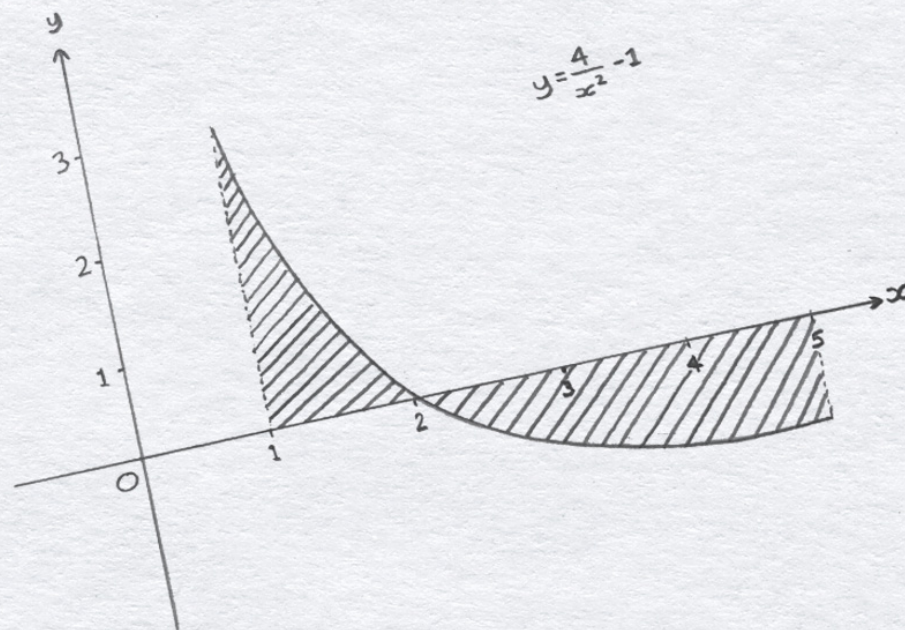


**STAY AHEAD OF**

**THE CURVE**

**About the new  
Further  
Mathematics B**



# ***FURTHER MATHS EXAMPLES***

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## **Example 1**

Core pure

+Mechanics major

+ Statistics minor

# FURTHER MATHS EXAMPLES

---

## Example 1

Core pure (FP1 + FP2 + FP3)

+Mechanics major (M2 + M3)

+ Statistics minor (S2)



# ***FURTHER MATHS EXAMPLES***

---

## **Example 1**

Core pure (FP1 + FP2 + FP3)

+Mechanics major (M2 + M3)

+ Statistics minor (S2)

## **Example 2**

Core pure

+ mechanics minor

+ stats minor

+ modelling with algorithms

# FURTHER MATHS EXAMPLES

---

## Example 1

Core pure (FP1 + FP2 + FP3)  
+Mechanics major (M2 + M3)  
+ Statistics minor (S2)

## Example 2

Core pure (FP1 + FP2 + FP3)  
+ mechanics minor (M2)  
+ stats minor (S2)  
+ modelling with algorithms (D1)



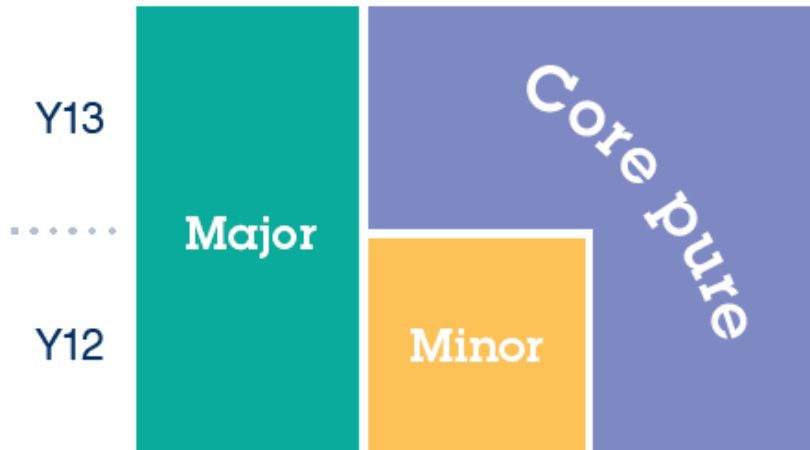
# A LEVEL FURTHER MATHEMATICS



If you want to teach A level Further Maths across two years, parallel to A level Maths, then:

- Imagine the two years split into 6 teaching slots
- The mandatory Core pure paper takes 3 slots, one in Y12 and two in Y13
- There are three slots for optional units

