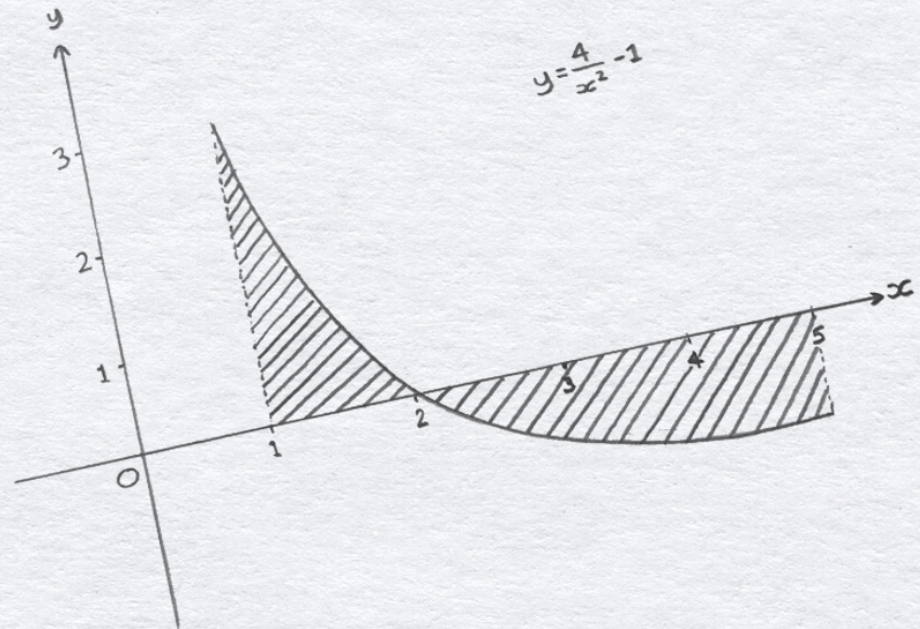


STAY AHEAD OF

THE CURVE

**About the new
Further
Mathematics B**



FURTHER MATHS EXAMPLES

Example 1

Core pure

+Mechanics major

+ Statistics minor

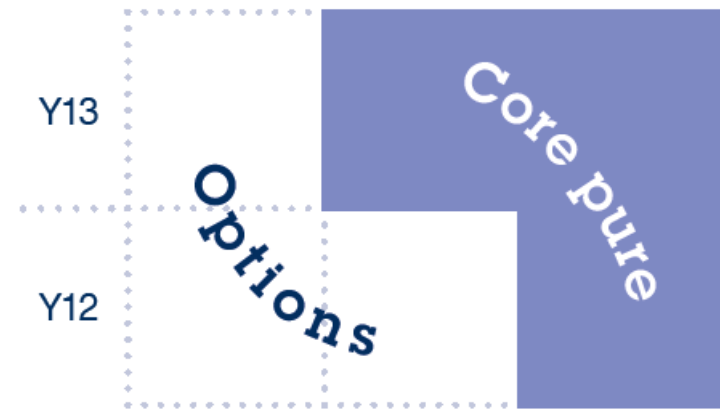
FURTHER MATHS EXAMPLES

Example 1

Core pure (FP1 + FP2 + FP3)

+Mechanics major (M2 + M3)

+ Statistics minor (S2)



FURTHER MATHS EXAMPLES

Example 1

Core pure (FP1 + FP2 + FP3)
+Mechanics major (M2 + M3)
+ Statistics minor (S2)

Example 2

Core pure
+ mechanics minor
+ stats minor
+ modelling with algorithms

FURTHER MATHS EXAMPLES

Example 1

Core pure (FP1 + FP2 + FP3)
+Mechanics major (M2 + M3)
+ Statistics minor (S2)

Example 2

Core pure (FP1 + FP2 + FP3)
+ mechanics minor (M2)
+ stats minor (S2)
+ modelling with algorithms (D1)



A LEVEL FURTHER MATHEMATICS



If you want to teach A level Further Maths across two years, parallel to A level Maths, then:

- Imagine the two years split into 6 teaching slots
- The mandatory Core pure paper takes 3 slots, one in Y12 and two in Y13
- There are three slots for optional units

