OCR
Oxford Cambridge and RSA

# GCE

# Statistics (MEI)

Unit **G243**: Statistics 3 (Z3)

Advanced Subsidiary GCE

# Mark Scheme for June 2015

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

## Annotations and abbreviations

| Annotation in scoris | Meaning |
|---|---|
| ✓and ✗ | |
| BOD | Benefit of doubt |
| FT | Follow through |
| ISW | Ignore subsequent working |
| M0, M1 | Method mark awarded 0, 1 |
| A0, A1 | Accuracy mark awarded 0, 1 |
| B0, B1 | Independent mark awarded 0, 1 |
| SC | Special case |
| ^ | Omission sign |
| MR | Misread |
| Highlighting | |
| | |
| **Other abbreviations in mark scheme** | **Meaning** |
| E1 | Mark for explaining |
| U1 | Mark for correct units |
| G1 | Mark for a correct feature on a graph |
| M1 dep* | Method mark dependent on a previous mark, indicated by * |
| cao | Correct answer only |
| oe | Or equivalent |
| rot | Rounded or truncated |
| soi | Seen or implied |
| www | Without wrong working |
| | |
| | |

## Subject-specific Marking Instructions for GCE Mathematics (MEI) Statistics strand

a Annotations should be used whenever appropriate during your marking.

**The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks.** It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

**M**
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

**A**
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

**B**
Mark for a correct result or statement independent of Method marks.

**E**

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

Candidates are expected to give numerical answers to an appropriate degree of accuracy. 3 significant figures may often be the norm for this, but this always needs to be considered in the context of the problem in hand. For example, in quoting probabilities from Normal tables, we generally expect *some* evidence of interpolation and so quotation to 4 decimal places will often be appropriate. But even this does not always apply – quotations of the standard critical points for significance tests such as 1.96, 1.645, 2.576 (maybe even 2.58 – but not 2.57) will commonly suffice, especially if the calculated value of a test statistic is nowhere near any of these values. Sensible discretion *must* be exercised in such cases.

Discretion must also be exercised in the case of small variations in the degree of accuracy to which an answer is given. For example, if 3 significant figures are expected (either because of an explicit instruction or because the general context of a

problem demands it) but only 2 are given, loss of an accuracy ("A") mark is likely to be appropriate; but if 4 significant figures are given, this should not normally be penalised. Likewise, answers which are slightly deviant from what is expected in a very minor manner (for example a Normal probability given, after an attempt at interpolation, as 0.6418 whereas 0.6417 was expected) should not be penalised. However, answers which are *grossly* over- or under-specified should normally result in the loss of a mark. This includes cases such as, for example, insistence that the value of a test statistic is (say) 2.128888446667 merely because that is the value that happened to come off the candidate's calculator. Note that this applies to answers that are given as final stages of calculations; intermediate working should usually be carried out, and quoted, to a greater degree of accuracy to avoid the danger of premature approximation.

The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h Genuine misreading (of numbers or symbols, occasionally even of text) occurs. If this results in the object and/or difficulty of the question being considerably changed, it is likely that all the marks for that question, or section of the question, will be lost. However, misreads are often such that the object and/or difficulty remain substantially unaltered; these cases are considered below.

The simple rule is that *all* method ("M") marks [and of course all independent ("B") marks] remain accessible but at least some accuracy ("A") marks do not. It is difficult to legislate in an overall sense beyond this global statement because misreads, even when the object and/or difficulty remains unchanged, can vary greatly in their effects. For example, a misread of 1.02 as 10.2 (perhaps as a quoted value of a sample mean) may well be catastrophic; whereas a misread of 1.6748 as 1.6746 may have so slight an effect as to be almost unnoticeable in the candidate's work.

A misread should normally attract *some* penalty, though this would often be only 1 mark and should rarely if ever be more than 2. Commonly in sections of questions where there is a numerical answer either at the end of the section or to be obtained and commented on (eg the value of a test statistic), this answer will have an "A" mark that may actually be designated as "cao" [correct answer only]. This should be interpreted *strictly* – if the misread has led to failure to obtain this value, then this "A" mark

must be withheld even if all method marks have been earned. It will also often be the case that such a mark is implicitly "cao" even if not explicitly designated as such.

On the other hand, we commonly allow "fresh starts" within a question or part of question. For example, a follow-through of the candidate's value of a test statistic is generally allowed (and often explicitly stated as such within the marking scheme), so that the candidate may exhibit knowledge of how to compare it with a critical value and draw conclusions. Such "fresh starts" are not affected by any earlier misreads.