# CHOOSE ONE MAJOR OPTION + ONE MINOR OPTION



## Major Options

Major

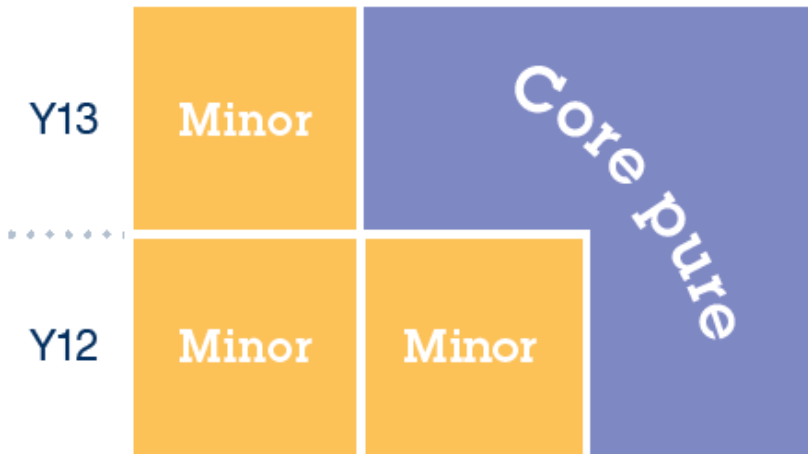
Mechanics major  
Statistics major

## Minor Options

Minor

Mechanics minor  
Statistics minor  
Modelling with algorithms  
Numerical methods  
Extra Pure  
Further pure with technology