CHOOSE ONE MAJOR OPTION + ONE MINOR OPTION



Major Options

Major

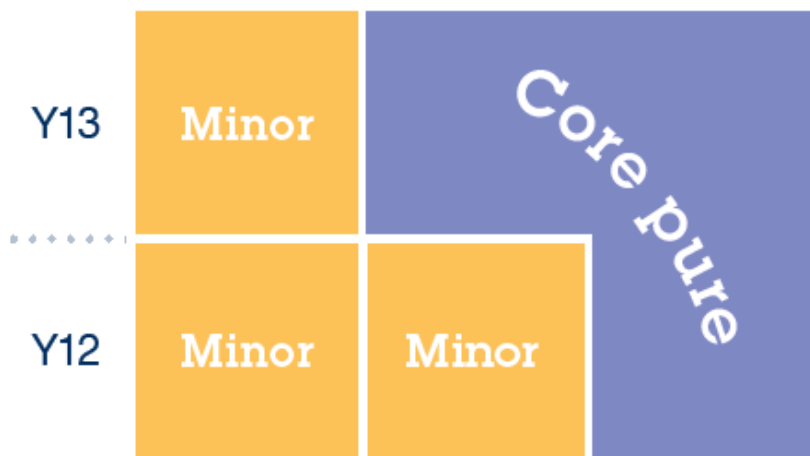
Mechanics major
Statistics major

Minor Options

Minor

Mechanics minor
Statistics minor
Modelling with algorithms
Numerical methods
Extra Pure
Further pure with technology