A misread may be of a symbol rather than a number – for example, an algebraic symbol in a mathematical expression. Such misreads are more likely to bring about a considerable change in the object and/or difficulty of the question; but, if they do not, they should be treated as far as possible in the same way as numerical misreads, *mutatis mutandis*. This also applied to misreads of text, which are fairly rare but can cause major problems in fair marking.

The situation regarding any particular cases that arise while you are marking for which you feel you need detailed guidance should be discussed with your Team Leader.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

| Question | | | Answer | Marks | Guidance | |
|---|---|---|---|---|---|---|
| 1 | (i) | | $\bar{x} = \frac{275.8}{50} = 5.516$ | B1e | Allow 5.52 or 5.5 with working | AO5 3 |
| | | | $s^2 = \frac{2910.5 - \frac{275.8^2}{50}}{49} = \frac{1389.19}{49} = 28.35$ | M1c | For $S_{xx}$ | |
| | | | $s = \sqrt{28.35} = 5.325$ | A1c<br>**[3]** | Allow answers in range 5.32 to 5.33 (from correct working) | |
| | (ii) | | $H_0$: $\mu_A = \mu_B$<br>$H_1$: $\mu_A > \mu_B$<br>Where $\mu_A$, $\mu_B$ denote the (population) **differences** in actual and scheduled arrival times for airlines A and B respectively | B1e<br>B1e<br>B1c | Condone absence of "population" if correct notation "$\mu$" has been used, but do NOT accept $\overline{X}$ and $\overline{Y}$ or similar unless explicitly stated to be population means.<br>Accept hypothesis explained in words, provided "population" appears.<br>Allow all remaining marks even if $H_1$: $\mu_A < \mu_B$ | AO1 2<br>AO2 1<br>AO3 5 |
| | | | 2–sample test based on N(0,1) soi | E1e | | AO4 2 |
| | | | Test statistic is<br>$\frac{5.516 - 3.74}{\sqrt{\frac{6.45^2}{50} + \frac{5.325^2}{50}}} = \frac{1.776}{1.183} = 1.501$ | M1e<br>M1c<br>A1c | Numerator<br>Denominator<br>CAO Allow TS in range 1.49 to 1.51 (from correct working) | AO5 1 |
| | | | 1–tailed 10% point of N(0,1) is 1.282 | B1e | No further A marks from here if CV incorrect | |
| | | | 1.501 > 1.282<br>Significant | M1e<br>A1c | FT Test statistic provided at least one method mark scored. | |
| | | | There is sufficient evidence to suggest that on average the flights for airline A is are earlier than those for airline B. (The differences in actual and scheduled arrival times for airline A is greater than that for airline B). | A1a<br>**[11]** | Condone statements such as: 'there is evidence to suggest that flights from airline A will arrive earlier than those from airline B'. | |
| | (iii) | | The null hypothesis is less likely to be rejected as the critical value will be higher. | B1e<br>E1a | Condone 'more likely to be accepted'<br>Or 'it will be rejected 10% of time with 10% level but only 1% | AO2 2 |

| Question | | | Answer | Marks | Guidance | |
|---|---|---|---|---|---|---|
| | | | | **[2]** | of time with 1% level.<br>Do not allow 'The TS will be greater than then CV' oe but allow CV would be 2.326. | |
| **2** | **(i)** | | This will allow a paired sample test to be carried out and the pairing will eliminate any differences in individual file download speeds and will therefore compare the download times onto the two hard drives | E1c<br>E1a<br>**[2]** | For 'paired'<br>Or for same file size on both computers or different copying speed. | **AO4 2** |
| | **(ii)** | | The population of differences of times must be Normally distributed | E1a<br>E1a<br>**[2]** | For population of differences<br>For Normally distributed<br>Ignore comments about variance etc. | **AO2 2** |
| | **(iii)** | | $H_0$: $\mu_D = 0$<br>$H_1$: $\mu_D \neq 0$ | B1e<br>B1e | No marks for $\mu$ unless defined as mean difference | **AO1 2** |
| | | | Where $\mu_D$ denotes the (population) mean for **differences**. | B1a | Condone absence of "population" if correct notation "$\mu$" has been used, but do NOT accept $\overline{D}$ or similar unless explicitly stated to be population means. | **AO2 1** |
| | | | No further marks unless paired comparison *t* test | | Hypotheses explained in words only must include "population" | **AO3 5** |
| | | | Differences e.g. (Jenna – Ronin) are<br>0.22 1.57 -1 0.76 0.64 -2.08<br>-1.72 -0.65 0.23 0.18 0.09 | M1e | For differences | **AO4 2** |
| | | | $\overline{d} = -0.16$ $s_{n-1} = 1.096$ | A1e | For both | **AO5 1** |
| | | | Test statistic is $\dfrac{-0.16-0}{1.096/\sqrt{11}} = -0.484$ | M1c<br>A1c | FT their $\overline{d}$ and $s_{n-1}$<br>CAO but FT from here if M1 awarded | |
| | | | Refer to $t_{10}$<br>Two tailed 5% critical value is –2.228 | M1e<br>A1e | For $t_{10}$ Can be implied by correct CV.<br>Must be minus 2.228 unless absolute values or (Ronin – Jenna) are being compared.<br>No further A marks from here if CV incorrect | |
| | | | –0.484 > –2.228 so not significant.<br>There is insufficient evidence to suggest that average copying speeds for the two hard drives are different | M1c<br>A1a | | |

