

## **Principal Learning**

## **Engineering**

Unit **F563**: Mathematical techniques and applications for engineers

OCR Level 3 Principal Learning

## **Mark Scheme for June 2015**

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All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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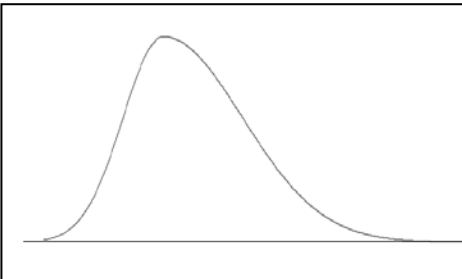
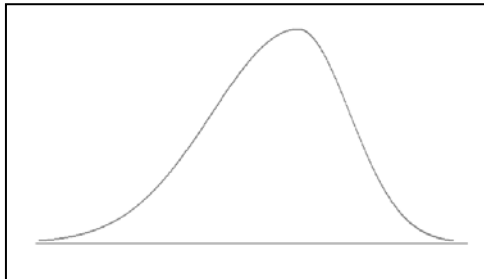
Annotations in scoris

The following annotations are available:

✓	correct response
✘	incorrect response
ecf	error carried forward
BOD	Benefit of doubt

Question	Expected Answer	Mark	Rationale/Additional Guidance
<b>Section A</b>			
1	$6x - (8x + 5) = 6x - 8x - 5 = -2x - 5$ $-2x$ $-5$	[1] [1]	
2	$x^2 + 5x - 14 = (x + 7)(x - 2)$ $(x + 7)$ $(x - 2)$	[1] [1]	
3	$[(x + 4)/8] - (x - 5)/2 = (x + 4 - 4x + 20)/8 \text{ or } ((2x + 8) - (8x - 40))/16$ $\text{or } (x + 4)/8 - (4x - 20)/8 \text{ or } (2x + 8)/16 - (8x - 40)/16$ $= (-3x + 24)/8 \text{ or } (-6x + 48)/16$ $(-3x + 24) \text{ or } (-6x + 48)$ $8 \quad \text{or } 16$	[1] [1]	<p>If denominator is subsequently removed from final answer i.e.</p> <p>final answer = <math>(-3x + 24)</math> or <math>(-6x + 48)</math></p> <p>award one mark only.</p> <p>Also accept <math>-3x/8 + 3</math> for 2 marks</p>
4	$6(3x - 5) = 2(4x - 7)$ $18x - 30 = 8x - 14$ $10x = 16$ $x = 1.6$	[1] [1]	<p>One mark for <math>18x - 30 = 8x - 14</math></p> <p>or <math>10x = 16</math></p> <p>or <math>-10x = -16</math></p> <p>CAO</p>
5	(a)	[1] [1]	<p>Award one mark for correct numerical results with or without the unit.</p> <p>Accept <math>\frac{2}{3}\pi</math> or answers between 2.05 and 2.15 radians.</p>
5	(b)		
	$\pi/3 \text{ radians} = \pi \times (360/6 \pi) = 60^\circ$ $120^\circ = 120 \times (2 \pi/360) = \frac{2}{3}\pi = 2.0944 \text{ radians.}$		

Question	Expected Answer	Mark	Rationale/Additional Guidance
<b>Section A</b>			
6	1 Sine wave.	[1]	Accept $y=\sin(x)$ or just $y=\sin$ or $\sin$
7	<p>In the triangle ABC angle <math>A = 30^\circ</math> and length of side 'a' = 100 mm.            Angle C = <math>90 - 30 = 60^\circ</math></p> <p>.Side <math>c = a/\tan A =</math>  <math>100/\tan 30^\circ = 173.2</math> mm.</p> <p>-----</p> <p>Alternative solution            Angle C = <math>90 - 30 = 60^\circ</math>  <math>100/\sin 30 = c/\sin 60</math>  <math>c = 100\sin 60/\sin 30 = 173.2</math> mm</p>	<p>[1]</p> <p>[1]</p> <p>-----</p> <p>or</p> <p>[1]</p> <p>[1]</p>	Accept answers between 173 and 173.5 mm. Unit not required for full marks.
8	<p>Given cosine <math>x = 3/5</math>. </p> <p>By observation the right-angled triangle will have sides of 3, 4 and 5 units long.            So sine <math>x = 4/5</math> or 0.8</p>	<p>[1]</p> <p>[1]</p>	One mark for triangle drawn with correct values. CAO.
9	<p>Given <math>y = 3 \cos x + 6x^3</math>            So <math>dy/dx = -3 \sin x + 18x^2</math></p> <p style="text-align: center;"><math>- 3 \sin x</math>  <math>+ 18x^2</math></p>	<p>[1]</p> <p>[1]</p>	<p>Allow <math>- 3 \sin + 18x^2</math></p> <p>Allow <math>3 (-) \sin x + 18x^2</math></p>
10	<p>Given <math>y = \sin x + (1/x) = \sin x + x^{-1}</math>            Then <math>dy/dx = \cos x - x^{-2} = \cos x - (1/x^2)</math></p>		Allow $\cos - x^{-2}$