OR CHOOSE THREE MINOR OPTIONS



Minor Options

Minor

Mechanics minor

Statistics minor

Modelling with algorithms

Numerical methods

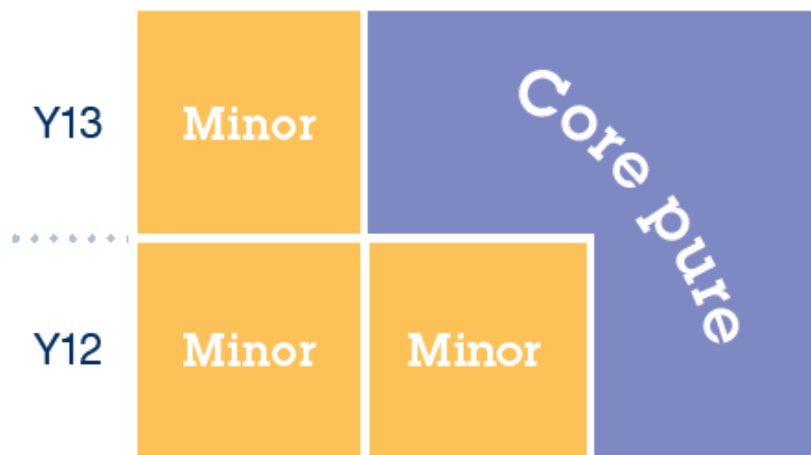
Extra Pure

Further pure with technology

ONE MAJOR + ONE MINOR OR THREE MINOR OPTIONS



or



Major Options

Major

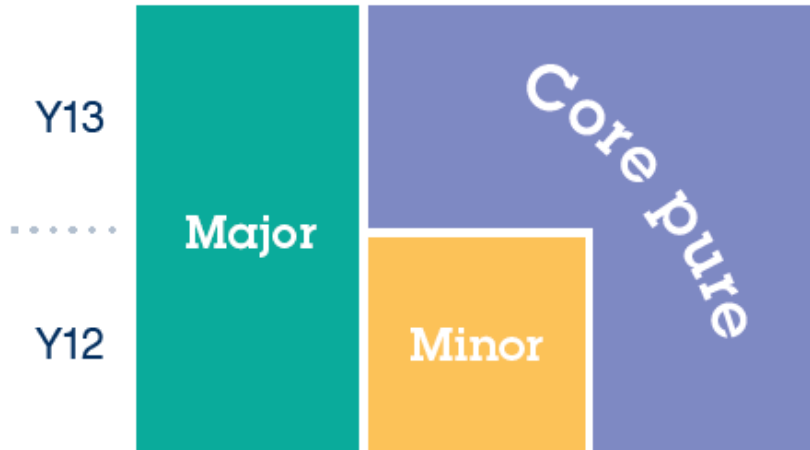
Mechanics major
Statistics major

Minor Options

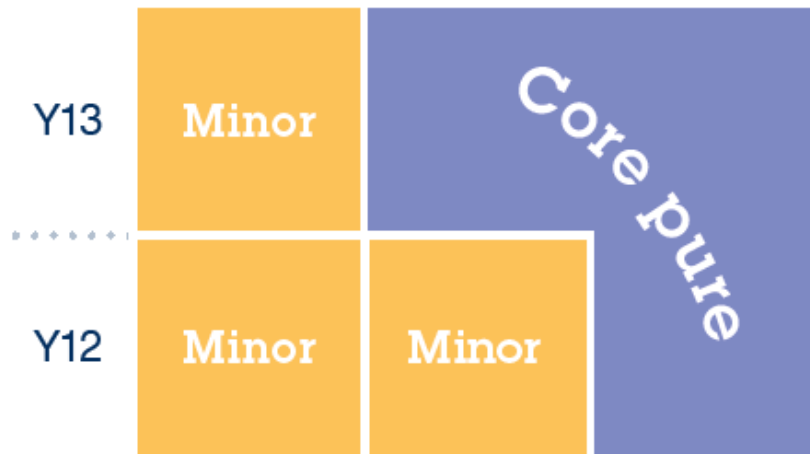
Minor

Mechanics minor
Statistics minor
Modelling with algorithms
Numerical methods
Extra Pure
Further pure with technology

ONE MAJOR + ONE MINOR OR THREE MINOR OPTIONS



or



Major Options

Major

Mechanics major
Statistics major

Minor Options

Minor

Mechanics minor
Statistics minor
Algorithms
Maths
Number
Further pure

Further pure with technology

**May sit extra minor units
– best combination counts**

AS FURTHER MATHEMATICS



AS Further Maths – anything you can sensibly teach in Year 12 of the two year A level, you can take as an AS unit.

AS Further Maths takes up three teaching slots

- The mandatory Core pure paper takes 1 slot; it's the same content as one-third of the A level Core pure unit
- There are two slots for two optional units

AS FURTHER MATHEMATICS



AS Options

AS option

- Mechanics a
- Statistics a
- Modelling with algorithms
- Numerical methods

AS FURTHER MATHEMATICS



AS Options

AS option

- Mechanics a
- Statistics a
- Modelling with algorithms
- Numerical methods
