# Teacher Delivery Guide Pure Mathematics: 1.04 Sequences and Series

| **OCR****Ref.** | **Subject Content** | **Stage 1 learners should…** | **Stage 2 learners additionally should…** | **DfE Ref.** |
| --- | --- | --- | --- | --- |
| **1.04 Sequences and Series** |
| 1.04a1.04c1.04b1.04d | Binomial expansion | a) Understand and be able to use the binomial expansion of  for positive integer  and the notations  and ,  or , with .*e.g. Find the coefficient of theterm in the expansion of* *Learners should be able to calculate binomial coefficients. They should also know the relationship of the binomial coefficients to Pascal’s triangle and their use in a binomial expansion.**They should also know that* .b) Understand and know the link to binomial probabilities.  | c) Be able to extend the binomial expansion of  to any rational , including its use for approximation. *Learners may be asked to find a particular term, but the general term will not be required.* *Learners should be able to write*  *in the form prior to expansion.*d) Know that the expansion is valid for .[*The proof is not required.*]*e.g. Find the coefficient of theterm in the expansion of and state the range of values for which the expansion is valid.* | MD1 |
| 1.04e1.04f |  Sequences |  | e) Be able to work with sequences including those given by a formula for the  term and those generated by a simple relation of the form .*Learners may be asked to generate terms, find*  *terms and comment on the mathematical behaviour of the sequence.*f) Understand the meaning of and work with increasing sequences, decreasing sequences and periodic sequences.*Learners should know the difference between and be able to recognise:**1. a sequence and a series,**2. finite and infinite sequences.* | MD2 |
| 1.04g | Sigma notation |  | g) Understand and be able to use sigma notation for sums of series. | MD3 |
| 1.04h | Arithmetic sequences |  | h) Understand and be able to work with arithmetic sequences and series, including the formulae for the  term and the sum to terms.*The term arithmetic progression (AP) may also be used. The first term will usually be denoted by , the last term by  and the common difference by . The sum to*  *terms will usually be denoted by* . | MD4 |
| 1.04i1.04j | Geometric sequences |  | i) Understand and be able to work with geometric sequences and series including the formulae for the  term and the sum of a finite geometric series.*Learners should know the difference between convergent and divergent geometric sequences and series.*j) Understand and be able to work with the sum to infinity of a convergent geometric series, including the use of  and the use of modulus notation in the condition for convergence.*The term geometric progression (GP) may also be used. The first term will usually be denoted by  and the common ratio by .The sum to* *terms will usually be denoted by  and the sum to infinity by* . | MD5 |
| 1.04k | Modelling |  | k) Be able to use sequences and series in modelling.*e.g. Contexts involving compound and simple interest on bank deposits, loans, mortgages, etc. and other contexts in which growth or decay can be modelled by an arithmetic or geometric sequence.**Includes solving inequalities involving exponentials and logarithms.* | MD6 |

**Thinking Conceptually**

**General approaches**

Prior to working with the subject content of this section of the specification, it is essential that learners have gained a thorough understanding of a number of topics such as the four rules of number including the priority of operations, signed numbers, rounding, algebra including substitution, bracket expansion and simplification of terms, products, factors, index notation and percentages. It would also be beneficial if learners have a good understanding of probability so that they can understand and appreciate the link to binomial probabilities. It is also essential that learners have the ability to recognize patterns.

Learners’ understanding should be deepened by a hands-on approach to this subject as they tend to struggle with the algebra involved. The use of multilink cubes or other similar building blocks can help provide a visual support of growing patterns and to highlight the progression from Key stage 3 and GCSE.

**Common misconceptions or difficulties learners may have**

The expansions of binomial expressions, sequences and series require learners to have a high level of skills in algebra. As the foundation of algebra is basic arithmetic, many misconceptions in algebra are found to be rooted in misconceptions in arithmetic.

There are many misconceptions concerning negative numbers as learners wrongly think that two negatives always make a positive when adding / subtracting negative numbers.

A common mistake is expanding  as if  had the value 1.

Learners often make mistakes with the signs when expanding, for example, 

Other common mistakes with expanding binomial expressions are raising only part of the term to the appropriate power and bracketing errors.

Also learners often waste time writing out the full expansion of the binomial expansion instead of just finding the coefficient of the required term.

Learners often find it difficult to remember the difference between series and sequences and there is a misconception that positive whole numbers are often the only numbers considered when generating sequences.

Learners often find it difficult to recognise when a sequence or series is arithmetic or geometric.

Learners struggle to correctly work out terms in a sequence by using the order of operations.

