

Principal Learning

Engineering

Unit **F563**: Mathematical techniques and applications for engineers

OCR Level 3 Principal Learning

Mark Scheme for January 2016

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Section A					
1			$4 + 5(6x - 7) + 3x = 4 + 30x - 35 + 3x = 33x - 31$ $33x$ $- 31$	[1] [1]	
2			$x^2 - 11x + 30 = (x - 6)(x - 5)$ $(x - 6)$ $(x - 5)$	[1] [1]	
3			$(3x - 4)/5 - (6x + 7)/8$ $= (24x - 32 - 30x - 35)/40 =$ $- 6x - 67$ 40	[1] [1]	
4			$(x + 2)/3 + (x + 3)/4 = (x + 4)/6$ $4x + 8 + 3x + 9 = 2x + 8$ $5x = - 9$ $x = - 1.8$	[1] [1]	Accept $x = 9/5$ or 1.8 with no minus for [1].
5			<p>Area of triangle = $\frac{1}{2}bc \sin A$</p> $= \frac{1}{2} \times 8 \times 10 \sin 70^\circ$ $= 37.59 \text{ m}^2$	[1] [1]	Accept answers between 37 and 38. Award one mark for correct numerical result with or without the unit.

Section A					
6			Given $\sin 20^\circ/\cos 70^\circ$ From complementary angles: $\cos 70^\circ = \sin (90^\circ - 70^\circ) = \sin 20^\circ$ $\sin 20^\circ/\cos 70^\circ = \sin 20^\circ/\sin 20^\circ = 1$	[1] [1]	Accept $\sin 20/\cos 70 = 0.342/0.342 = 1$ for [1]
7		(a)	$30^\circ = 30 (\pi/180) = \pi/6$ radians or 0.5236 radians	[1]	Accept $\pi/6$ or answers between 0.5 and 0.55 with or without the unit..
		(b)	$3\pi/4$ radians = $(3\pi/4)(180/\pi) = 135^\circ$	[1]	Award one mark for correct numerical result with or without the unit.
8			Given $a = 13$ m, $b = 5$ m and $c = 12$ m $\sin C = 12/13$ or $\cos C = 5/13$ or $\tan C = 12/5$ Angle $C = 67.38^\circ$	[1] [1]	Accept answers for angle C between 67 and 68. Accept any alternative method e.g. $\operatorname{cosec} C = 13/12 = 1.083$ Angle $C = \operatorname{cosec}^{-1} 13/12 = 67.38^\circ$
9			Given $y = 6 \cos 3x$ $dy/dx = -3 (6 \sin 3x)$ $= -18 \sin 3x$	- 18 sin 3x [1] [1]	
10			Given $y = 5 \ln 4x$ $dy/dx = 5/x$ 5 1/x	[1] [1]	

Section A					
11			$\int \sqrt{x^3} dx = \int x^{1.5} dx = (x^{2.5})/2.5 + C = [0.4 \sqrt{x^5}] + C$ $[0.4 \sqrt{x^5}]$ C	[1] [1]	Accept $2/5x^{5/2}$
12			$\text{Area under curve} = \int_0^{30} x^2 dx = [x^3/3]_0^{30} = 9000 - 0 = 9000$ $[x^3/3]$ 0 $=9000$	[1] [1]	
13			<p>Given 3 2 7 1 9 6 Ascending order 1 2 3 6 7 9 Median is between 3 and 6 Median = 4.5</p>	[1] [1]	
14			<p>Both black = $(3 \times 2)/(8 \times 7) = 6/56$ Both blue = $(5 \times 4)/(8 \times 7) = 20/56$ Total = $(6/56) + (20/56) = 26/56$</p>	[1] [1]	Both 6/56 and 20/56 correct for [1]
15			A quartile is one of three points along the scale of a frequency distribution that divides the distribution into four equal parts.	[1] [1]	Award one mark for reference to 'three points' and award one mark for reference to 'four equal parts'. Allow BOD e.g. 'points that divide data into four (equal) parts'
			Total	[30]	