# OR CHOOSE THREE MINOR OPTIONS



## Minor Options

Minor

Mechanics minor

Statistics minor

Modelling with algorithms

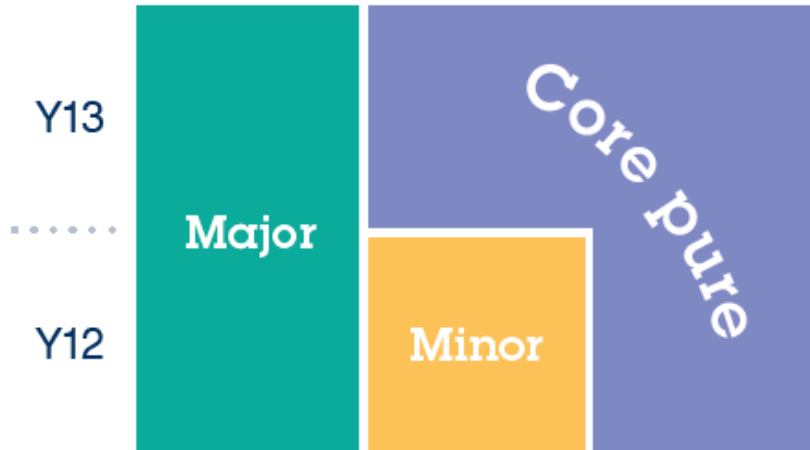
Numerical methods

Extra Pure

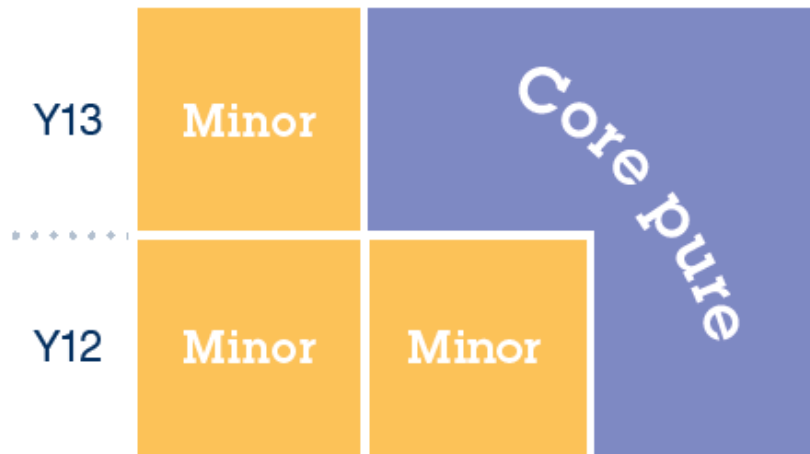
Further pure with technology



# ONE MAJOR + ONE MINOR OR THREE MINOR OPTIONS



or



## Major Options

Major

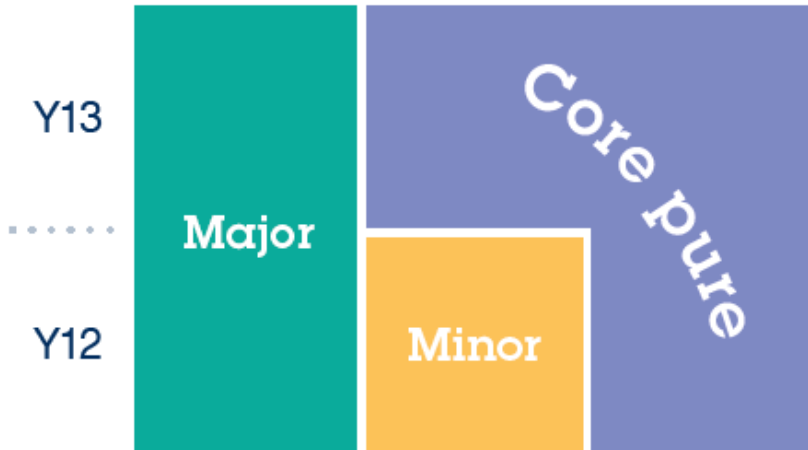
Mechanics major  
Statistics major

## Minor Options

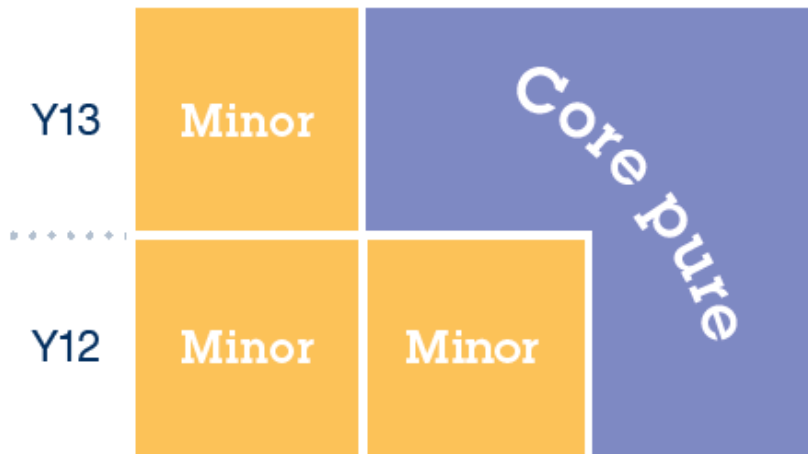
Minor

Mechanics minor  
Statistics minor  
Modelling with algorithms  
Numerical methods  
Extra Pure  
Further pure with technology

# ONE MAJOR + ONE MINOR OR THREE MINOR OPTIONS



or



## Major Options

Major

Mechanics major  
Statistics major

## Minor Options

Minor

Mechanics minor  
Statistics minor  
Algorithms  
Maths  
Number  
Further pure

Further pure with technology

**May sit extra minor units  
– best combination counts**

# AS FURTHER MATHEMATICS

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AS Further Maths – anything you can sensibly teach in Year 12 of the two year A level, you can take as an AS unit.

AS Further Maths takes up three teaching slots


- The mandatory Core pure paper takes 1 slot; it's the same content as one-third of the A level Core pure unit
- There are two slots for two optional units

# AS FURTHER MATHEMATICS

---



## AS Options



Mechanics a  
Statistics a  
Modelling with algorithms  
Numerical methods

# AS FURTHER MATHEMATICS



## AS Options

AS option

- Mechanics a
- Statistics a
- Modelling with algorithms
- Numerical methods
- Mechanics b
- Statistics b

# H645 A level Further Mathematics B (MEI)

## Mandatory unit

**Y420** Core pure

**2hr 40mins**

**144 marks**

**50%** of A level  
(after scaling)

## Major options

**Y421** Mechanics major

**Y422** Statistics major

**2hr 15mins**

**120 marks**

**33 $\frac{1}{3}$ %** of A level  
(after scaling)

## Minor options

**Y431** Mechanics minor

**Y432** Statistics minor

**Y433** Modelling with  
algorithms

**Y434** Numerical  
methods

**Y435** Extra pure

**Y436** Further pure  
with technology

**1 hr 15mins**

[Y436 1hr 45mins]

**60 marks**

**16 $\frac{2}{3}$ %** of A level  
(after scaling)

## H635 AS Further Mathematics B (MEI)

### Mandatory unit

**Y410** Core pure

**1 hr 15 mins**

**60 marks**

**33 $\frac{1}{3}$ %** of AS level

### Optional units

**Y411** Mechanics a

**Y412** Statistics a

**Y413** Modelling with algorithms

**Y414** Numerical methods

**Y415** Mechanics b

**Y416** Statistics b

**1 hr 15 mins**

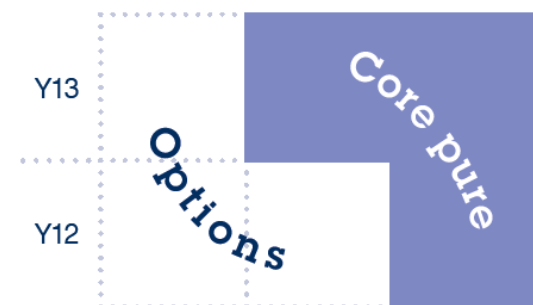
**60 marks**

**33 $\frac{1}{3}$ %** of AS level

# PURE CONTENT

---

- National compulsory content 50% of A level Further Mathematics.
- At least 30% of AS Further Mathematics must be based on the compulsory content – we have made it one third.