Question	Expected Answer	Mark	Rationale/Additional Guidance
<b>Section A</b>			
	$\cos x$ $-(1/x^2) \text{ or } -x^{-2}$	<p>[1]</p> <p>[1]</p>	
11	$\int 2 \sin 5x \, dx = -0.4 \cos 5x + C$ $-0.4 \cos 5x \text{ or } -2/5 (\cos 5x)$ $+C$	<p>[1]</p> <p>[1]</p>	
12	$\int_2^5 4x^3 \, dx = \left[ x^4 \right]_2^5 = 5^4 - 2^4 = 625 - 16 = 609$ $\left[ x^4 \right]_2^5$ $5^4 - 2^4$ $609$	<p>[1]</p> <p>[1]</p> <p>[1]</p>	<p>Do not accept error carried forward if incorrect integration is used.</p>
13	<p>In a statistical distribution, the curve appears distorted either to the left or to the right.</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  <p>Positive skew</p> </div> <div style="text-align: center;">  <p>Negative skew</p> </div> </div>	<p>[2]</p>	<p>Diagram 1 mark</p> <p>Explanation 1 mark</p> <p>Award 2 marks for a clearly annotated diagram</p>

Question	Expected Answer	Mark	Rationale/Additional Guidance
<b>Section A</b>			
14	<p>Given 6 9 7 11 8 10 6</p> <p>Mode = 6</p> <p>Values arranged in order of magnitude:</p> <p style="padding-left: 40px;">6 6 7 <u>8</u> 9 10 11</p> <p>Median point = 8</p>	<p>[1]</p> <p>[1]</p>	
15	<p>P(A) = probability that event A happens.</p> <p>.</p> <p>P(B) = probability that event B happens.</p> <p>.</p>	<p>[1]</p> <p>[1]</p>	<p>Accept similar explanations but these MUST include reference 'Probability' or 'Likelihood' or 'possibility' and must mention (event) A (and B).</p> <p>eg</p> <p>'likelihood of (event) A (happening)'</p> <p>Do not accept eg:</p> <p>'A will happen' or 'A happens' etc</p>
<b>Total</b>		<b>[30]</b>	

Question		Expected Answer	Mark	Rationale/Additional Guidance
<b>Section B</b>				
1	(a)	Given $W = (mv^2) / 2$ Substitute $m = 5$ and $v = 10$ . Then $W = (5 \times 10^2) / 2$ $= 250$	[1]	
1	(b)	Given that $A = B(1 + 2CD)$ Open the brackets then $A = B + 2BCD$ Then $2BCD = A - B$ So $C = (A - B) / 2BD$ ----- Alternative solution $A/B = (1 + 2CD)$ $A/B - 1 = 2CD$ $C = (A/B - 1) / 2D$	[1] [1] [1] ----- or [1] [1] [1]	Accept any other correct method.
1	(c)	Given $v^2 = u^2 + 2as$ Subtract both sides by $2as$ Then $u^2 = v^2 - 2as$ Square root both sides So $u = \sqrt{v^2 - 2as}$	[1]  [1]	Accept any other correct method.



Question	Expected Answer	Mark	Rationale/Additional Guidance
<b>Section B</b>			
1	(d)	Given $A = B/(B + 2)$ Multiply both sides by $(B + 2)$ then $AB + 2A = B$ Subtract both sides by $B$ then $AB - B = - 2A$ Then $B(A - 1) = - 2A$ So $B = - 2A/(A - 1)$ or $2A/(1 - A)$	[1] [1] [1] [1] Accept any other correct method.
		<b>Total</b>	[10]