- Mechanics b
- Statistics b

AS Further Mathematics



**These units are available
as standalone Level 3
Certificates**

AS Core pure
Mechanics a
Statistics a
Modelling with algorithms
Numerical methods

AS Options

AS
option

Mechanics a
Statistics a
Modelling with algorithms
Numerical methods
Mechanics b
Statistics b

H645 A level Further Mathematics B (MEI)

Mandatory unit

Y420 Core pure

2hr 40mins

144 marks

50% of A level
(after scaling)

Major options

Y421 Mechanics major

Y422 Statistics major

2hr 15mins

120 marks

33 $\frac{1}{3}$ % of A level
(after scaling)

Minor options

Y431 Mechanics minor

Y432 Statistics minor

Y433 Modelling with
algorithms

Y434 Numerical
methods

Y435 Extra pure

Y436 Further pure
with technology

1 hr 15mins

[Y436 1hr 45mins]

60 marks

16 $\frac{2}{3}$ % of A level
(after scaling)

H635 AS Further Mathematics B (MEI)

Mandatory unit

Y410 Core pure

1 hr 15 mins

60 marks

33 $\frac{1}{3}$ % of AS level

Optional units

Y411 Mechanics a

Y412 Statistics a

Y413 Modelling with algorithms

Y414 Numerical methods

Y415 Mechanics b

Y416 Statistics b

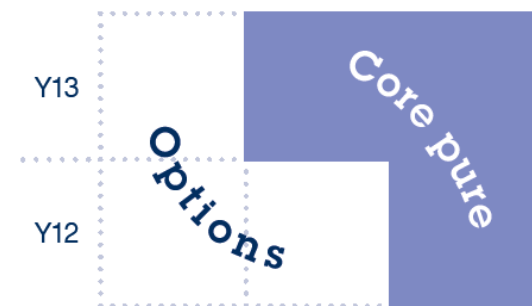
1 hr 15 mins

60 marks

33 $\frac{1}{3}$ % of AS level

PURE CONTENT

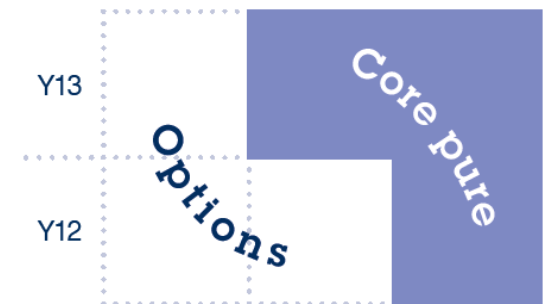
- National compulsory content 50% of A level Further Mathematics.
- At least 30% of AS Further Mathematics must be based on the compulsory content – we have made it one third.