# MECHANICS/ STATISTICS

---

For A level Further Maths you can do

no mechanics *or*

one-sixth mechanics (minor option) *or*

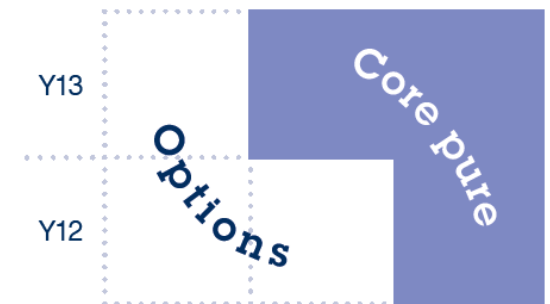
one-third mechanics (major option)

For AS Further Maths you can do

no mechanics *or*

one-third mechanics *or*

two-thirds mechanics



# MECHANICS

---

- Dimensional analysis
- Forces, moments and equilibrium
- Work, energy, power
- Momentum & impulse: direct impact collisions
- Centre of mass: system of particles
  
- Oblique impact collisions
- Circular motion
- Hooke's law
- Vectors and variable forces: projectile up plane, SHM
- Centre of mass using calculus

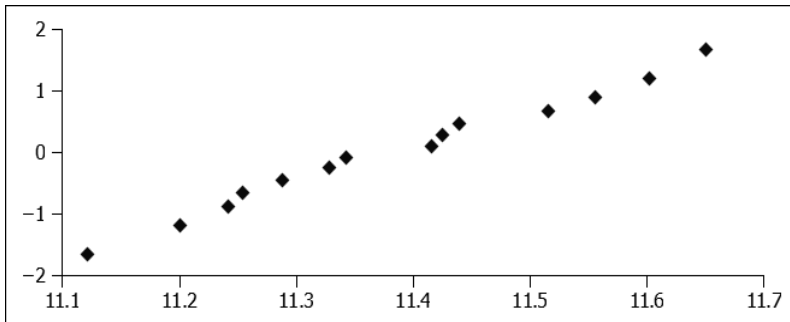
# STATISTICS

---

- Sampling
- Discrete random variables: uniform, Poisson, geometric
- Bivariate data: correlation and regression
- Chi-squared tests for contingency table and goodness of fit
  
- Continuous random variables
- Confidence intervals based on Normal and  $t$ -distributions
- Wilcoxon test for single sample, compared with corresponding  $t$ -test and Normal test
- Simulation

# TECHNOLOGY: CONFIDENCE INTERVALS

Using  $t$  or Normal distribution, as appropriate, to produce and interpret single sample, two-sample and paired sample confidence intervals



A Normal probability plot  
The closer the points to a straight line  
the better a Normal distribution fits

A confidence interval based on the  $t$  distribution,  
assuming the underlying distribution is Normal

T Estimate of a Mean

Confidence Level

Sample

Mean

s

N

Result

T Estimate of a Mean

Mean	11.385
s	0.1592
SE	0.0425
N	14
df	13
Lower Limit	11.2568
Upper Limit	11.5132
Interval	11.385 ± 0.1282

# TECHNOLOGY: SIMULATION

Hui's score is the total of 10 dice.  
 What is the probability that her score is greater than 35?