Question	Expected Answer	Mark	Rationale/Additional Guidance
2 (a)	Area of triangle $A = \frac{1}{2}bh$ So perpendicular height $h = 2A/b$ $= (2 \times 1000)/40$ $= 50 \text{ mm.}$	[1]  [1]	Unit not required for full marks
2 (b)	Given $x = 8 \text{ m}$ , $y = 10 \text{ m}$ and $z = 12 \text{ m}$ .  Area of triangle $A = \sqrt{[s(s-x)(s-y)(s-z)]}$ where $s = (x + y + z)/2$ So $s = (8 + 10 + 12)/2 = 15$ Then Area $A = \sqrt{[15(15-8)(15-10)(15-12)]}$ $= \sqrt{1575}$ $= 39.69 \text{ m}^2$  -----  Alternative solutions  $x^2 = y^2 + z^2 - 2yz\cos X$ $\cos X = (x^2 - y^2 - z^2) / (-2yz)$ $\cos X = (8^2 - 10^2 - 12^2) / (-2 \times 10 \times 12) = 0.75$ $X = 41.41^\circ$ Area = $(10 \times 12 \times \sin 41.41) / 2$ $= 39.69 \text{ m}^2$  $y^2 = x^2 + z^2 - 2xz\cos Y$ $\cos Y = (y^2 - x^2 - z^2) / (-2xz)$ $\cos Y = (10^2 - 8^2 - 12^2) / (-2 \times 8 \times 12) = 0.56$ $Y = 55.77$ Area = $(8 \times 12 \times \sin 55.77) / 2$ $= 39.69 \text{ m}^2$	[1] [1] [1] [1] [1]  ----- or [1] [1] [1] [1] or [1] [1] [1] [1]	BEWARE $\frac{1}{2}(8 \times 10) = 40$ (no marks for this)  Accept answers between $39 \text{ m}^2$ and $40 \text{ m}^2$ provided correct working is seen  Unit not required for full marks

Question	Expected Answer	Mark	Rationale/Additional Guidance
	$z^2 = x^2 + y^2 - 2xy\cos Z$ $\cos Z = (z^2 - x^2 - y^2) / (-2xy)$ $\cos Z = (12^2 - 8^2 - 10^2) / (-2 \times 8 \times 10) = 0.125$ $Z = 82.82^\circ$ $\text{Area} = (8 \times 10 \times \sin 82.82) / 2$ $= 39.69 \text{ m}^2$	or [1] [1] [1] [1] [1]	
2 (c)	Given angle $B = 30^\circ$ , side $a = 12 \text{ m}$ and side $c = 16 \text{ m}$ .  Area of triangle $A = \frac{1}{2}ac \sin B$ $= \frac{1}{2} \times 12 \times 16 \sin 30^\circ$ $= 96 \times 0.5$ $= 48 \text{ m}^2$	[1] [1] [1]	Unit not required for full marks.
	<b>Total</b>	<b>[10]</b>	

Question	Expected Answer	Mark	Rationale/Additional Guidance
3 (a)	Given $s = 60(20e^{-t/20} + t - 20)$ . Velocity = $ds/dt = 60(-e^{-t/20} + 1)$ Acceleration = $dv/dt = d^2s/dt^2 = 60(e^{-t/20}/20)$ When $t = 10s$ Acceleration = $3(e^{-10/20})$ = $1.82 \text{ ms}^{-2}$	[1] [1] [1] [1]	Unit not required for full marks
3 (b)	Given $S = 2\pi r^2 + 400/r = 2\pi r^2 + 400r^{-1}$ $ds/dr = 4\pi r - 400r^{-2} = 4\pi r - 400/r^2$ Surface area is a minimum when $ds/dr = 0$ So $4\pi r - 400/r^2 = 0$ $4\pi r = 400/r^2$ $4\pi r^3 = 400$ $r^3 = 400/4\pi$ So $r = 3.17$	[1] [1] [1] [1] [1]	Allow solutions by trial and error (4 marks) for answers between 3.0 and 3.3 (2 marks)
	<b>Total</b>	<b>[10]</b>	

Question			Expected Answer			Mark	Rationale/Additional Guidance
4	(a)	(i)	Time (minutes)	Tally	Cumulative frequency (f)		
			$0 < t \leq 20$	8	8		
			$20 < t \leq 40$	18	26		
			$40 < t \leq 60$	15	41		
			$60 < t \leq 80$	5	46		
			$80 < t \leq 100$	3	49		
			$100 < t \leq 120$	1	50		