Question	Expected Answer	Mark	Rationale/Additional Guidance
Section B			
1	(a)		
		[1] [1]	Accept answers between 35342.5 and 35343.5.
1	(b)		
		[1] [1] [1] [1]	Accept alternative correct responses e.g. $g = 4\pi^2L/T^2$ $g = L(T/2\pi)^{-2}$
	(c)		
		[1] [1] [1] [1]	
		[1]	
		Total	[10]

Question	Expected Answer	Mark	Rationale/Additional Guidance
2 (a) (i)	$(4/x) - (1/y) = 13$ Eqn. 1 $(3/x) - (2/y) = 6$ Eqn. 2 Multiply eqn.1 by 2 $(8/x) - (2/y) = 26$ Eqn. 3 $(3/x) - (2/y) = 6$ Eqn. 2 Subtract Eqn. 2 from Eqn. 3 $(5/x) = 20$ $x = 5/20 = 0.25 = 1/4$ Substitute $x = 1/4$ into Eqn. 1 $(4/x) - (1/y) = 13$ $16 - (1/y) = 13$ Then $(1/y) = 16 - 13 = 3$ $y = 1/3$	[1] [1] [1] [1] [1]	Accept ecf for x. Accept correct value of y for any x [3]

Question			Expected Answer	Mark	Rationale/Additional Guidance
2	(a)	(ii)	Substitute $x = \frac{1}{4}$ and $y = \frac{1}{3}$ into Eqn. 1 Then $(4/\frac{1}{4}) - (1/\frac{1}{3}) = 13$ So $16 - 3 = 13$ QED and Substitute $x = \frac{1}{4}$ and $y = \frac{1}{3}$ into Eqn.2 Then $(3/\frac{1}{4}) - (2/\frac{1}{3}) = 6$ So $12 - 6 = 6$ QED	[1]	Both substitutions needed for one mark.
2	(b)		Given $5(3a - 6) - 4(6a - 4) = 8 - (2a + 1)$. Then $15a - 30 - 24a + 16 = 8 - 2a - 1$ So $15a - 24a + 2a = 8 - 1 + 30 - 16$ Then $-7a = 21$ $a = -3$	[1] [1] [1]	Accept 7a or -7a [1] -21 or 21 [1] -3 with ECF [1]
			Total	[10]	

Question		Expected Answer	Mark	Rationale/Additional Guidance
3	(a)	$AD = 80 \times \tan 60^\circ = 138.56 \text{ mm}$	[1] [1]	Accept $80\sqrt{3}$ or answers between 138 and 139. Award one mark for correct numerical result with or without the unit.
3	(b)	$BD = 80/\cos 60^\circ$ or $\sqrt{(80^2 + 138.56^2)}$ $= 160 \text{ mm}$	[1] [1]	Award one mark for correct numerical result with or without the unit.
3	(c)	length of the arc DE = $[\pi (BD) \Theta^\circ]/180$ $= [\pi \times 160 \times 60]/180$ $= 167.55 \text{ mm}$	[1] [1]	Accept $160\pi/3$ or answers between 167 and 168. Award one mark for correct numerical result with or without the unit.
3	(d)	area of one right-angled triangle = $\frac{1}{2} (AB)(AD)$ $= \frac{1}{2} \times 80 \times 138.56$ $= 5542.4 \text{ mm}^2$	[1] [1]	Accept answers between 5542 and 5543. Award one mark for correct numerical result with or without the unit.
3	(e)	area of the sector of the circle = $[\pi (BD^2) \Theta^\circ]/360$ $= [\pi \times 160^2 \times 60]/360$ $= 13404.13 \text{ mm}^2$	[1] [1]	Accept answers between 13404 and 13405. Award one mark for correct numerical result with or without the unit.
Total			[10]	