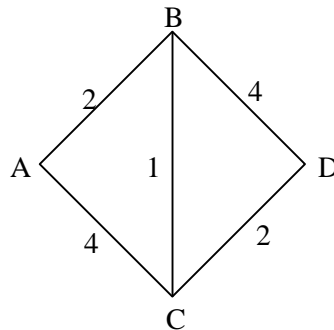


	A	B	C	D	E	F	G	H	I	J	K	L	M	N	
1		Throw of dice										Lili's	Hui's		
2		1	2	3	4	5	6	7	8	9	10		score	score	
3	Game 1	3	5	2	1	1	3	1	1	1	4		30	22	
4	Game 2	6	3	2	4	4	3	5	3	3	5		60	38	
5	Game 3	6	4	2	6	5	2	1	5	2	3		60	36	
6	Game 4	1	5	1	6	6	3	1	4	6	2		10	35	
7	Game 5	4	4	3	1	6	4	4	1	6	2		40	35	
8	Game 6	2	1	5	1	2	5	1	5	2	3		20	27	
9	Game 7	1	1	3	4	4	5	6	3	4	2		10	33	
10	Game 8	1	1	3	6	3	4	4	5	2	3		10	32	
11	Game 9	2	2	2	4	3	2	1	5	5	6		20	32	
12	Game 10	3	5	3	3	5	3	4	3	1	1		30	31	
13	Game 11	5	3	6	5	5	4	2	1	1	5		50	37	
14	Game 12	6	4	3	2	4	1	3	3	5	3		60	34	
15	Game 13	2	3	2	1	2	2	2	2	2	1		20	19	
16	Game 14	4	1	3	3	1	2	6	6	1	3		40	30	
17	Game 15	5	1	2	6	3	4	6	3	6	4		50	40	
18	Game 16	3	6	1	1	5	3	1	3	3	3		30	29	
19	Game 17	5	2	5	2	4	5	2	2	3	4		50	34	
20	Game 18	3	6	3	5	5	2	3	1	1	2		30	31	
21	Game 19	6	6	3	1	5	6	3	4	1	6		60	41	
22	Game 20	2	6	4	5	6	5	2	4	3	3		20	40	
23	Game 21	5	3	5	4	5	3	3	6	6	1		50	41	
24	Game 22	6	3	5	5	6	3	5	6	1	1		60	41	
25	Game 23	5	4	5	5	6	4	2	1	3	6		50	41	
26	Game 24	3	5	2	3	2	4	3	2	3	3		30	30	
27	Game 25	5	2	4	2	4	5	2	2	5	2		50	33	
28															
29													mean	37.60	33.68
30													sd	17.39	5.77

# MODELLING WITH ALGORITHMS

- Network & network algorithms
- Algorithms
- Linear programming: solving using graphical approach, simplex and software
- Reformulating network problems as LPs and using software to solve

