Oxford Cambridge and RSA

Wednesday 24 June 2015 – Morning

A2 GCE MATHEMATICS (MEI)

4798/01 Further Pure Mathematics with Technology (FPT)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4798/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator
- Computer with appropriate software

Duration: Up to 2 hours

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

COMPUTING RESOURCES

- Candidates will require access to a computer with a computer algebra system, a spreadsheet, a programming language and graph-plotting software throughout the examination.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Turn over

1 This question concerns the family of curves with parametric equations

$$x = a\cos t + 3\cos\frac{2t}{3}, \quad y = a\sin t - 3\sin\frac{2t}{3}$$

where $0 \leqslant t < 6\pi$.

(i) Sketch the curves in the cases $a = 2$, $a = 3$ and $a = 4$ on separate axes.

State one common feature of these three curves.

State one distinctive feature of the curve for the case $a = 2$. **[7]**

(ii) For the case $a = 2$, find the values of t in the range $0 \leqslant t < 2\pi$ at the points where the curve intersects the y-axis.

Hence find the coordinates of the points of intersection with the y-axis for the complete curve. **[5]**

(iii) The distance from the origin of a point on a curve in this family is denoted by r.

Show that

$$r^2 = (6a\cos\frac{5t}{3}) + a^2 + 9.$$

Show that the values of t for which the curve has maximum and minimum distance from the origin are independent of a.

Find the maximum and minimum distance from the origin for a point on the curve for the case $a = 2$. **[7]**

(iv) For the case $a = 2$, confirm the feature of the curve at the point where $t = \frac{6\pi}{5}$ by investigating the gradient as $t \to \frac{6\pi}{5}$. **[5]**

2 This question concerns the functions $f(z) = \sin z$ and $g(z) = z - \frac{z^3}{6}$ where $z \in \mathbb{C}$.

(i) Find the root of the equation $z - \frac{z^3}{6} = 0.3 + 0.4i$ for which $0 < \text{Re}(z) < \pi$ and $\text{Im}(z) > 0$. **[2]**

(ii) It is to be investigated whether this root provides a good approximation for a root of the equation $\sin z = 0.3 + 0.4i$.

Find the root of the equation $\sin z = 0.3 + 0.4i$ for which $0 < \text{Re}(z) < \pi$ and $\text{Im}(z) > 0$. Hence find the errors in the real and imaginary parts of the approximation. **[3]**

(iii) The function $g(z)$ is used to approximate $f(z)$ for $z = a + 0.4i$, where $a > 0$. Construct a spreadsheet that will calculate the error in the real part of this approximation for different values of a. State the formulae you have used in your spreadsheet.

Use your spreadsheet to find, correct to 1 decimal place, the minimum positive value of a such that the real part of $g(z)$ exceeds the real part of $f(z)$ by more than 0.001. **[7]**

(iv) Use the Newton-Raphson method to find a numerical solution to the equation $\sin z = 0.3 + 0.4i$ with starting value $z_0 = 0$.

Show sufficient iterations to establish the result with both real and imaginary parts correct to 4 decimal places. **[5]**

(v) Give the Maclaurin expansions for $\sin z$ and $\sinh z$ and hence state the relationship between $\sin z$ and $\sinh z$.

Find the root of the equation $\sinh z = 0.4 - 0.3\mathrm{i}$ for which $\mathrm{Re}(z) > 0$ and $-\pi < \mathrm{Im}(z) < 0$. Show that this root is consistent with the relationship. **[6]**

3 **(i)** Create a program to find all the values of n such that $2^n \equiv 2 \pmod{n}$ with $3 \leqslant n \leqslant 30$.

You should write out your program in full and list all the values it gives. **[5]**

(ii) Edit your program to find a value of n such that $2^n \equiv 2 \pmod{n}$ and n is not prime with $3 \leqslant n \leqslant 500$.

State the changes you have made to your program and the value you have found.

State Fermat's Little Theorem and explain why this value does not disprove it. **[5]**

(iii) Create a program to find all the pairs of values of a and b such that $2^a \equiv 2 \pmod{b}$ and $2^b \equiv 2 \pmod{a}$, where a and b are distinct primes with $3 \leqslant a \leqslant 100$ and $3 \leqslant b \leqslant 100$.

You should write out your program in full and list all the pairs of values it gives.

Check one of your pairs of values by calculating 2^a and 2^b. **[9]**

(iv) Show that, if a and b are distinct primes, $2^a \equiv 2 \pmod{b} \Rightarrow 2^{ab} \equiv 2 \pmod{b}$.

Given that $2^a \equiv 2 \pmod{b}$ and $2^b \equiv 2 \pmod{a}$, where a and b are distinct primes, show that $2^{ab} \equiv 2 \pmod{ab}$.

Find a value of n that is not prime for which $2^n \equiv 2 \pmod{n}$ and $n > 500$. **[6]**

END OF QUESTION PAPER