# Section Check In – 4.05 Further Algebra

## Questions

1. The roots of the equation  are  and . Find the value of .

2. The roots of the equation  are and . Find the equation whose roots are  and .

3.\* Express  in partial fractions.

4.\* Express  in partial fractions.

5. The equation  has roots and.

(i) Show that the substitution  transforms the equation to

.

(ii) Deduce the value of

(a) ,

(b) .

6.\* Show that .

7.\* The roots of the equation  are three consecutive terms of an arithmetic progression. Find the roots.

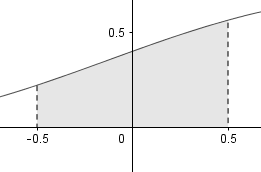
8.\* The expression  is defined by . By first expressing  in

partial fractions, find the value of the constant  (where ) such that .

9. (i) The equation  has roots  and . The equation is transformed   
 by the substitution .  
 List the roots of the new equation in terms of  and .

(ii) Analysis for a precision engineering company project requires the exact real root of the equation  to be found. Use the substitution  to find the exact value of  satisfying the equation.

10.\*



*x*

*y*

*R*

The rational expression  is defined by . The diagram shows the curve  for . The region  is bounded by the curve and the lines  and . The region  is rotated completely about the -axis to form a solid. This solid is the model for an industrial component. Units are metres and it is required to find the volume of the solid.

(i) Express  in partial fractions and hence find the power series for  up to and including the term in .

(ii) Using the power series as an approximation to , determine the approximate volume of the solid.

[You may use the result that the volume is given by .]

**Extension**

1. The roots of the equation  are three consecutive terms of a

geometric progression. Find the roots.

2. The roots of the equation  are  and . Find the equation whose roots are

(i)  and ,

(ii)  and ,

(iii)  and .

## Worked solutions

1. Transform the equation using substitution 





Required value is sum of roots of this equation, i.e. 

2. From given equation,  and 

For new equation:

sum of roots 

product of roots 

= 

New equation is 

3. Note that numerator has degree 4 and denominator has degree 3 so division will lead to a linear expression

Let 

Multiplying leads to 



Putting  gives  and so 

Putting  gives  and so 

Considering coefficient of  leads to 

Putting  gives 

Considering coefficient of  leads to  and so  and 

Answer is 

4. Let 

Multiplying leads to 

Putting  gives  and so 

Putting  gives  and so 

Considering coefficient of  leads to  and so 

Answer is 

5. (i) Substituting  leads to 

Multiplying by  gives 

Expanding gives 

Simplifying gives 

(ii) Root of new equation corresponding to  is value of  where 

Rearranging gives  and so  or 

Roots of new equation are 

(a) Sum of roots of new equation 

(b) Product of roots of new equation 

6. Let 

Multiplying leads to 

Comparing coefficients or putting particular values of  gives 

So integral is  





7. Let roots be  so that sum of roots 

From equation, sum of roots  so that  and 

Sum of roots two at a time 

From equation,  giving  and so 

Taking  gives roots  and therefore the roots are 

(Taking  gives same roots but in the reverse order)

Alternatives

After line 2 above, substituting  in the equation enables the value of  to be found  and then the equation can be solved by factorising the left-hand side (knowing already that one of the factors is ).

Calling the three roots  at the beginning leads to equivalent work but with slightly more involved algebra.

8. Let 

Multiplying, 

Comparing coefficients and/or substituting particular values gives 

So  giving 

Using limits, 

 gives 

Simplifying gives  and hence 

9. (i) Root of new equation corresponding to  given by 

Rearranging,  and so roots of new equation are 

(ii) Applying substitution  gives 

Expanding and simplifying give  and 

So  and the exact  value required is 

10. (i) Expressing  in partial fractions gives 

Rearranging, 

Expanding as far as  terms, 

Simplifying, 

(ii) This power series is suitable approximation provided  and , i.e. provided  which will be the case here as the integration limits are  and 

Volume is 

Applying the limits gives  

Volume of the solid is approximately m3

**Extension**

1. If roots are  their product is 

From equation, product is  giving  and so 

Considering sum of roots,  and solving this gives  or 

Using either value of , the three roots are 

2. From the equation,  and 

(i) Sum of roots 

Product of roots 

Equation is 

(ii) First, 

For new equation, sum of roots 

Product of roots 

Equation is 

(iii) Sum of roots 

Product of roots 

Equation is 

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