Find shortest path  
from A to D



Minimise

$$2AB + 4BD + 4AC + 2CD + BC + CB$$

subject to

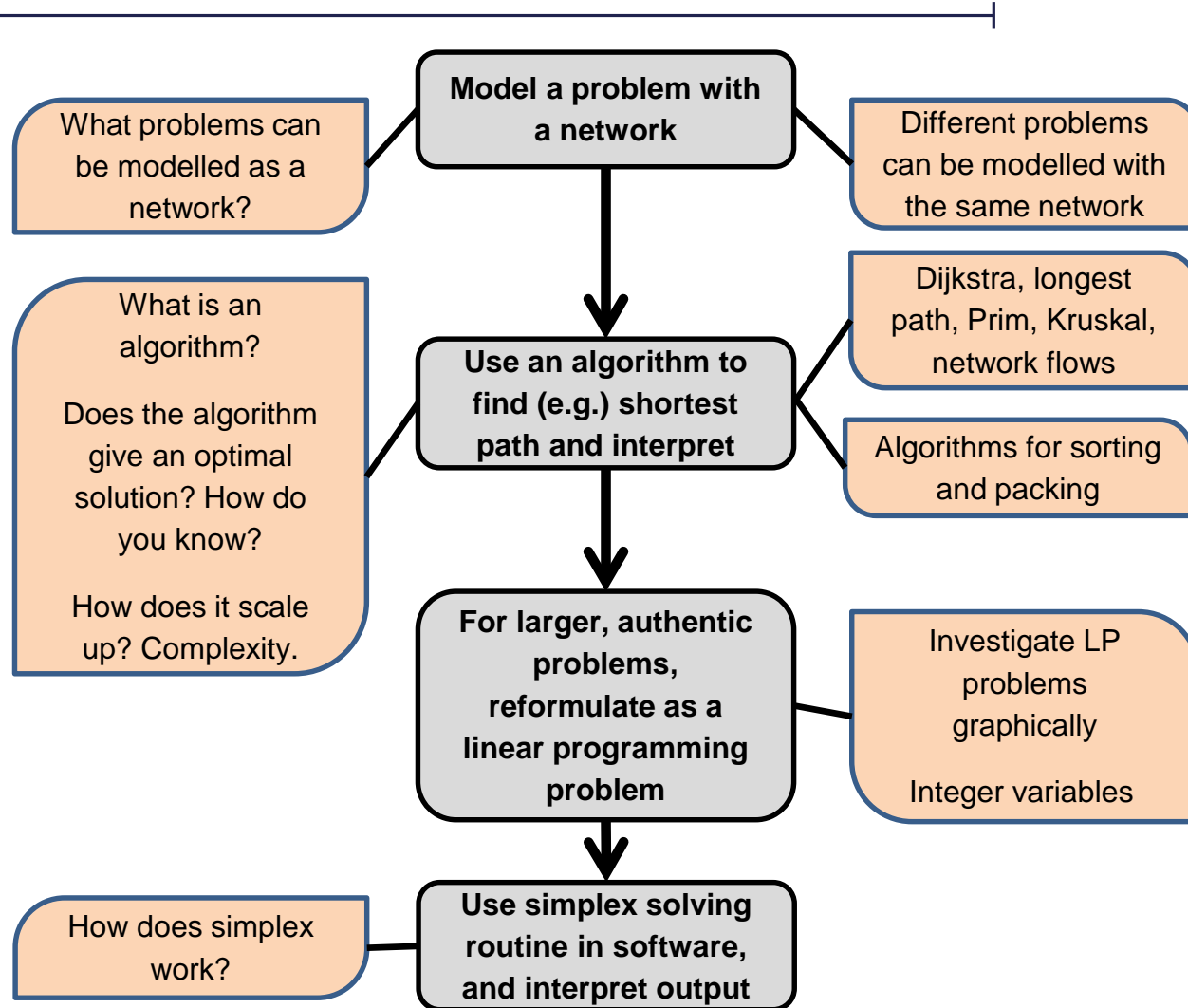
$$AB + AC = 1$$

$$AB + CB - BC - BD = 0$$

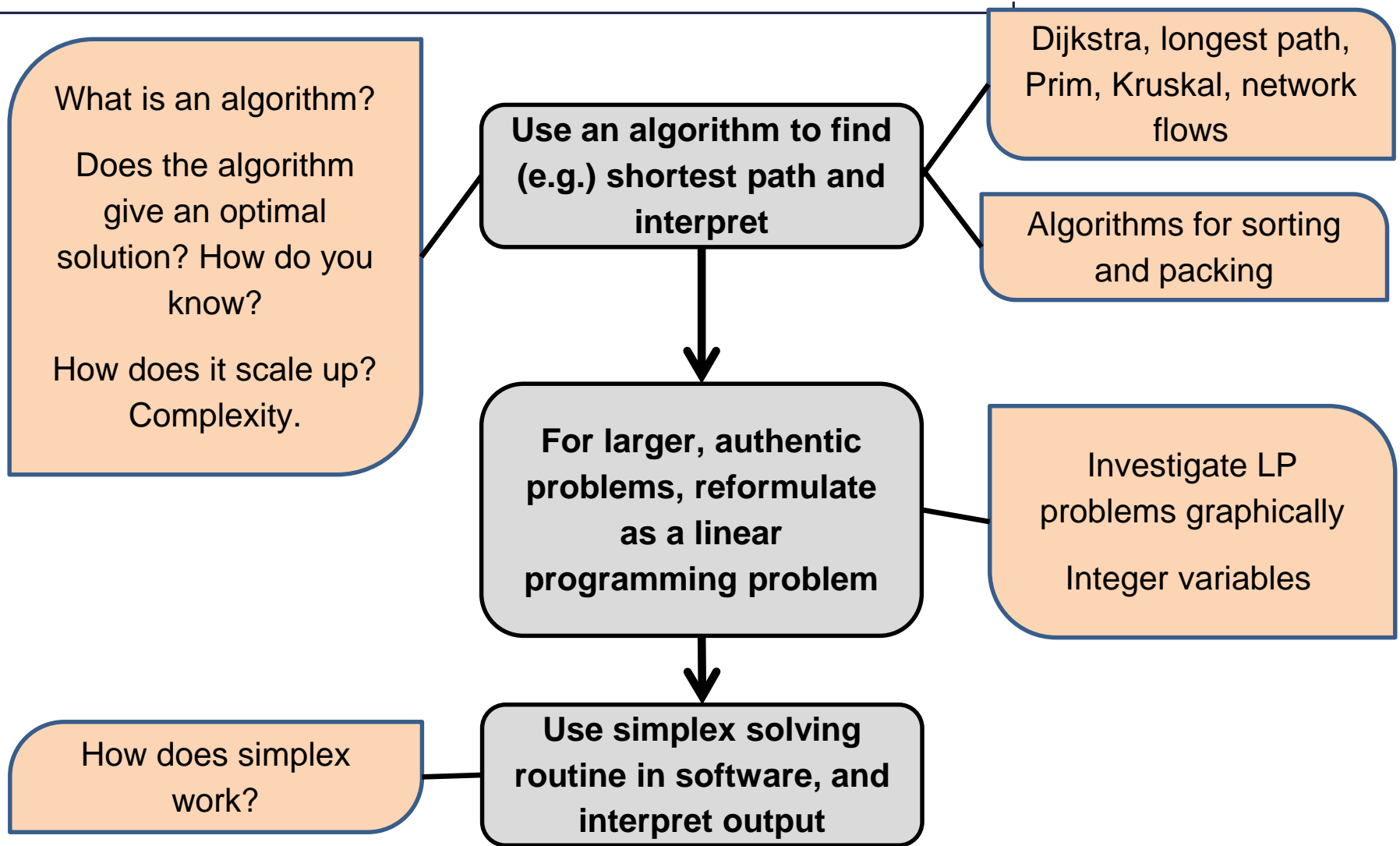
$$AC + BC - CB - CD = 0$$

$$BD + CD = 1$$

# JOURNEY THROUGH MODELLING WITH ALGORITHMS



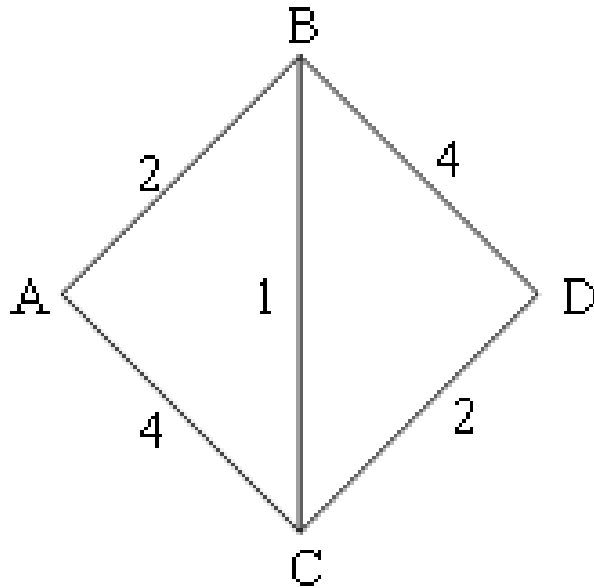
# JOURNEY THROUGH MODELLING WITH ALGORITHMS





# NEW TOPICS IN MWA

Reformulating a network problem as an LP



Minimise

$$2AB + 4BD + 4AC + 2CD + BC + CB$$

subject to

$$AB + AC = 1$$

$$AB + CB - BC - BD = 0$$

$$AC + BC - CB - CD = 0$$

$$BD + CD = 1$$

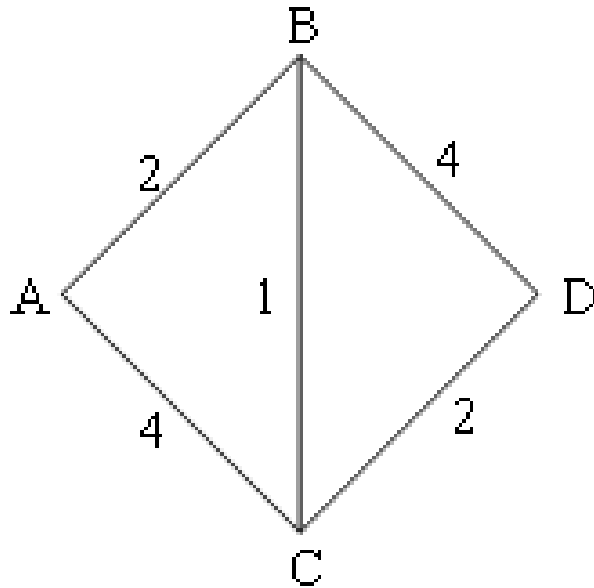
**Find the shortest path from A to D**

AB is a variable. It takes the value 1 if the path from A to D uses the arc from A to B. Otherwise it takes the value 0.

**So for the path A to B to D,  $AB = 1, BD = 1, AC = CD = BC = CB = 0$**

# NEW TOPICS IN MWA

Using software to solve an LP



Minimise

$$2AB + 4BD + 4AC + 2CD + BC + CB$$

subject to

$$AB + AC = 1$$

$$AB + CB - BC - BD = 0$$

$$AC + BC - CB - CD = 0$$

$$BD + CD = 1$$

$$AB = 1 \quad BD = 0 \quad AC = 0 \quad CD = 1 \quad BC = 1 \quad CD = 0$$

$$\text{Objective} = 5$$

# ***NUMERICAL METHODS***

---

- Content as for current Numerical Methods
  - Use of technology
  - Errors
  - Solution of Equations
  - Numerical differentiation
  - Numerical integration
  - Approximation to functions
- No coursework but interpretation of output from spreadsheets will be expected