Also learners find it difficult to correctly find the value of, the common ratio, in geometric sequences.

In addition, learners find it difficult to use and apply the correct formulae for arithmetic and geometric series.

**Conceptual links to other areas of the specification**

1.02a Indices – learners need a good understanding of the laws of indices when expanding binomial expressions.

1.02g-i Inequalities – learners need to be able to use sequences and series in modelling and this sometimes involves solving inequalities involving exponentials and logarithms.

1.02j Polynomials – learners need to have a good understanding of polynomials to appreciate that a binomial is a polynomial that is the sum of two terms.

1.02l The modulus function – learners need to understand and be able to work with the sum to infinity of a convergent geometric series and this includes the use of modulus notation in the condition for convergence.

1.02m-t Graphs – learners need to a good understanding of graphs as often sequences and series can be analysed using graphs to determine whether they are converging or diverging.

1.06a Exponentials – learners need to have a good understanding of exponentials. The behaviour of a geometric sequence depends on the value of the common ratio. If the common ratio is greater than 1, there will be exponential growth towards positive or negative infinity (depending on the sign of the initial term) and if it is between −1 and 1 but not zero, there will be exponential decay towards zero. If the common ration is less than −1 then the magnitude of terms grows exponentially but their signs oscillate.

1.06g Logarithms – learners need to be able to use sequences and series in modelling and this involves solving inequalities involving exponentials and logarithms.

2.04c Probability – the expansion of binomial expressions has a similar structure to the calculation of binomial probabilities.

**Thinking Contextually**

Links to a range of resources that can be used to enhance and support the delivery of the ‘Sequences and Series’ topic are provided below.

Video tutorials, presentation slides, worked examples, exercises and interactive diagnostic tests are all used to reinforce the understanding of the different content areas.

Many learners fail to make connections between what they are learning and how that knowledge will be used. They struggle to understand the concepts in mathematics unless they can see the relevance to their everyday lives.

Learners will be more successful if they investigate mathematics through real life scenarios as they can see how these concepts are actually used outside of the classroom. They will then be able to discover the meaningful relationship between abstract ideas and practical applications in the real world. This in turn, will lead to greater motivation, enjoyment through discovery, improved confidence, independent thinking and better retention of skills.

# Past paper examples

[2018 H240/01](https://www.ocr.org.uk/Images/535607-question-paper-pure-mathematics.pdf) Q7: Geometric progression question set in context.

[2018 H240/01](https://www.ocr.org.uk/Images/535607-question-paper-pure-mathematics.pdf) Q8: Binomial expansion problem with an interesting twist in part (ii)

