

GCE

Mathematics (MEI)

Unit 4777: Numerical Computation

Advanced GCE

Mark Scheme for June 2016

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations and abbreviations

Annotation in scoris	Meaning
BP	Blank Page - this annotation must be used on all blank pages within an answer booklet (structured or
	unstructured) and on each page of an additional object where there is no candidate response.
√and ≭	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
٨	Omission sign
MR	Misread
Highlighting	
Other abbreviations	Meaning
in mark scheme	
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

Α

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

Е

Mark for a correct result or statement independent of Method marks.

Ε

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored.

Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.
 - Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.
- g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question	Answer	Marks	Guidance
1 (i)	Write equations for x_1 and x_2 . Correct algebra to obtain α and k	M1 A1 A1 [3]	
(ii)	25 20 15 10 10 -4 -3 -2 -1 0 1 2 3 4 Convincing explanation (e.g. one curve rises the other is bounded)	G2 G1 E1 [4]	One graph correct And the other

Question			Ar	nswer	Marks	Guidance
(iii)	Roots at abou				B1	
	_	nning at 1.5: i	teration begin	-		
		.5		0.5	M2	Setting up a correct spreadsheet
	1.561			82549		spreadsheet
	1.56650			84951	A1	
	1.5667			24218		First iteration
	1.566			31446	A1	
	1.566			50233		Second iteration
	1.566			65851		
	1.566			66745		
	1.566			66788		
	1.566	79	1.	56679		
	G '11 1				E1	
	Sensible expl New iteration					
	New Iteration	0.5		k		
		0.582549	0.082549	κ		
		0.784951	0.082349	2.451894	M1	Modify spreadsheet to
	$\alpha =$	0.784931	0.202402	2.431034	M1	calculate k
	α –	0.445144	-0.03642			Modify spreadsheet to calculate α
		0.400727	-0.03042	3.707646		Calculate a
	$\alpha =$	0.271700	-0.13302	3.707040	A1	One successful iteration
	α –	0.450394	-0.00486			
		0.43173	-0.00486	3.290369	A1	Correct use of α to restart the
	$\alpha =$	0.458717	-0.010	3.270307		iteration
	α –	0.458624	-9.3E-05			
		0.458324	-0.0003	3.234158		
	$\alpha =$	0.458758	-0.0003	3.43+130		
	α –	0.458758	-3.4E-08			
		0.458758	-3.4E-08 -1.1E-07	3.23308	A1	Obtain root
		0.430/30	-1.1E-U/	3.43300	[11]	
					[11]	

Question	Answer	Marks	Guidance		
(iv)	Set up spreadsheet to calculate both roots with a user specified c . E.g. for $c = 5$:	M3			
	5 0.5 k 1.5 k				
	0.582549 0.082549 1.56116 0.06116				
	0.784951 0.202402 2.451894 1.5665044 0.0053444 0.087383	86			
	0.443144 1.5670161				
	0.406727 -0.03642 1.5668012 -0.0002149				
	0.271706	<u>1</u>			
	0.456594 1.5667905				
	0.45173 -0.00486 1.5667905 -1.32E-09				
	0.435728 -0.016 3.290369 1.5667905 -6.304E-11 0.04774	9			
	0.458717 1.5667905				
	0.458624 -9.3E-05 1.5667905 0				
	0.458324 -0.0003 3.234158 1.5667905 0 #DIV/0	<u>!</u>			
	Difference between roots: 1.1310627	A1			
	Trial and error to obtain $c = 4.71$	M1A1 [6]			
2 (i)	f(x) = 1 2h = 2a + b	M1A1			
	$(f(x) = x, x^3 \text{ give } 0 = 0, \text{ not reqd.})$	2/141			
	$f(x) = x^2 \qquad 2h^3/3 = 2a\alpha^2$	M1A1 M1A1			
	$f(x) = x^4 \qquad 2h^5/5 = 2a\alpha^4$	M1A1			
	Convincing algebra to verify given results	[8]			
(ii)	Show that $f(x) = x^5$ gives correct answer $(0 = 0)$	B1			
	Show that $f(x) = x^6$ gives $2h^7/7 = 2a\alpha^6$ That is, $2h^7/7 = 6h^7/25$	B1 B1			
	So the local error is of order h^7 , the global error is of order h^6 .	B1			
	, 5	[4]			

Q	uestion					Α	nswer					Marks	Guidance
	(iii)	h	m - a	m	m + a	f(m - a)	f(m)	f(m+a)		I	k =		
		1.570796	0.354063	1.570796	2.78753	1.414408	2.718282	1.414408		6.264046	1	M1A1	First application
		0.785398	0.177031	0.785398	1.393765	1.192567	2.028115	2.676128	3.103928				
		0.785398	1.747828	2.356194	2.964561	2.676128	2.028115	1.192567	3.103928	6.207856		M1A1	Halving the interval and
		0.392699	0.088516	0.392699	0.696882	1.092425	1.466214	1.899955	1.164641				applying the rule twice
		0.392699	0.873914	1.178097	1.482281	2.152967	2.519044	2.707661	1.939737				
		0.392699	1.659312	1.963495	2.267679	2.707661	2.519044	2.152967	1.939737				
		0.392699	2.44471	2.748894	3.053077	1.899955	1.466214	1.092425	1.164641	6.208756			
		0.19635	0.044258	0.19635	0.348441	1.045237	1.215421	1.406963	0.479624				
		0.19635	0.436957	0.589049	0.74114	1.526816	1.742935	1.964287	0.68502				
		0.19635	0.829656	0.981748	1.133839	2.091119	2.296692	2.474513	0.898881				
		0.19635	1.222355	1.374447	1.526538	2.559741	2.666549	2.715621	1.040853				
		0.19635	1.615054	1.767146	1.919238	2.715621	2.666549	2.559741	1.040853				
		0.19635	2.007753	2.159845	2.311937	2.474513	2.296692	2.091119	0.898881				
		0.19635	2.400452	2.552544	2.704636	1.964287	1.742935	1.526816	0.68502			M1A1	Iterate to convergence
		0.19635	2.793151	2.945243	3.097335	1.406963	1.215421	1.045237	0.479624	6.208758		WIIAI	nerate to convergence
		G , ,		6.200	0. 5 6							A1	
		So $k = 1$	gives I	= 6.208	8 to 5 sf							M1A1	Setting up spreadsheet for new
		And b-	1 ~;	og I = 1 '	7460							A1	k
		And k =	- 1 giv	es I = 1.	7402							[10]	
	(iv)	Trial an	d error,	varying	the valu	e of k							
				73 for <i>I</i>								M1A1	
												[2]	
3	(i)	Use cen	tral diff	erence fo	ormula f	or y"						M1	
		Use cen	tral diff	erence fo	ormula f	or y'						M1	
		Convin	cing alge	ebra to e	quation	for y_{r+1}						M1A1	
		Obtain	two equa	ations fo	$r y_1$ in te	rms of y	-1					M1A1	
		Elimina	te to ob	tain equ	ation for	y_1						M1A1	
												[8]	

