OCR
Oxford Cambridge and RSA

Tuesday 21 June 2016 – Morning

AS GCE MATHEMATICS (MEI)

4776/01 Numerical Methods

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4776/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 The expression $\sqrt{\frac{n}{n-1}}$ is sometimes approximated by 1 when n is large.

(i) Find the absolute and the relative error in this approximation when $n = 40$. **[2]**

(ii) Using trial and error or otherwise, find the smallest integer n for which the magnitude of the relative error is less than 1%. **[2]**

2 You are given that the equation

$$x^3 + x - 3 = 0$$

has a single real root α, where $1 < \alpha < 2$.

(i) Use the Newton-Raphson method with $x_0 = 1.5$ to find α correct to 5 decimal places. **[5]**

(ii) By considering ratios of differences, show that the Newton-Raphson method is faster than first order. **[3]**

3 A function f(x) has the following values, correct to 5 decimal places. (The values of x are exact.)

x	0.5	1	1.5	2	2.5
f(x)	0.958 85	0.841 47	0.665 00	0.454 65	0.239 39

(i) Obtain two Simpson's rule estimates of $I = \int_{0.5}^{2.5} \mathrm{f}(x)\,\mathrm{d}x$. **[3]**

(ii) State the order of Simpson's rule and hence estimate the value of I that would be obtained if f(x) were known at $x = 0.75, 1.25, 1.75, 2.25$. **[4]**

(iii) Give the value of I to the accuracy that is justified. **[1]**

4 **(i)** State the orders of accuracy of the forward difference and central difference formulae for numerical differentiation. Explain what this means in practice. **[3]**

(ii) A function $g(x)$ has the following values, correct to 5 decimal places. (The values of x are exact.)

x	−0.2	−0.1	0	0.1	0.2
$g(x)$	0.755 60	0.876 86	1	1.123 14	1.244 40

Obtain two estimates of $g'(0)$ using the forward difference formula, and two estimates of $g'(0)$ using the central difference formula.

Comment on your estimates. **[5]**

5 A function $h(x)$ has values as shown in the table.

x	$h(x)$
0	1.357 01
0.5	1.413 33
1	1.381 77
1.5	1.264 31

(i) Show, by means of a difference table, that $h(x)$ can be well approximated by a quadratic. **[3]**

(ii) Use Newton's forward difference interpolation formula with $x_0 = 0$ to write down an expression for the quadratic approximation to $h(x)$. (You do not need to simplify this expression.) **[3]**

(iii) Find the error in the quadratic approximation at $x = 1.5$. **[2]**

Section B (36 marks)

6 **(i)** Show, by means of a sketch graph, that the equation

$$kx = 3^{-x}, \quad (*)$$

where $k > 0$, has exactly one root. **[4]**

(ii) Show numerically that the iterative formula

$$x_{r+1} = \tfrac{1}{k}3^{-x_r}, \quad (**)$$

with $x_0 = 1$,

(A) converges in the case $k = 0.5$,

(B) diverges in the case $k = 0.4$.

Explain why it would *not* be a good idea to use (**) in the case $k = 0.5$. **[7]**

(iii) Show that (*) may be rearranged as

$$x = 0.5\left(x + \tfrac{1}{k}3^{-x}\right).$$

Use an iteration based on this rearrangement to find the root of (*), correct to 4 decimal places, in the cases

(A) $k = 0.5$,

(B) $k = 0.4$. **[7]**

7 Let S_n be the sum of the first n terms in the series

$$1 - \tfrac{1}{2} + \tfrac{1}{3} - \tfrac{1}{4} + \cdots. \quad (*)$$

It is known that S_n converges to a limit S as n tends to infinity. A spreadsheet is used to investigate the rate of convergence of S_n to S.

(i) The spreadsheet gives $S_{1000} = 0.692\,647$, hence 0.692 65 to 5 decimal places.

Find S_{1001} and S_{1002} correct to 5 decimal places. Comment on the rate of convergence of S_n. **[4]**

(ii) Show, by combining adjacent terms, that (*) may be written as

$$\tfrac{1}{2} + \tfrac{1}{12} + \cdots. \quad (**)$$

State the next two terms in this series. **[3]**

Let T_n be the sum of the first n terms of (**). A spreadsheet is used to investigate the rate of convergence of T_n.

(iii) Explain why T_{500} will be 0.692 65 correct to 5 decimal places.

Find T_{501} and T_{502} correct to 5 decimal places. Comment on the rate of convergence of T_n. **[4]**

An improved method for summing (**) is to add a 'correction term' as follows.

$$T_n + \tfrac{1}{4n+2} \quad (***)$$

(iv) Evaluate (***) correct to 5 decimal places for $n = 500$ and $n = 501$.

Comment on your answers. **[4]**

(v) Discuss briefly what your answers to parts **(i)**, **(iii)** and **(iv)** indicate about convergence when successive answers agree to a certain number of decimal places.

Explain which, if any, of the sums calculated you would regard as the value of S correct to 5 decimal places. **[3]**

END OF QUESTION PAPER

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