# TOPICS IN NUMERICAL METHODS

---

- Use of technology
  - spreadsheets in the classroom; calculators
- **Dealing with errors**
  - **how they arise, propagate and can be analysed**
- Solution of equations
  - 5 methods, failure, order of convergence; relaxation
- Numerical differentiation
  - forward difference and central difference
- Numerical integration
  - midpoint, trapezium, Simpson's
- Approximation to functions
  - Newton, Lagrange

# ***NEW TOPICS IN NUMERICAL METHODS***

---

- No coursework, so clearer expectations about the use of technology
- One small topic – relaxation
- Clearer descriptions in the spec of (e.g.)
  - order of convergence and order of method
  - what error analysis is expected
  - notation for Simpson, trapezium and midpoint rules
- Some modelling in exam questions

# SPREADSHEETS IN THE EXAM

The spreadsheet printout shows the application of the secant method starting with  $x_0 = 0$  and  $x_1 = 1$ . Successive approximations to the root are in column E.

(ii) What feature of column B shows that this application of the secant method has been successful? [1]

(iii) Write down a suitable spreadsheet formula to obtain the value in cell E2. [2]

	A	B	C	D	E
1	$x_n$	$f(x_n)$	$x_{n+1}$	$f(x_{n+1})$	$x_{n+2}$
2	0	-1	1	0.6156265	0.6189549
3	1	0.6156265	0.6189549	-0.175846	0.7036139
4	0.6189549	-0.1758461	0.7036139	-0.025245	0.7178053
5	0.7036139	-0.0252451	0.7178053	0.0011619	0.7171808
6	0.7178053	0.0011619	0.7171808	-7.4E-06	0.7171848
7	0.7171808	-7.402E-06	0.7171848	-2.16E-09	0.7171848
8	0.7171848	-2.16E-09	0.7171848	3.997E-15	0.7171848

# A NEW TOPIC - RELAXATION

$$x_{n+1} = \exp(-(x_n)^2)$$

	$\lambda=0.5$	$\lambda=0.3$	$\lambda=0.1$	$\lambda=0.7$
1	1	1	1	1
2	0.367879441	0.68394	0.810364	0.936788
3	0.873423018	0.655168	0.722824	0.884688
4	0.466327189	0.653084	0.683892	0.841938
5	0.804558944	0.652931	0.666655	0.806964
6	0.523449304	0.65292	0.659016	0.77841
7	<b>0.652919</b>	0.652919	0.655626	0.755126
8	0.652919	0.652919	0.654121	0.736154
9	0.652919	0.652919	0.653453	0.720702
10	0.652919	0.652919	0.653156	0.708119
11	0.652919	0.652919	0.653024	0.697873
12	0.652919	0.652919	0.652965	0.689531
13	0.652919	0.652919	0.652939	0.682738
14	0.652919	0.652919	0.652928	0.677207
15	0.682229284	0.652919	0.652923	0.672702
16	0.627860798	0.652919	0.65292	0.669034
17	0.674213008	0.652919	<b>0.652919</b>	0.666046
18	0.634725168	0.652919	0.652919	0.663613
19	0.66839495	0.652919	0.652919	0.66163
20	0.639702657	0.652919	0.652919	0.660016
21	0.664168438	0.652919	0.652919	0.6587

For the iteration  
 $x_{n+1} = g(x_n)$   
the relaxed iteration is  
 $x_{n+1} = (1 - \lambda)x_n + \lambda g(x_n)$

# ***EXTRA PURE***

---

- Recurrence relations
- Groups
- Multivariable calculus: surfaces
- Matrices: eigenvectors/values
  
- No choice of questions



# ***FURTHER PURE WITH TECHNOLOGY***

---

- Investigation of curves
  - use a graph plotter to explore curves, a computer algebra system (CAS), to explore tangents, length of curves
- Number theory
  - use simple programming skills to investigate results in number theory
- Differential equations
  - use a graph plotter to draw tangent fields, CAS to produce analytical solutions where they exist, a spreadsheet to use numerical methods

Recommend Geogebra & Python

# ***SUPPORT***

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You can get more information or support at

<http://mei.org.uk/2017-mei-maths-spec>

<http://mei.org.uk/2017-mei-furthermaths-spec>

or by emailing [keith.proffitt@mei.org.uk](mailto:keith.proffitt@mei.org.uk)

# NEXT STEPS

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- 1 Download** our accredited specifications  
[ocr.org.uk/alevelmaths](https://ocr.org.uk/alevelmaths)
- 2 Book** Get Started CPD Training  
[ocr.org.uk/alevelmathscpd](https://ocr.org.uk/alevelmathscpd)
- 3 Sign up** to teach - [ocr.org.uk/teachwithocr](https://ocr.org.uk/teachwithocr)

**4 Talk to us**

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