876	0.3384	0.3721
20	0.3783	0.4438	0.5155	0.5614	0.2403	0.2845	0.3348	0.3683
21	0.3687	0.4329	0.5034	0.5487	0.2377	0.2816	0.3314	0.3646
22	0.3598	0.4227	0.4921	0.5368	0.2353	0.2787	0.3281	0.3610
23	0.3515	0.4132	0.4815	0.5256	0.2329	0.2759	0.3249	0.3575
24	0.3438	0.4044	0.4716	0.5151	0.2306	0.2732	0.3218	0.3542
25	0.3365	0.3961	0.4622	0.5052	0.2284	0.2706	0.3188	0.3509
26	0.3297	0.3882	0.4534	0.4958	0.2262	0.2681	0.3158	0.3477
27	0.3233	0.3809	0.4451	0.4869	0.2241	0.2656	0.3129	0.3445
28	0.3172	0.3739	0.4372	0.4785	0.2221	0.2632	0.3102	0.3415
29	0.3115	0.3673	0.4297	0.4705	0.2201	0.2609	0.3074	0.3385
30	0.3061	0.3610	0.4226	0.4629	0.2181	0.2586	0.3048	0.3357
					0.2162	0.2564	0.3022	0.3328
					0.2144	0.2542	0.2997	0.3301

Critical values for Spearman's rank correlation coefficient,  $r_s$ 

$n$	1-Tail Test				2-Tail Test			
	5%	2½%	1%	½%	10%	5%	2%	1%
1	-	-	-	-	-	-	-	-
2	-	-	-	-	-	-	-	-
3	-	-	-	-	-	-	-	-
4	1.0000	-	-	-	-	-	-	-
5	0.9000	1.0000	1.0000	-	-	-	-	-
6	0.8286	0.8857	0.9429	1.0000	0.3009	0.3550	0.4158	0.4556
7	0.7143	0.7857	0.8929	0.9286	0.2960	0.3494	0.4093	0.4487
8	0.6429	0.7381	0.8333	0.8810	0.2913	0.3440	0.4032	0.4421
9	0.6000	0.7000	0.7833	0.8333	0.2869	0.3388	0.3972	0.4357
10	0.5636	0.6485	0.7455	0.7939	0.2826	0.3338	0.3916	0.4296
11	0.5364	0.6182	0.7091	0.7545	0.2785	0.3291	0.3862	0.4238
12	0.5035	0.5874	0.6783	0.7273	0.2746	0.3246	0.3810	0.4182
13	0.4835	0.5604	0.6484	0.7033	0.2709	0.3202	0.3760	0.4128
14	0.4637	0.5385	0.6264	0.6791	0.2673	0.3160	0.3712	0.4076
15	0.4464	0.5214	0.6036	0.6536	0.2638	0.3120	0.3665	0.4026
16	0.4294	0.5029	0.5824	0.6353	0.2605	0.3081	0.3621	0.3978
17	0.4142	0.4877	0.5662	0.6176	0.2573	0.3044	0.3578	0.3932
18	0.4014	0.4716	0.5501	0.5996	0.2542	0.3008	0.3536	0.3887
19	0.3912	0.4596	0.5351	0.5842	0.2512	0.2973	0.3496	0.3843
20	0.3805	0.4466	0.5218	0.5699	0.2483	0.2940	0.3457	0.3801
21	0.3701	0.4364	0.5091	0.5558	0.2455	0.2907	0.3420	0.3761
22	0.3608	0.4252	0.4975	0.5438	0.2429	0.2876	0.3384	0.3721
23	0.3528	0.4160	0.4862	0.5316	0.2403	0.2845	0.3348	0.3683
24	0.3443	0.4070	0.4757	0.5209	0.2377	0.2816	0.3314	0.3646
25	0.3369	0.3977	0.4662	0.5108	0.2353	0.2787	0.3281	0.3610
26	0.3306	0.3901	0.4571	0.5009	0.2329	0.2759	0.3249	0.3575
27	0.3242	0.3828	0.4487	0.4915	0.2306	0.2732	0.3218	0.3542
28	0.3180	0.3755	0.4401	0.4828	0.2284	0.2706	0.3188	0.3509
29	0.3118	0.3685	0.4325	0.4749	0.2262	0.2681	0.3158	0.3477
30	0.3063	0.3624	0.4251	0.4670	0.2241	0.2656	0.3129	0.3445
					0.2221	0.2632	0.3102	0.3415
					0.2201	0.2609	0.3074	0.3385
					0.2181	0.2586	0.3048	0.3357
					0.2162	0.2564	0.3022	0.3328
					0.2144	0.2542	0.2997	0.3301

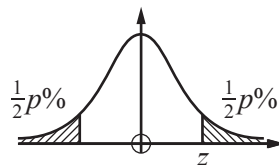
Percentage points of the  $\chi^2$  (chi-squared) distribution