Question			Expected Answer	Mark	Rationale/Additional Guidance
4	(a)	(ii)	<p>Cf</p> <p>Time</p>	[3]	<p>Consider ECF provided there is an obvious upward trend of values up to about 50 in the table.</p> <p>Award one mark for six correctly plotted points</p> <p>Award one mark for both correct axes.</p> <p>Award one mark for lines or curves joining points.</p>
4	(a)	(iii)	median number = 39	[1]	Accept answers between 36 and 42
4	(a)	(iv)	60 <sup>th</sup> percentile = 45	[1]	Accept answers between 43 and 47
4	(b)		$\Sigma x = 27 + 35 + 23 + 40 + 35 = 160$ Mean = $\Sigma x/n = 160/5 = 32$ $\Sigma x^2 = 27^2 + 35^2 + 23^2 + 40^2 + 35^2 = 5308$ Standard deviation = $\sqrt{\{(\Sigma x^2)/n\} - \{[\Sigma x/n]^2\}}$ $= \sqrt{\{5380/5\} - (160/5)^2}$ $= 6.13$	[1] [1] [1] [1]	Accept answers between 6 and 6.5
<b>Total</b>				[10]	

Question	Expected Answer	Mark	Rationale/Additional Guidance
5 (a) (i)	Given $v = u + at$ . $v = 60$ when $t = 10$ and $v = 30$ when $t = 4$ . So $60 = u + 10a$ Equation 1 And $30 = u + 4a$ Equation 2 Subtract eqn 2 from eqn 1 Then $30 = 6a$ So $a = 30/6 = 5$ Substitute $a = 5$ into eqn 1 So $60 = u + (10 \times 5)$ Then $u = 60 - 50 = 10$	  [1] [1]  [1] [1]  [1] [1]	
5 (a) (ii)	When $t = 6$ use equation $v = 10 + 5t$ $v = 10 + (5 \times 6)$ $= 40$	  [1]	
5 (b)	Given $3x^2 + 14x + 8 = 0$ . By inspection $3x^2 = (3x)(x)$ And $8 = (1)(8)$ or $(2)(4)$ By factorisation $(3x + 2)(x + 4) = 0$ So $(3x + 2) = 0$ or $(x + 4) = 0$ When $3x + 2 = 0$ Then $3x = -2$ so $x = -2/3$ or $0.666\dots$ or $0.67$	  [1]   [1]	

Question	Expected Answer	Mark	Rationale/Additional Guidance
	Likewise when $(x + 4) = 0$ Then $x = -4$ ----- Alternative method $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-14 \pm \sqrt{14^2 - 4 \times 3 \times 8}}{2 \times 3}$ $x = \frac{-14 \pm \sqrt{14^2 - 4 \times 3 \times 8}}{2 \times 3}$ $x = \frac{-14 \pm \sqrt{100}}{6}$ $x = \frac{-14 \pm 10}{6}$ $x = -4/6$ or $-2/3$ or $0.666\dots$ or $0.67$ $x = -24/6$ or $-4$	[1] ----- or [1] [1] [1] [1]	
	<b>Total</b>	<b>[10]</b>	



Question	Expected Answer	Mark	Rationale/Additional Guidance
6 (a)	<p>In a triangle ABC.            AB = 5 m, AC = 8 m and angle ABC = <math>42^\circ</math>.</p> <p>Let angle ABC be angle B.            Let angle ACB be angle C.</p> <p>Sine Rule: <math>AC / \sin B = AB / \sin C</math>            so <math>\sin C = (AB \sin B) / AC</math>            Then <math>\sin C = (5 \sin 42^\circ) / 8 = 0.4182</math>            So <math>C = \sin^{-1} 0.4182 = 24.72^\circ</math>            So Angle ACB is <math>24.72^\circ</math></p>	<p>[1]            [1]            [1]            [1]</p>	<p>Accept answers between <math>24</math> and <math>25^\circ</math>.</p>
6 (b)	<p>In a triangle ABC.            Side a = 40 mm, side b = 70 mm and angle C = <math>50^\circ</math></p> <p>Cosine Rule: <math>\cos C = (a^2 + b^2 - c^2) / 2ab</math>            So <math>c^2 = a^2 + b^2 - 2ab \cos C</math>  <math>= (40^2 + 70^2 - [2 \times 40 \times 70 \cos 50^\circ])</math>  <math>= 1600 + 4900 - [5600 \times 0.6428]</math>  <math>= 1600 + 4900 - 3600</math>  <math>= 2900</math>            So <math>c = \sqrt{2900} = 53.85</math> mm.</p>	<p>[1]            [1]            [1]            [1]</p>	<p>Accept answers between 53 and 54mm.</p>
6 (c)	<p>Given: <math>\sin 30^\circ + \cos 60^\circ + \tan 45^\circ = 2</math>            LHS = <math>0.5 + 0.5 + 1 = 2</math>            RHS = 2            Therefore <math>2 = 2</math> QED</p>	<p>[1]            [1]</p>	
	<b>Total</b>	<b>[10]</b>	