MECHANICS/ STATISTICS

For A level Further Maths you can do
no mechanics *or*
one-sixth mechanics (minor option) *or*
one-third mechanics (major option)

For AS Further Maths you can do
no mechanics *or*
one-third mechanics *or*
two-thirds mechanics



MECHANICS

- Dimensional analysis
- Forces, moments and equilibrium
- Work, energy, power
- Momentum & impulse: direct impact collisions
- Centre of mass: system of particles

- Oblique impact collisions
- Circular motion
- Hooke's law
- Vectors and variable forces: projectile up plane, SHM
- Centre of mass using calculus

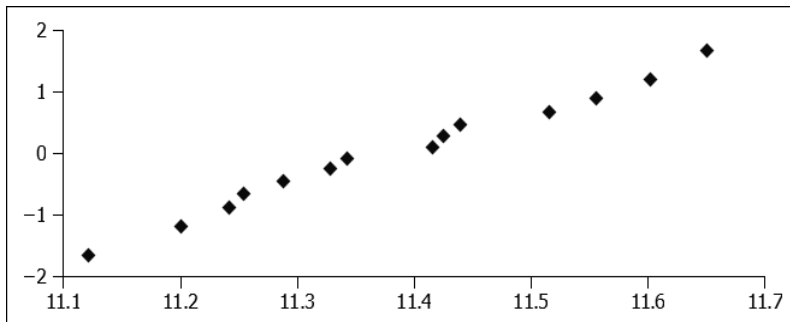
STATISTICS

- Sampling
- Discrete random variables: uniform, Poisson, geometric
- Bivariate data: correlation and regression
- Chi-squared tests for contingency table and goodness of fit

- Continuous random variables
- Confidence intervals based on Normal and t -distributions
- Wilcoxon test for single sample, compared with corresponding t -test and Normal test
- Simulation

TECHNOLOGY: CONFIDENCE INTERVALS

Using t or Normal distribution, as appropriate, to produce and interpret single sample, two-sample and paired sample confidence intervals



A Normal probability plot
The closer the points to a straight line
the better a Normal distribution fits

A confidence interval based on the t distribution,
assuming the underlying distribution is Normal

T Estimate of a Mean

Confidence Level

Sample

Mean

s

N

Result

T Estimate of a Mean

Mean	11.385
s	0.1592
SE	0.0425
N	14
df	13
Lower Limit	11.2568
Upper Limit	11.5132
Interval	11.385 ± 0.1282

TECHNOLOGY: SIMULATION

Hui's score is the total of 10 dice.
 What is the probability that her score is greater than 35?

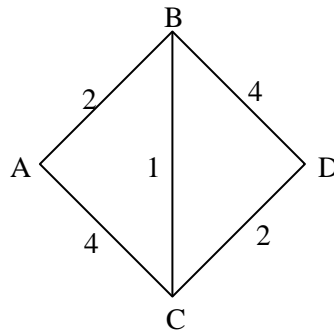


	A	B	C	D	E	F	G	H	I	J	K	L	M	N	
1		Throw of dice										Lili's	Hui's		
2		1	2	3	4	5	6	7	8	9	10		score	score	
3	Game 1	3	5	2	1	1	3	1	1	1	4		30	22	
4	Game 2	6	3	2	4	4	3	5	3	3	5		60	38	
5	Game 3	6	4	2	6	5	2	1	5	2	3		60	36	
6	Game 4	1	5	1	6	6	3	1	4	6	2		10	35	
7	Game 5	4	4	3	1	6	4	4	1	6	2		40	35	
8	Game 6	2	1	5	1	2	5	1	5	2	3		20	27	
9	Game 7	1	1	3	4	4	5	6	3	4	2		10	33	
10	Game 8	1	1	3	6	3	4	4	5	2	3		10	32	
11	Game 9	2	2	2	4	3	2	1	5	5	6		20	32	
12	Game 10	3	5	3	3	5	3	4	3	1	1		30	31	
13	Game 11	5	3	6	5	5	4	2	1	1	5		50	37	
14	Game 12	6	4	3	2	4	1	3	3	5	3		60	34	
15	Game 13	2	3	2	1	2	2	2	2	2	1		20	19	
16	Game 14	4	1	3	3	1	2	6	6	1	3		40	30	
17	Game 15	5	1	2	6	3	4	6	3	6	4		50	40	
18	Game 16	3	6	1	1	5	3	1	3	3	3		30	29	
19	Game 17	5	2	5	2	4	5	2	2	3	4		50	34	
20	Game 18	3	6	3	5	5	2	3	1	1	2		30	31	
21	Game 19	6	6	3	1	5	6	3	4	1	6		60	41	
22	Game 20	2	6	4	5	6	5	2	4	3	3		20	40	
23	Game 21	5	3	5	4	5	3	3	6	6	1		50	41	
24	Game 22	6	3	5	5	6	3	5	6	1	1		60	41	
25	Game 23	5	4	5	5	6	4	2	1	3	6		50	41	
26	Game 24	3	5	2	3	2	4	3	2	3	3		30	30	
27	Game 25	5	2	4	2	4	5	2	2	5	2		50	33	
28															
29													mean	37.60	33.68
30													sd	17.39	5.77