**3 (i)**

The population should have a bivariate Normal distribution. In this case the points appear to lie in two separate clusters rather than in an elliptical pattern. This suggests that the population may not have a bivariate Normal distribution.

**[3]**

E1a For bivariate Normal
E1a For two clusters Allow 'two islands'
E1a For not elliptical Allow 'not oval'

**AO2 3**

**(ii)**

| Org | Inorg | R Org | R Inorg | $d$ | $d^2$ |
|---|---|---|---|---|---|
| 85 | 95 | 10 | 9 | -1 | 1 |
| 83 | 100 | 9 | 11 | 2 | 4 |
| 73 | 97 | 7 | 10 | 3 | 9 |
| 20 | 22 | 1 | 1 | 0 | 0 |
| 29 | 39 | 2 | 3 | 1 | 1 |
| 88 | 115 | 11 | 12 | 1 | 1 |
| 94 | 93 | 12 | 8 | -4 | 16 |
| 45 | 40 | 5 | 4 | -1 | 1 |
| 41 | 31 | 4 | 2 | -2 | 4 |
| 81 | 69 | 8 | 7 | -1 | 1 |
| 56 | 42 | 6 | 6 | 0 | 0 |
| 40 | 41 | 3 | 5 | 2 | 4 |

M1e For ranking (allow all ranks reversed)

M1e For $d^2$

$\Sigma d^2 = 42$

A1e For $\Sigma d^2$

$$r_s = 1 - \frac{6 \times 42}{12 \times 143} = 1 - 0.147 = 0.853$$

M1e For method for $r_s$
A1c FT their ranks provided $|r_s| < 1$
NB No ranking scores zero.
**[5]**

**AO1 4**
**AO5 1**

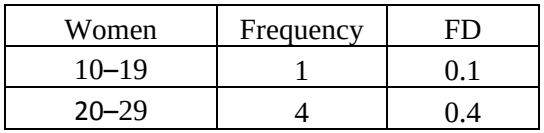
**(iii)**

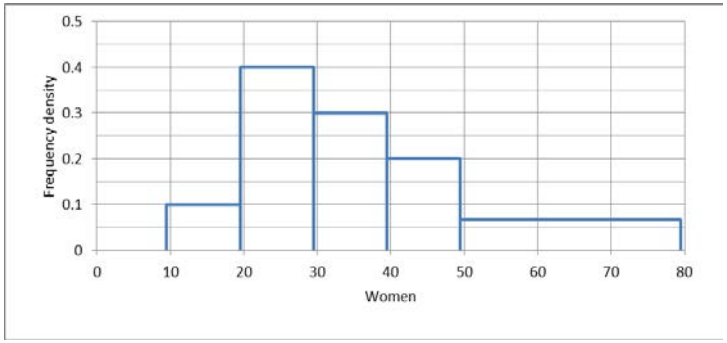
$H_0$: no association between levels of organic and inorganic phosphorus in the population

$H_1$: positive association between levels of organic and inorganic phosphorus in the population

B1e
B1e NB $H_0$ $H_1$ not ito $\rho$
B1c For mention of population, either in hypotheses or in concluding statement