Question	Answer	Marks	Guidance	
(ii)	E.g. set up the iterations:			
	h r x_r y_r 0.1 0 1 1 0.1 1 1.1 1.095 0.1 2 1.2 1.179835 0.1 3 1.3 1.254121 0.1 4 1.4 1.317387 0.1 5 1.5 1.369189 0.1 6 1.6 1.409184 0.1 7 1.7 1.437196 0.1 8 1.8 1.453257 0.1 9 1.9 1.45763	M4 A2	Set up of spreadsheet Correct answers	
(iii)	etc. And graph the solution: 1.6 1.4 1.2 1 0.8 0.6 0.4 0.2 0 0 2 4 6 8 10	G2		
	Repeat with reducing values of h to obtain maximum at $x = 1.89$ and $y = 1.46$	12 [8] M1A1 A1 A1 [4]		

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Q	uestion			Ans	swer		Marks	Guidance
	(iv)	Construct differe	ence table and	d ratios for y(3). E.g.		M1	
		h	y(3)	diffs	ratios			
		0.1	1.001764					
		0.05	1.002479	0.000715			A1	y(3) values
		0.025	1.002657	0.000178	0.249067		۸.1	Differences and action
		0.0125	1.002701	4.45E-05	0.249767		A1	Differences and ratios
		Ratios approxim					E1	
		Ratios approxim	attery 0.23 50	second order			[4]	
4	(i)	Gauss-Seidel giv	es. e.g.				[-7]	
_	(-)	x1	x2	х3	x4			
		1	1	1	1		M2	Setting up spreadsheet
		-5	-28	-8.33333	75.33333		A2	Accurate iterations
		-37.6667	-349.333	33.22222	295.7778			
		806.7778	3473.556	1748.407	-12076.7			
		10260.26	77201.19	1579.247	-104035			
		-100734	-294025	-306079	1719813			
		-2263524	-1.5E+07	-1581484	25916318			
		8420677	-1.1E+07	48062389	-2E+08			
		4.41E+08	2.65E+09	4.87E+08	-5.5E+09		E1	Consincing demonstration
		Which is clearly	diverging				EI	Convincing demonstration
		To be diagonally	dominant th	a modulus of	anch alamant	n the leading diagonal of	E1	Dominance
		To be diagonally dominant the modulus of each element on the leading diagonal of the coefficient matrix has to be greater than or equal to the sum of the moduli of the				0 0	LI	Dominance
		other coefficients in that row. Strict diagonal dominance, where at least on inequality						Strict dominance
		is strict, is a suff					E1 E1	Condition
		15 561700, 15 6 5611	1010111 0 00 1101	incompany of			[8]	

Question			Ans	swer	Marks	Guidance	
(ii)	Interchanging	rows 1 and 2, a	and interchang	ging rows 3 and 4 g	ives diagonal	M1	
	dominance. Ga	uss-Seidel the	n give, e.g.	A1			
	x1	x2	х3	x4			
	1	1	1	1		M2	Re-enter equations into
	0.8	-0.45	0.4625	0.80625			spreadsheet
	0.54	-0.28813	0.600469	0.848255			
	0.561583	-0.31452	0.585775	0.845308			
	0.558064	-0.31217	0.587684	0.84587			
	0.558377	-0.31253	0.587476	0.845826			
	0.558329	-0.3125	0.587503	0.845834			
	0.558334	-0.3125	0.5875	0.845833			
	0.558333	-0.3125	0.5875	0.845833		A2	
	Which is clear	y converging					
	Evidence of su	bstituting solu	tion back into	equations as a che	ck	M1A1 [8]	

Question	Answer	Marks	Guidance
(iii)	For $k = 1, 2,, 6$ the solutions are x1 x2 x3 x4 1 2.576923 -0.63462 -0.25 0.480769 2 1.353535 -0.43939 0.257576 0.70202 3 0.917808 -0.36986 0.438356 0.780822 4 0.694301 -0.3342 0.531088 0.821244 5 0.558333 -0.3125 0.5875 0.845833 6 0.466899 -0.29791 0.625436 0.862369	M1 A4	
	Graph: 3 2.5 2 1.5 1 0.5 0 -0.5 -1 —Series1 —Series2 —Series3 —Series4	G3	
		[8]	

OCR (Oxford Cambridge and RSA Examinations) 1 Hills Road Cambridge **CB1 2EU**

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