# Resources

| Title | Organisation | Description | Ref |
| --- | --- | --- | --- |
| [Sequences and series](https://www.ocr.org.uk/Images/400648-section-check-in-1.04-sequences-and-series.docx) | OCR | Questions relating to section 1.04 of the new AS/A Level Maths specification. | 1.04 |
| [Pascal’s triangle and the Binomial Theorem](http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-pascal-2009-1.pdf) | maths centre | Set of notes, examples and practice exercises. | 1.04a |
| [The mathematical secrets of Pascal’s triangle](https://www.youtube.com/watch?v=XMriWTvPXHI) | TED-Ed | This interesting video resource is aimed at AS level and demonstrates how Pascal’s triangle is used to expand a Binomial expression and looks at patterns within Pascal’s triangle. It also demonstrates how the Binomial expansion is used in probability.  | 1.04a |
| [Binomial Expansion: tutorial 1](https://www.youtube.com/watch?v=h3wdStMSfXk) | Exam Solutions | This youtube video is aimed at AS level and introduces learners to the binomial expansion for positive integer powers. | 1.04a |
| [Binomial Theorem](http://www.mathsisfun.com/algebra/binomial-theorem.html) | Math is Fun | This resource is aimed at AS level and demonstrates how to use the binomial expansion of positive integer powers. It includes ten interactive practice questions with worked answers for learners to reveal once they have attempted each question.  | 1.04a |
| [Binomial Theorem](http://ibmathstuff.wikidot.com/binomial-theorem) | IB Math Stuff | This excellent resource is aimed at AS level and demonstrates how to use the binomial expansion for positive integer powers and looks at the relationship of the binomial coefficients to Pascal’s triangle. This resource offers the opportunity to practice the important skills involved in binomial expansion to help address some common misconceptions and difficulties | 1.04a |
| [The Binomial Theorem](https://maths.mq.edu.au/numeracy/web_mums/module4/Worksheet412/module4.pdf) | Macquarie University | This excellent resource is aimed at AS level and demonstrates how to use the binomial expansion for positive integer powers and looks at the relationship of the binomial coefficients to Pascal’s triangle. This resource offers the opportunity to practice the important skills involved in binomial expansion to help address some common misconceptions and difficulties.  | 1.04a |
| [Genetics: Binomial Expansion](https://www.youtube.com/watch?v=mP3pBjHx7oo) | Christian D | This video resource is aimed at AS level and demonstrates how the binomial theorem is used in genetics. | 1.04a |
| [Can we use a binomial expansion to evaluate](https://undergroundmathematics.org/counting-and-binomials/r7978)   | Underground Mathematics | UCLES A level Mathematics 1 QP 186, 1951, Q3 requiring learners to determine the number of terms required in an expansion in order to obtain the requested accuracy. | 1.04a |
| [Binomial Expansion nCr method](https://www.youtube.com/watch?v=_SMFX9c8BLI) | Exam Solutions | This excellent video resource is aimed at AS level and demonstrates how to expand a Binomial expression for positive powers using the nCr method. | 1.04a  |
| [nCr, What is it?](https://www.youtube.com/watch?v=MqOhV9110y0) | Exam Solutions | This excellent short video resource is aimed at AS level and demonstrates how to calculate nCr with and without a calculator. | 1.04a |
| [The Binomial Expansion (AS)](http://mei.org.uk/files/sow/08-binomial-expansion.pdf) | MEI | A commentary of the underlying mathematics, a sample resource, a use of technology, links with other topics, common errors, opportunities for proof and questions to promote mathematical thinking. | 1.04a and 1.04b |
| [More Binomial Expansions](http://www.chartwellyorke.com/teachalevel/vol2sample2.ppt) | Mathematics Software for Education | This excellent resource is a Powerpoint presentation aimed at A level. The slides demonstrate how to use the binomial expansion for positive, fractional and negative powers.  | 1.04a, 1.04c and 1.04d |
| [Probability: Binomial Theorem](https://www.youtube.com/watch?v=W1_cSGUZbdY) | Rahul Patwari | This excellent video resource is aimed at AS level and demonstrates how the binomial theorem is used in probability. | 1.04b |
| [The Binomial Theorem](http://www.a-levelmathstutor.com/binomial.php) | A-Level Maths Tutor | This resource is aimed at A level as it demonstrates how to use the binomial expansion for positive integer powers and looks at the relationship of the binomial coefficients to Pascal’s triangle. It then looks at the binomial expansion of a fractional power. | 1.04c |
| [Binomial expansion – negative power](https://www.youtube.com/watch?v=DFRsdGigHis) | A Level Maths Help | This video resource is aimed at A level and demonstrates how to use the binomial expansion for negative powers.  | 1.04c and 1.04d |
| [The Binomial Theorem](https://www.intmath.com/series-binomial-theorem/4-binomial-theorem.php) | Interactive Mathematics | This excellent interactive resource is aimed at A level and demonstrates how to use the binomial expansion for positive integer powers and fractional powers. This resource offers the opportunity to practice the important skills involved in binomial expansion to help address some common misconceptions and difficulties.  | 1.04c and 1.04d |
| [Binomial expansion – fractional power](https://www.youtube.com/watch?v=1__zL-aU5p4) | A Level Maths Help | This video resource is aimed at A level and demonstrates how to use the binomial expansion for fractional powers | 1.04c and 1.04d |