Question	Expected Answer	Mark	Rationale/Additional Guidance
7 (a)	Integrate $\cos 2x + 1/x^2 + \sqrt{x}$ with respect to $x$ . So $\int \cos 2x + 1/x^2 + \sqrt{x} \, dx = \int \cos 2x + x^{-2} + x^{0.5} \, dx$ $= \frac{1}{2} \sin 2x + (x^{-1})/-1 + (x^{1.5})/1.5 + C$ $= \frac{1}{2} \sin 2x - 1/x + \frac{2}{3} \sqrt{x^3} + C$	[5]	Award one mark for $\int \cos 2x + x^{-2} + x^{0.5} \, dx$ . Award one mark for $\frac{1}{2} \sin 2x$ Award one mark for $-1/x$ or $-x^{-1}$ Award one mark for $+ \frac{2}{3}$ or $1/1.5$ Award one mark for $\sqrt{x^3}$ or $x^{3/2}$ or $x^{1.5}$ +C is not required for full marks
7 (b)	Given $v = 4 + 6t$ where $t_1 = 0$ and $t_2 = 5$ . $\text{Distance} = \int_0^5 (4 + 6t) \, dt$ $= [4t + 3t^2]_0^5$ $= (\{4 \times 5\} + \{3 \times 5^2\}) - 0$ $= 95 \text{ metres}$	[1]  [1]  [1]	$\text{Award one mark for Distance} = \int_0^5 (4 + 6t) \, dt.$ $\text{Award one mark for } [4t + 3t^2]_0^5$ Unit not required for full marks

Question	Expected Answer	Mark	Rationale/Additional Guidance
7 (c)	<p>Given <math>F = 8s - s^2</math> where <math>s_1 = 2</math> and <math>s_2 = 6</math></p> $\text{Work done} = \int_2^6 (8s - s^2) ds$ $= \left[ 4s^2 - \frac{1}{3}s^3 \right]_2^6$ $= ((4 \times 6^2) - \{\frac{1}{3} \times 6^3\}) - ((4 \times 2^2) - \{\frac{1}{3} \times 2^3\})$ $= (144 - 72) - (16 - \frac{2}{3})$ $= 72 - 13\frac{1}{3}$ $= 58\frac{2}{3} \text{ or } 58.666\dots \text{ or } 58.67 \text{ newtons}$	<p>[1]</p> <p>[1]</p> <p>Total [10]</p>	<p>Award one mark for <math>\left[ 4s^2 - \frac{1}{3}s^3 \right]_2^6</math></p> <p>Unit (newtons) not required for full marks</p>
Question	Expected Answer	Mark	Rationale/Additional Guidance

Question	Expected Answer	Mark	Rationale/Additional Guidance
8 (a)	<p>An event that is affected by previous events.</p> <p>Suitable example involving an experiment 'without replacement' or equivalent.</p>	<p>[1]</p> <p>[1]</p>	
8 (b)	<p>Total number of screws = <math>110 + 120 + 70 = 300</math>.            Assume that three separate draws take place.            Draw One:            Probability of selecting a steel screw is <math>120/300</math>.            Draw Two:            There are now 119 steel screws in a total of 299            Probability of selecting a steel screw is <math>119/299</math>.            Draw Three:            There are now 118 steel screws in a total of 298.            Probability of selecting a steel screw is <math>118/298</math></p> <p>The probability of choosing a steel screw on the first draw and the second and the third draw is:  <math>(120/300) \times (119/299) \times (118/298)</math> is 0.063</p>	<p>[1]</p> <p>[1]</p> <p>[1]</p> <p>[1]</p>	<p>Accept answers between 0.062 and 0.064</p>
8 (c)	<p>Mutually exclusive</p> <p>Not Mutually exclusive</p> <p>Not Mutually exclusive</p> <p>Mutually exclusive</p>	<p>[4]</p>	<p>Award one mark for each correct response.</p>
	<p><b>Total</b></p>	<p>[10]</p>	

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