**AO3 2**
**AO4 4**
**AO5 1**

| Question | | | Answer | Marks | Guidance | |
|---|---|---|---|---|---|---|
| | | | One tail test critical value at 1% level is 0.6783 | B1e | For 0.6783 Do not allow –0.6783 even if their $r_s$ is negative<br>No further A marks from here if CV incorrect | |
| | | | Since 0.853 > 0.6783,<br>There is sufficient evidence to reject $H_0$.<br>Thus there is enough evidence to suggest positive association between levels of organic and inorganic phosphorus in the population. | M1e<br>A1c<br>A1a<br>**[7]** | For comparison with c.v., provided $\lvert r_s \rvert < 1$ leading to conclusion<br>For conclusion in words in context – FT their $r_s$ | |
| **4** | **(i)** | | EG: A census involves the whole population and so inference is not necessary and the results are certain.<br>EG: A sample survey involves less work. | E1a<br>E1a<br>**[2]** | Must mention whole population oe<br>Allow other sensible comments | **AO1 2** |
| | **(ii)** | | The first 12 women who take the test on a particular day might not be representative of all women who take the test.<br><br>For example women who take the test early in the day may be women who go out to work later on, whereas women who take it later may be retired. | E1e<br><br>E1c<br>**[2]** | Allow 'sample may be biased'<br><br>Allow any sensible alternative about time of day and particular day<br>Allow E1 for comment about not being random and E2 if go on to comment that statistical inference cannot be carried out. | **AO4 2** |
| | **(iii)** | | Allocate numbers 1 to 960 to the women.<br><br>Randomly select one of the first 80 women.<br>Then select every eightieth woman. | E1e<br><br>E1e<br>E1e<br>**[3]** | Allow 'put the women in order' or 'make a list of the women' oe<br>If just state 'select one at random' then need to say 'go back to starting point when reach end of list' | **AO3 3** |
| | **(iv)** | | The two populations must be Normally distributed.<br>The two populations must have equal variances. | E1e<br>E1a<br>**[2]** | | **AO3 2** |
| | **(v)** | |  <br>Women / Frequency / FD<br>10–19 / 1 / 0.1<br>20–29 / 4 / 0.4 | B1e | For frequency densities (all correct) ft their frequencies<br>Mark for FDs can be gained from graph if not given explicitly<br><br>Condone final fd of 0.07 | **AO3 4**<br>**AO4 1** |

| Question | | | Answer | Marks | Guidance | |
|---|---|---|---|---|---|---|
| | | | 30–39 \| 3 \| 0.3<br>40–49 \| 2 \| 0.2<br>50–79 \| 2 \| 0.06667 | | | |
| | | |  | M1e<br><br>A1c | For attempt at histogram (with fd's, not frequencies)<br><br>Histogram all correct | |
| | | | Positive skewness<br>Because the data are skewed | B1e<br>E1a<br>**[5]** | Allow any suitable alternative | |
| | **(vi)** | | $H_0$: the medians of the two populations are the same<br>$H_1$: the median two populations are not the same | B1e<br>B1a | For medians<br>Need population for second mark<br>Note: Explicit statement re shapes of distributions (eg that they are the same shape) is not required. | **AO1 2**<br>**AO3 5**<br>**AO4 2**<br>**AO5 1** |
| | | | Wilcoxon rank sum test (or Mann-Whitney form thereof) | M1e | SOI<br>**NB No marks for *t* test** | |
| | | | Ranks are<br><br>Rank female: 3 4 8 9 11 14 16 17 18 21 23 24<br>Rank male: 1 2 5 6 7 10 12 13 15 19 20 22 | M1e | Combined ranking | |
| | | | Smaller rank sum (for male) is 132 | A1e | FT | |

| Question | | | Answer | Marks | Guidance | |
|---|---|---|---|---|---|---|
| | | | | | (M–W stat = 0+0+2+2+2+4+5+5+6+9+9+10 = 54) | |
| | | | Refer to (12,12) table | B1e | | |
| | | | 2–tail 5% critical value is 115 [or 37 for M–W] | B1e | No further A marks from here if CV incorrect | |
| | | | 132>115 | M1c | FT their Wilcoxon CV for M mark, but not Mann-Whitney if Wilcoxon test statistic calculated | |
| | | | Not significant | A1c | | |
| | | | There is insufficient evidence to suggest that, on average, the (typing test) results for women and men are different. | A1a<br>**[10]** | | |
| | **(vii)** | | If the researcher took only 1 person of each sex, then there would be no evidence of the variability of the data and therefore statistical inference would be impossible. | E1c<br>E1a<br>**[2]** | Allow any sensible alternative<br>Allow E1 for larger sample is likely to be more representative of the data. | **AO4 2** |

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