| [A2 Binomial Expansion: Negative Fractional Powers](http://www.alevelmathsnotes.com/2014/01/a2-binomial-expansion-negative.html) | A Level Maths Notes | This short resource is aimed at A level and demonstrates how to use the binomial expansion for negative fractional powers by giving four worked examples. | 1.04c and 1.04d |
| [Sequences](https://www.stem.org.uk/resources/elibrary/resource/32141/sequences) | STEM Learning | These resources are aimed at A level and allow learners to become more familiar with sequences including Fibonacci and quadratic sequences. They include games, investigations, worksheets and practical activities. The resources are free but a login is required. | 1.04e |
| [Sequences](http://www.cimt.org.uk/primaryske/B1/Text.pdf) | CIMT | This resource is aimed at A level and provides the opportunity for learners to practice the important skills involved in sequences to help address some common misconceptions and difficulties. There are numerous exercises available for learners to attempt. | 1.04e |
| [Fibonacci Surprises](http://nrich.maths.org/11164) | NRICH | This interesting resource invites learners to investigate the Fibonacci sequence. | 1.04e |
| [Sequences and Series](http://mei.org.uk/files/sow/23-sequences-and-series.pdf) | MEI | A commentary of the underlying mathematics, a sample resource, a use of technology, links with other topics, common errors, opportunities for proof and questions to promote mathematical thinking. | 1.04e, 1.04f, 1.04g, 1.04h, 1.04i, 1.04j and 1.04k |
| [Sigma Notation](http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/sigma.pdf) | maths centre | Set of notes, examples and exercises. | 1.04g |
| [Series](https://revisionmaths.com/advanced-level-maths-revision/pure-maths/algebra/series) | Revision Maths | This excellent resource is aimed at A level and introduces learners to the sigma notation and explains the difference between a series and a sequence. It provides examples of arithmetic sequences (progressions) including the formulae for the nth term and the sum to n terms. It also provides examples of geometric sequences (progressions) including formulae for the nth term and the sum to n terms.  | 1.04g, 1.04h and 1.04i |
| [Arithmetic Sequences Harder Questions AS A2 Maths](https://www.youtube.com/watch?v=uYq4Uu0ZaG8) | UK Maths Teacher | This challenging resource is aimed at A level and demonstrates how to find the nth term of an arithmetic sequence using sequence formulae and simultaneous equations. This resource is aimed at learners studying at A level. | 1.04h |
| [Arithmetic Sequences and Series](https://www.slideshare.net/itutor/arithmetic-sequence-and-series) | Slide Share | This resource is aimed at at A level and includes fourteen professional looking slides on arithmetic sequences. The slides include how to find the nth term and the sum to n terms by showing worked examples. | 1.04h  |
| [Sequences and Series (2) – Introduction to Arithmetic Sequences](https://www.youtube.com/watch?v=DoZCRAW_vRc) | UK Maths Teacher | This excellent video resource is aimed at A level and introduces the learner to arithmetic sequences. Learners are asked to identify arithmetic sequences, generate terms and find nth terms of increasing and decreasing sequences. | 1.04h  |
| [Arithmetic Sequence Calculator](https://www.geogebra.org/m/eK2b6XtA) | Geogebra | Two demonstrations using geogebra software.Question 1 allows learner to enter First term and common difference and nth term rule, in the form , is generated.Question 2 allows learner to specify two specific terms and the nth term rule, in the form , is generated. | 1.04h  |
| [The nth term for Fractional Sequences](https://corbettmaths.com/2012/08/20/the-nth-term-for-fractional-sequences/) | Corbettmaths | This video resource is aimed at A level and demonstrates how to calculate the nth term for fractional arithmetic sequences. | 1.04h  |
| [What’s the next number to occur in both sequences](https://undergroundmathematics.org/divisibility-and-induction/r9916) | Underground Mathematics | UKMT SMC 2005 Q7 problem looking at common differences of arithmetic sequences | 1.04h |
| [Arithmetic sequence real life car rental](https://www.youtube.com/watch?v=zxQPCLa9UhI) | Heritagealgebra | This video resource is aimed at A level and demonstrates real – life practical applications of using arithmetic sequences by investigating car rental. | 1.04h and 1.04k |
| [Arithmetic and Geometric Progressions](http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-apgp-2009-1.pdf) | maths centre | This resource is aimed at A level and offers the opportunity to practice the important skills involved in arithmetic and geometric sequences to help address some common misconceptions and difficulties. There are exercises to complete along with answers. | 1.04h, 1.04i and 1.04j |
| [What is means for a Sequence to Converge, Diverge, be Periodic or Oscillate](https://www.youtube.com/watch?v=thzoShIFMDQ) | Jack Brown | This short video resource is aimed at A level and explains the differences between converging, diverging and periodic sequences.  | 1.04i |
| [Solving Sequences, Converging or Diverging? Calculus Tips](https://www.youtube.com/watch?v=0_D66IgOWoI) | Straighter Line | This short youtube video is aimed at A level as it introduces learners to the difference between convergent and divergent geometric sequences and series.  | 1.04i |