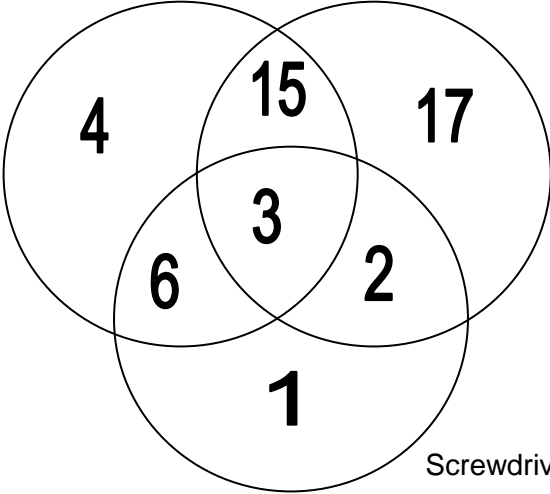
Question		Expected Answer	Mark	Rationale/Additional Guidance
4	(a)	$y = \sin x$ $y = \cos x$ $y = \tan x$	[3]	Award one mark for each correctly labelled graph. Allow sin, cos, tan with no x
4	(b)	Given $\sin^2 x (\operatorname{cosec}^2 x + \sec^2 x) \equiv 1/\cos^2 x$. LHS = $\sin^2 x (\operatorname{cosec}^2 x + \sec^2 x)$ But $\operatorname{cosec} x = 1/\sin x$ LHS = $\sin^2 x (1/\sin^2 x + \sec^2 x)$ But $\sec x = 1/\cos x$ LHS = $\sin^2 x (1/\sin^2 x + 1/\cos^2 x)$ $= (\sin^2 x/\sin^2 x) + (\sin^2 x/\cos^2 x)$ $= 1 + (\sin^2 x/\cos^2 x)$ $= (\cos^2 x + \sin^2 x)/\cos^2 x$ But $\sin^2 x + \cos^2 x = 1$ LHS = $1/\cos^2 x$ RHS = $1/\cos^2 x$ LHS = RHS This proves that $\sin^2 x (\operatorname{cosec}^2 x + \sec^2 x) \equiv 1/\cos^2 x$	 [1] [1] [1] [1] [1] [1] [1]	 An alternative response could be $\sin^2 x (1/\sin^2 x + 1/\cos^2 x)$ [3] Common denominator $\sin^2 x (\cos^2 x + \sin^2 x)/(\cos^2 x \sin^2 x)$ [1] cancel $\sin^2 x$ $(\cos^2 x + \sin^2 x)/(\cos^2 x)$ [1] $\cos^2 x + \sin^2 x = 1$ [1] $1/\cos^2 x$ [1]
Total			[10]	

Question	Expected Answer	Mark	Rationale/Additional Guidance
5 (a)	$\tan 35^\circ = 0.5w/h$ $w = 2h \tan 35^\circ$ $= 1.4 h$	 [1] [1]	Accept any correct alternative solution e.g $w/2 \sin 35 = h/\sin 55$ $w = 2h \sin 35/\sin 55$
5 (b)	$V = \text{length} \times \text{cross-sectional area}$ $= 6 \times 0.5 wh$ $= 3 wh$ but $w = 1.4 h$ So $V = 3h \times 1.4h = 4.2 h^2$	 [1] [1]	
5 (c)	$dV/dh = 8.4h$	[1]	

Question	Expected Answer	Mark	Rationale/Additional Guidance
5 (d)	<p>Given $dV/dt = (dV/dh) \times (dh/dt)$</p> <p>So $dV/dt = 8.4h \times (dh/dt)$</p> <p>Water is entering at the rate of $1\text{ m}^3 \text{ min}^{-1}$</p> <p>After 5 minutes the volume of water is 5 m^3</p> <p>From $V = 4.2 h^2$, $h = \sqrt{V/4.2}$</p> <p>Then $h = \sqrt{5/4.2} = 1.0911 \text{ m}$</p> <p>We have $dV/dh = 8.4h = 8.4 \times 1.0911 = 9.165$</p> <p>Because water is entering at the rate of $1 \text{ m}^3 \text{ min}^{-1}$ then $dV/dt = 1$</p> <p>We have $dV/dt = (dV/dh) \times (dh/dt)$</p> <p>So $1 = 9.165 \times (dh/dt)$</p> <p>So $dh/dt = 1/9.165 = 0.1091$</p> <p>The depth is increasing at a rate of $0.1091 \text{ m min}^{-1}$</p>	<p>[1]</p> <p>[1]</p> <p>[1]</p> <p>[1]</p> <p>[1]</p>	
	Total	[10]	