MODELLING WITH ALGORITHMS

- Network & network algorithms
- Algorithms
- Linear programming: solving using graphical approach, simplex and software
- Reformulating network problems as LPs and using software to solve

Find shortest path
from A to D



Minimise

$$2AB + 4BD + 4AC + 2CD + BC + CB$$

subject to

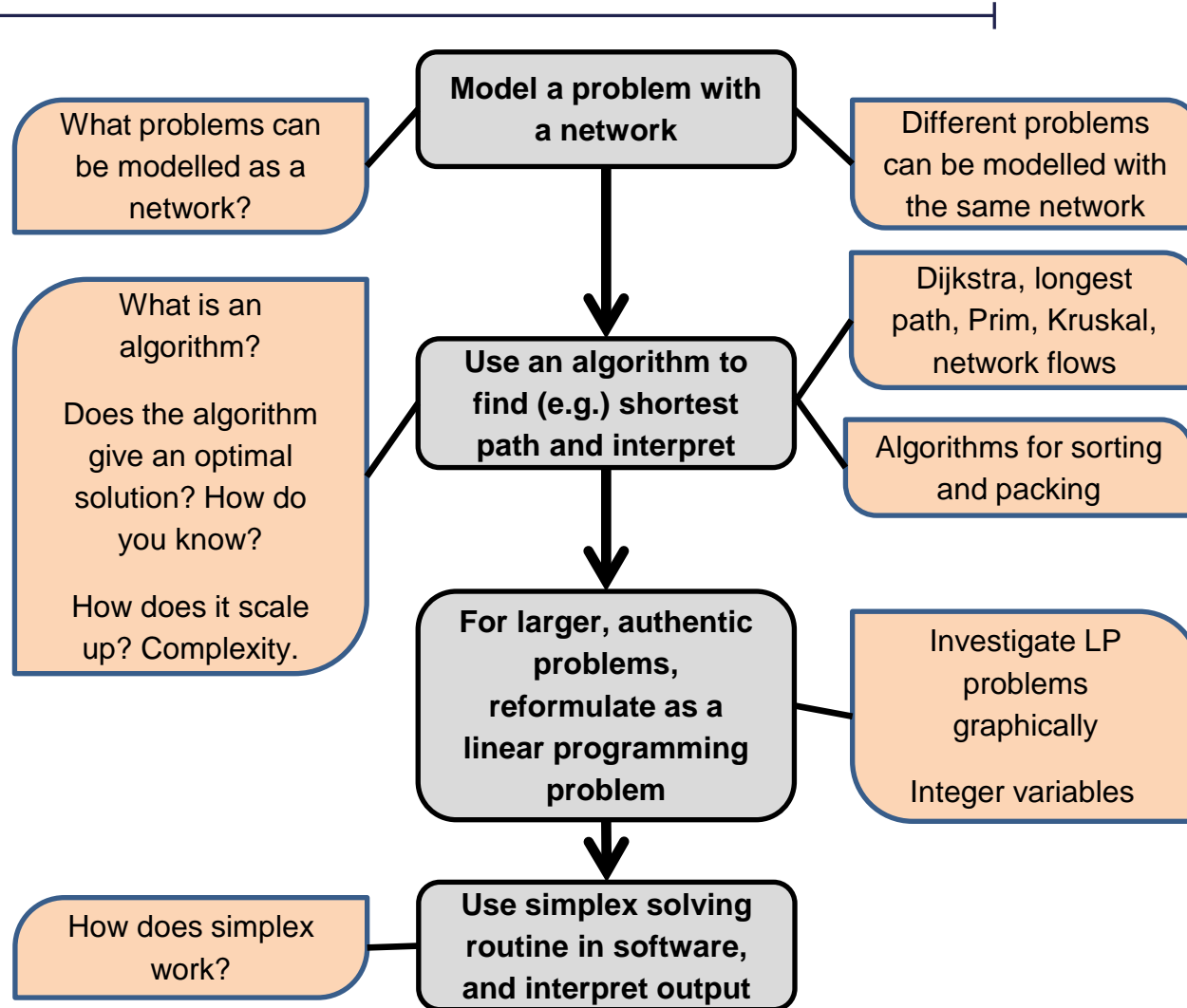
$$AB + AC = 1$$

$$AB + CB - BC - BD = 0$$

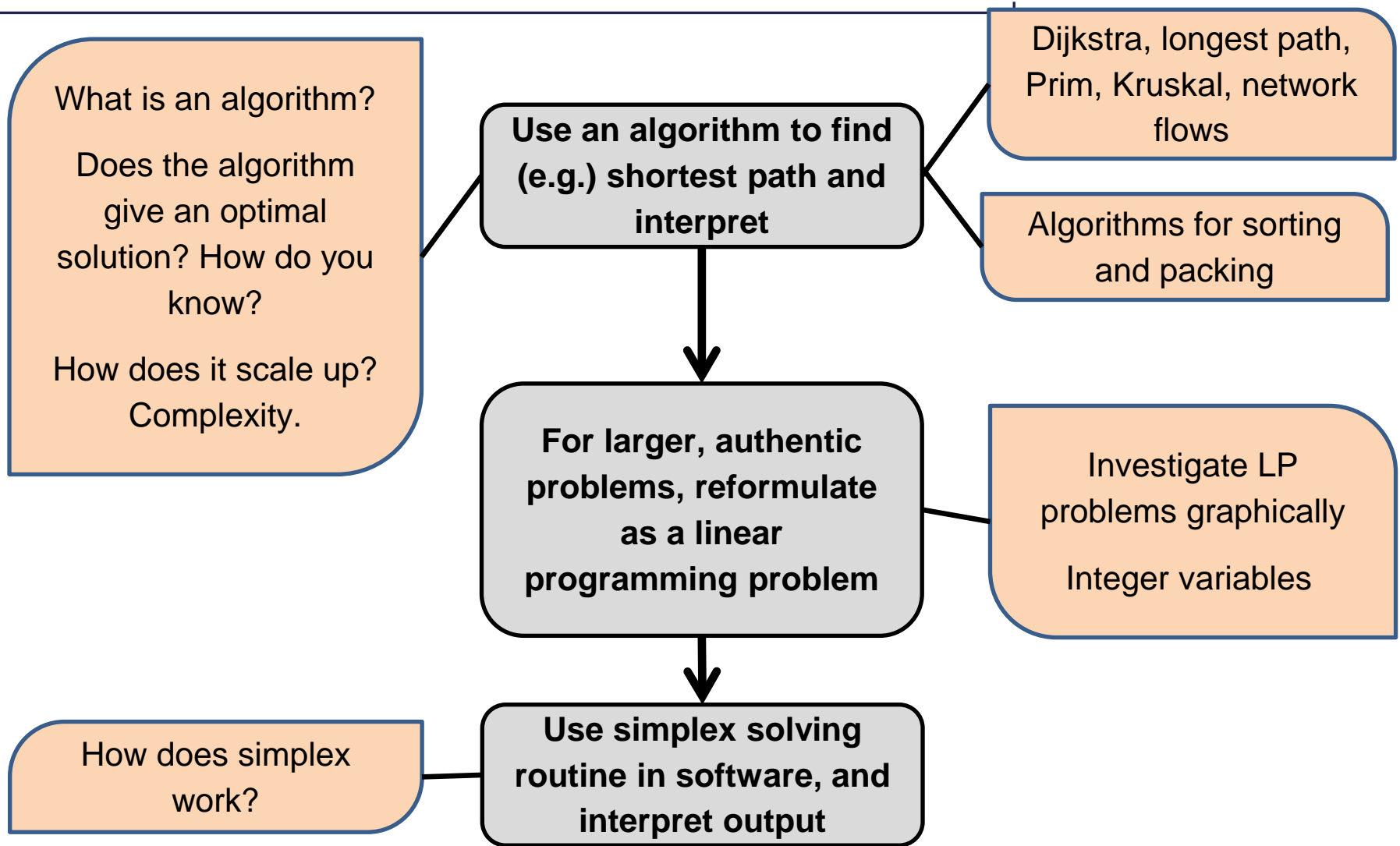
$$AC + BC - CB - CD = 0$$

$$BD + CD = 1$$

JOURNEY THROUGH MODELLING WITH ALGORITHMS

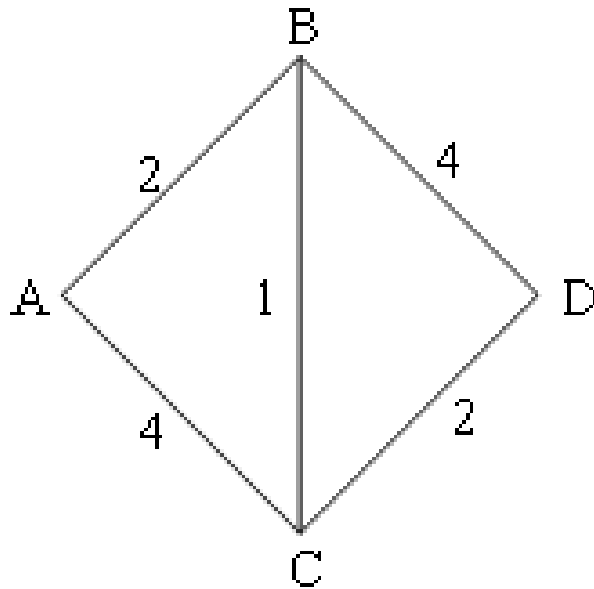


JOURNEY THROUGH MODELLING WITH ALGORITHMS



NEW TOPICS IN MWA

Reformulating a network problem as an LP



Minimise

$$2AB + 4BD + 4AC + 2CD + BC + CB$$

subject to

$$AB + AC = 1$$

$$AB + CB - BC - BD = 0$$

$$AC + BC - CB - CD = 0$$

$$BD + CD = 1$$