| [Staircase sequences](https://undergroundmathematics.org/thinking-about-numbers/staircase-sequences) | Underground Mathematics | Open ended sequence problem that leads into ideas of convergence.  | 1.04i |
| [Geometric sequence calculator](https://www.geogebra.org/m/TVdvyvgy) | Geogebra | Two demonstrations using geogebra software. Question 1 allows learner to enter first term and common ratio and the nth term rule, in the form  , is generated.Question 2 allows learner to specify two specific terms and the nth term rule, in the form  , is generated. | 1.04i |
| [Compound interest: establishing a formula](https://www.youtube.com/watch?v=luQ-4tJzC30) | Metal Maths Project | This excellent video resource shows an example of where sequences can be used in the real world. The video gives the example of a student who is saving for a round-the-world trip and shows how to calculate how long this will take. | 1.04i |
| [Compound Interest](https://justmaths.co.uk/Worksheets/Number/Compound%20Interest%20-%20EXAM%20QUESTIONS.pdf) | Just Maths | This resource invites learners to answer questions on compound interest and reviews GCSE content. | 1.04i |
| [Double a Penny Everyday for 31 Days -Learn The Power of Compound Interest](https://www.youtube.com/watch?v=XUCYdGAWcbo) | Your Plan B Income | This short video resource looks at the “power” of compound interest. | 1.04i |
| [Finite geometric series word problem: mortgage](https://www.khanacademy.org/math/algebra2/sequences-and-series/copy-of-finite-geometric-series-word-problems/v/geometric-series-sum-to-figure-out-mortgage-payments) | Khan Academy | This video resource is aimed at A level and looks at how finite geometric series are used in the real world by looking at mortgage calculations. | 1.04i |
| [Geometric Sequences and Series (1) – Sequences Brief Introduction](https://www.youtube.com/watch?v=ryWO3wYJ8Iw) | UK Maths Teacher | This excellent video resource is aimed at A level and introduces the learner to geometric sequences and series and explains the difference between the two. It demonstrates how to find the nth term of increasing and decreasing geometric sequences and also looks at converging and diverging sequences. | 1.04i and 1.04j |
| [Geometric Sequence](https://www.slideshare.net/chelseaanneberganio/geometric-sequence-51673284?next_slideshow=1) | Slide Share | This resource is aimed at A level and includes forty one professional looking slides on finite and infinite geometric sequences. The slides include how to find the nth term, the sum of a finite series and the sum to infinity of convergent geometric series. It includes questions for learners to attempt along with worked answers. | 1.04i and 1.04j |
| [Geometric Series](http://www.projectmaths.ie/documents/T%26L/GeometricSeries.pdf) | Project Maths | This challenging resource (pages 20 - 24) is aimed at A level and offers further practice in solving real life problems using geometric sequences. | 1.04i, 1.04j and 1.04k |
| [Sequences word problems](https://www.khanacademy.org/math/algebra/sequences/modeling-with-sequences/v/modeling-situations-with-arithmetic-and-geometric-sequences) | Khan Academy | This video resource is aimed at A level and demonstrates how to solve problems with modelling real life situations using arithmetic and geometric sequences. | 1.04k |
| [The Geometric Series in Finance](http://www.konfluence.org/geomser/geomser.html) | Konfluence Research Institute | This challenging resource is aimed at A level and demonstrates how geometric series are used in finance by looking at mortgages and investments. | 1.04k |
| [Finance and Growth](http://www.metalproject.co.uk/METAL/Resources/Teaching_learning/55848.pdf) | Metal Project | This challenging resource looks at how arithmetic and geometric series can be used to calculate simple and compound interest, the time value of money, investment appraisal and the NPV rule and the IRR rate of return rule. There are worked examples and exercises for learners to complete along with links to video resources. The content is at A level standard. | 1.04k |
| [Applications of Sequences and Series](http://www.algebralab.org/lessons/lesson.aspx?file=algebra_seqseriesapps.xml)  | Algebra LAB | This interactive resource is aimed at A level and offers further practice in solving real life problems using arithmetic and geometric sequences. | 1.04k |
| [Geometric sequence real life retirement](https://www.youtube.com/watch?v=-Wk2pJj3FH4) | Heritagealgebra | This video resource demonstrates real – life practical applications of using geometric sequences by investigating retirement figures. It is aimed at A level. | 1.04k |
| [Geometric Sequences and Series](http://www.hec.ca/en/cam/help/topics/geometric_sequences_and_series.pdf) | HEC Montréal | This challenging resource is aimed at A level and looks at geometric sequences and series applications in financial mathematics. | 1.04k |
| [Multiplying bacteria - a geometric sequence problem](https://www.bbc.co.uk/programmes/p00zk1hz) | BBC | This short video resource poses a real – life problem using sequences. The video can be paused so that learners can discuss and attempt to solve the problem. | 1.04k |
| [Sequences and Series](http://tentotwelvemath.com/classroom-resources/sequences-and-series-2/) | 10 to 12 | Set of notes, demonstrations and activities on AP and GP  | 1.04k |

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