Question	Expected Answer	Mark	Rationale/Additional Guidance
6 (a)	Given $\int 3x^2 + 6\sqrt{x} - (8/x^2) dx$ Rewrite $\int 3x^2 + 6x^{1/2} - 8x^{-2} dx$ $= x^3 + (6x^{1.5})/1.5 - (8x^{-1})/-1 + (C)$ $= x^3 + 4x^{1.5} + (8/x) + (C)$ $= x^3 + 4\sqrt{x^3} + (8/x) + (C)$	[1] [1] [1] [1]	Do not award any marks for constant C. This has been tested in section A x^3 [1] $4x^{3/2}$ [1] $8/x$ [1] $x^3 + 4\sqrt{x^3} + (8/x)$ [1]
6 (b)	$\int_0^{\pi/2} (\sin 4x - \cos 4x) dx$ $= \left[\frac{-\cos 4x}{4} - \frac{\sin 4x}{4} \right]_0^{\pi/2}$ $= \left[\frac{-\cos 2\pi}{4} - \frac{\sin 2\pi}{4} \right] - \left[\frac{-\cos 0}{4} - \frac{\sin 0}{4} \right]$ $= -\frac{1}{4} - 0 + \frac{1}{4} + 0$ $= 0$	[2] [3] [1]	$\frac{-\cos 4x}{4}$ [1] $-\frac{\sin 4x}{4}$ [1] $\frac{-\cos 2\pi}{4} - \frac{\sin 2\pi}{4}$ [1] $-$ [1] $\frac{-\cos 0}{4} - \frac{\sin 0}{4}$ [1] $-\frac{1}{4} - 0 + \frac{1}{4} + 0 = 0$ [1]
	Total	[10]	

Question		Expected Answer					Mark	Rationale/Additional Guidance
7	(a)	x	f	fx	x – mean	$(x - mn)^2$	$f(x - mn)^2$	<p>Award marks as shown</p> <p>[7]</p> <p>fx column [1] $\Sigma fx = 4270$ [1] mean = 42.7 [1] x–mean column [1] $(x - \text{mean})^2$ [1] $f(x - \text{mean})^2$ column [1] $\Sigma f(x - \text{mean})^2 = 229$ [1]</p>
		40	10	400	– 2.7	7.29	72.9	
		41	14	574	– 1.7	2.89	40.46	
		42	19	798	– 0.7	0.49	9.31	
		43	23	989	0.3	0.09	2.07	
		44	21	924	1.3	1.69	35.49	
		45	13	585	2.3	5.29	68.77	
			100	4270			229	
		Mean = $\Sigma fx/f = 4270/100 = 42.7$						
	(b)	Variance = $229/100 = 2.29$ Standard deviation = $\sqrt{2.29} = 1.51$					[2] [1]	$v = (\Sigma f(x - \text{mean})^2)/\Sigma f$ [1] $119/100 = 2.29$ [1]
Total						[10]		

Question	Expected Answer	Mark	Rationale/Additional Guidance
8 (a)	<p style="text-align: center;">Hammer Tenon saw</p>  <p style="text-align: center;">Screwdriver</p>	[7]	Award one mark for each correct number in the correct circle.
8 (b)	2	[1]	
8 (c) (i)	Probability = $48/50 = 24/25 = 0.96$	[1]	Accept ECF e.g. 50 –value from 8(b)
8 (c) (ii)	Probability = $22/50 = 11/25 = 0.44$	[1]	Accept ECF Accept $1 - (2/50) = 1 - 0.04 = 0.96$
	Total	[10]	

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