$p\%$	99	97.5	95	90		10	5	2.5	1	0.5
$v = 1$	.0001	.0010	.0039	.0158		2.706	3.841	5.024	6.635	7.879
2	.0201	.0506	0.103	0.211		4.605	5.991	7.378	9.210	10.60
3	0.115	0.216	0.352	0.584		6.251	7.815	9.348	11.34	12.84
4	0.297	0.484	0.711	1.064		7.779	9.488	11.14	13.28	14.86
5	0.554	0.831	1.145	1.610		9.236	11.07	12.83	15.09	16.75
6	0.872	1.237	1.635	2.204		10.64	12.59	14.45	16.81	18.55
7	1.239	1.690	2.167	2.833		12.02	14.07	16.01	18.48	20.28
8	1.646	2.180	2.733	3.490		13.36	15.51	17.53	20.09	21.95
9	2.088	2.700	3.325	4.168		14.68	16.92	19.02	21.67	23.59
10	2.558	3.247	3.940	4.865		15.99	18.31	20.48	23.21	25.19
11	3.053	3.816	4.575	5.578		17.28	19.68	21.92	24.72	26.76
12	3.571	4.404	5.226	6.304		18.55	21.03	23.34	26.22	28.30
13	4.107	5.009	5.892	7.042		19.81	22.36	24.74	27.69	29.82
14	4.660	5.629	6.571	7.790		21.06	23.68	26.12	29.14	31.32
15	5.229	6.262	7.261	8.547		22.31	25.00	27.49	30.58	32.80
16	5.812	6.908	7.962	9.312		23.54	26.30	28.85	32.00	34.27
17	6.408	7.564	8.672	10.09		24.77	27.59	30.19	33.41	35.72
18	7.015	8.231	9.390	10.86		25.99	28.87	31.53	34.81	37.16
19	7.633	8.907	10.12	11.65		27.20	30.14	32.85	36.19	38.58
20	8.260	9.591	10.85	12.44		28.41	31.41	34.17	37.57	40.00
21	8.897	10.28	11.59	13.24		29.62	32.67	35.48	38.93	41.40
22	9.542	10.98	12.34	14.04		30.81	33.92	36.78	40.29	42.80
23	10.20	11.69	13.09	14.85		32.01	35.17	38.08	41.64	44.18
24	10.86	12.40	13.85	15.66		33.20	36.42	39.36	42.98	45.56
25	11.52	13.12	14.61	16.47		34.38	37.65	40.65	44.31	46.93
26	12.20	13.84	15.38	17.29		35.56	38.89	41.92	45.64	48.29
27	12.88	14.57	16.15	18.11		36.74	40.11	43.19	46.96	49.64
28	13.56	15.31	16.93	18.94		37.92	41.34	44.46	48.28	50.99
29	14.26	16.05	17.71	19.77		39.09	42.56	45.72	49.59	52.34
30	14.95	16.79	18.49	20.60		40.26	43.77	46.98	50.89	53.67
35	18.51	20.57	22.47	24.80		46.06	49.80	53.20	57.34	60.27
40	22.16	24.43	26.51	29.05		51.81	55.76	59.34	63.69	66.77
50	29.71	32.36	34.76	37.69		63.17	67.50	71.42	76.15	79.49
100	70.06	74.22	77.93	82.36		118.5	124.3	129.6	135.8	140.2

Percentage points of the  $t$  distribution

$v \backslash p\%$	10	5	2	1	
1	6.314	12.71	31.82	63.66	
2	2.920	4.303	6.965	9.925	
3	2.353	3.182	4.541	5.841	
4	2.132	2.776	3.747	4.604	
5	2.015	2.571	3.365	4.032	
6	1.943	2.447	3.143	3.707	
7	1.895	2.365	2.998	3.499	
8	1.860	2.306	2.896	3.355	
9	1.833	2.262	2.821	3.250	
10	1.812	2.228	2.764	3.169	
11	1.796	2.201	2.718	3.106	
12	1.782	2.179	2.681	3.055	
13	1.771	2.160	2.650	3.012	
14	1.761	2.145	2.624	2.977	
15	1.753	2.131	2.602	2.947	
20	1.725	2.086	2.528	2.845	
30	1.697	2.042	2.457	2.750	
50	1.676	2.009	2.403	2.678	
100	1.660	1.984	2.364	2.626	
$\infty$	1.645	1.960	2.326	2.576	= percentage points of the Normal distribution $N(0, 1)$

## Percentage points of the normal distribution



$p$	10	5	2	1
$z$	1.645	1.960	2.326	2.576

## Critical values for the Wilcoxon Single Sample test

1-tail	5%	2½%	1%	½%	1-tail	5%	2½%	1%	½%
2-tail	10%	5%	2%	1%	2-tail	10%	5%	2%	1%
<i>n</i>					<i>n</i>				
2	–	–	–	–	26	110	98	84	75
3	–	–	–	–	27	119	107	92	83
4	–	–	–	–	28	130	116	101	91
5	0	–	–	–	29	140	126	110	100
6	2	0	–	–	30	151	137	120	109
7	3	2	0	–	31	163	147	130	118
8	5	3	1	0	32	175	159	140	128
9	8	5	3	1	33	187	170	151	138
10	10	8	5	3	34	200	182	162	148
11	13	10	7	5	35	213	195	173	159
12	17	13	9	7	36	227	208	185	171
13	21	17	12	9	37	241	221	198	182
14	25	21	15	12	38	256	235	211	194
15	30	25	19	15	39	271	249	224	207
16	35	29	23	19	40	286	264	238	220
17	41	34	27	23	41	302	279	252	233
18	47	40	32	27	42	319	294	266	247
19	53	46	37	32	43	336	310	281	261
20	60	52	43	37	44	353	327	296	276
21	67	58	49	42	45	371	343	312	291
22	75	65	55	48	46	389	361	328	307
23	83	73	62	54	47	407	378	345	322
24	91	81	69	61	48	426	396	362	339
25	100	89	76	68	49	446	415	379	355
					50	466	434	397	373

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