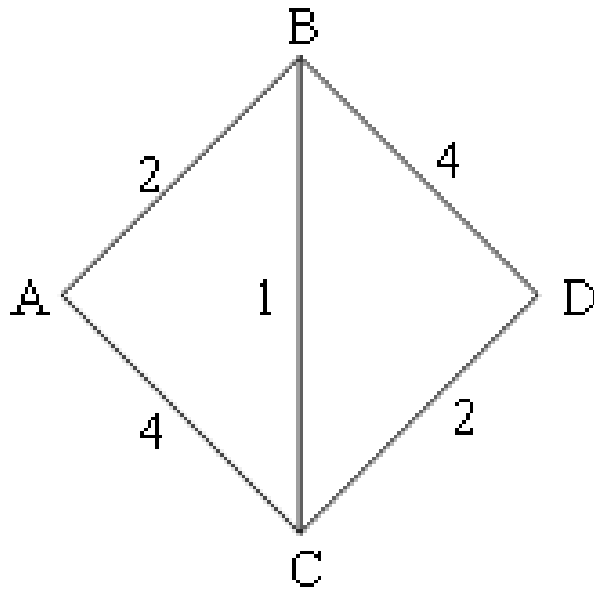
Find the shortest path from A to D

AB is a variable. It takes the value 1 if the path from A to D uses the arc from A to B. Otherwise it takes the value 0.

So for the path A to B to D, $AB = 1$, $BD=1$, $AC=CD=BC=CB=0$

NEW TOPICS IN MWA

Using software to solve an LP



Minimise

$$2AB + 4BD + 4AC + 2CD + BC + CB$$

subject to

$$AB + AC = 1$$

$$AB + CB - BC - BD = 0$$

$$AC + BC - CB - CD = 0$$

$$BD + CD = 1$$

$$AB = 1 \quad BD = 0 \quad AC = 0 \quad CD = 1 \quad BC = 1 \quad CD = 0$$

$$\text{Objective} = 5$$

NUMERICAL METHODS

- Content as for current Numerical Methods
 - Use of technology
 - Errors
 - Solution of Equations
 - Numerical differentiation
 - Numerical integration
 - Approximation to functions
- No coursework but interpretation of output from spreadsheets will be expected

TOPICS IN NUMERICAL METHODS

- Use of technology
 - spreadsheets in the classroom; calculators
- **Dealing with errors**
 - **how they arise, propagate and can be analysed**
- Solution of equations
 - 5 methods, failure, order of convergence; relaxation
- Numerical differentiation
 - forward difference and central difference
- Numerical integration
 - midpoint, trapezium, Simpson's
- Approximation to functions
 - Newton, Lagrange

NEW TOPICS IN NUMERICAL METHODS

- No coursework, so clearer expectations about the use of technology
- One small topic – relaxation
- Clearer descriptions in the spec of (e.g.)
 - order of convergence and order of method
 - what error analysis is expected
 - notation for Simpson, trapezium and midpoint rules
- Some modelling in exam questions

SPREADSHEETS IN THE EXAM

The spreadsheet printout shows the application of the secant method starting with $x_0 = 0$ and $x_1 = 1$. Successive approximations to the root are in column E.

(ii) What feature of column B shows that this application of the secant method has been successful? [1]

(iii) Write down a suitable spreadsheet formula to obtain the value in cell E2. [2]

	A	B	C	D	E
1	x_n	$f(x_n)$	x_{n+1}	$f(x_{n+1})$	x_{n+2}
2	0	-1	1	0.6156265	0.6189549
3	1	0.6156265	0.6189549	-0.175846	0.7036139
4	0.6189549	-0.1758461	0.7036139	-0.025245	0.7178053
5	0.7036139	-0.0252451	0.7178053	0.0011619	0.7171808
6	0.7178053	0.0011619	0.7171808	-7.4E-06	0.7171848
7	0.7171808	-7.402E-06	0.7171848	-2.16E-09	0.7171848
8	0.7171848	-2.16E-09	0.7171848	3.997E-15	0.7171848

A NEW TOPIC - RELAXATION

$$x_{n+1} = \exp(-(x_n)^2)$$

	$\lambda=0.3$	$\lambda=0.5$	$\lambda=0.3$	$\lambda=0.1$	$\lambda=0.7$
1	1	1	1	1	1
2	0.367879441	0.68394	0.810364	0.936788	0.557516
3	0.873423018	0.655168	0.722824	0.884688	0.680245
4	0.466327189	0.653084	0.683892	0.841938	0.644766
5	0.804558944	0.652931	0.666655	0.806964	0.655334
6	0.523449304	0.65292	0.659016	0.77841	0.652201
7		0.652919	0.655626	0.755126	0.653132
8		0.652919	0.654121	0.736154	0.652855
9		0.652919	0.653453	0.720702	0.652937
10		0.652919	0.653156	0.708119	0.652913
11		0.652919	0.653024	0.697873	0.65292
12		0.652919	0.652965	0.689531	0.652918
13		0.652919	0.652939	0.682738	0.652919
14		0.652919	0.652928	0.677207	0.652919
15	0.682229284	0.652919	0.652923	0.672702	0.652919
16	0.627860798	0.652919	0.65292	0.669034	0.652919
17	0.674213008	0.652919	0.652919	0.666046	0.652919
18	0.634725168	0.652919	0.652919	0.663613	0.652919
19	0.66839495	0.652919	0.652919	0.66163	0.652919
20	0.639702657	0.652919	0.652919	0.660016	0.652919
21	0.664168438	0.652919	0.652919	0.6587	0.652919

For the iteration
 $x_{n+1} = g(x_n)$
 the relaxed iteration is
 $x_{n+1} = (1 - \lambda)x_n + \lambda g(x_n)$

EXTRA PURE

- Recurrence relations
- Groups
- Multivariable calculus: surfaces
- Matrices: eigenvectors/values

- No choice of questions

FURTHER PURE WITH TECHNOLOGY

- Investigation of curves
 - use a graph plotter to explore curves, a computer algebra system (CAS), to explore tangents, length of curves
- Number theory
 - use simple programming skills to investigate results in number theory
- Differential equations
 - use a graph plotter to draw tangent fields, CAS to produce analytical solutions where they exist, a spreadsheet to use numerical methods

Recommend Geogebra & Python

SUPPORT

You can get more information or support at

<http://mei.org.uk/2017-mei-maths-spec>

<http://mei.org.uk/2017-mei-furthermaths-spec>

or by emailing keith.proffitt@mei.org.uk

NEXT STEPS

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