

Cambridge Technicals

Engineering

Unit **23**: Applied mathematics for engineering

Level 3 Cambridge Technical Certificate/Diploma in Engineering
05822 - 05825

Mark Scheme for January 2017

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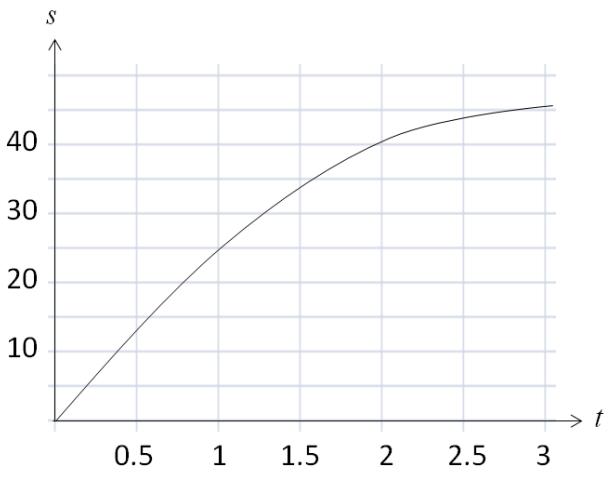
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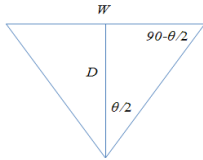
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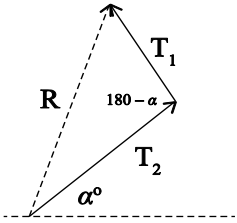
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Question	Answer	Marks	Guidance
1 (a)		<p>B1</p> <p>B1</p> <p>B1 FT</p> <p>[3]</p>	<p>Suitable scales marked on s and t axes</p> <p>Plotting of points; allow one error</p> <p>Smooth curve through points plotted; allow FT if correct general shape</p>
(b) (i)	<p>Using $t = 1$, $25 = A + B$ Using $t = 2$, $40 = 4A + 2B$ $100 = 4A + 4B$ $60 = 2B$ $B = 30$, $A = 25 - 30 = -5$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>Two correct equations</p> <p>Eliminate one variable; allow any appropriate method</p> <p>Correct values for A and B</p>
(b) (ii)	<p>$30 = -5t^2 + 30t$ $5t^2 - 30t + 30 = 0$ $t^2 - 6t + 6 = 0$ $t = \frac{6 \pm \sqrt{6^2 - 4 \times 1 \times 6}}{2 \times 1} = \frac{6 \pm \sqrt{12}}{2}$ Obtain $t = 1.27$</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>Form quadratic using their A and B from (i);</p> <p>Use quadratic formula oe to solve their equation;</p> <p>And no others 1.2679(49...); allow exact equivalent $3 - \sqrt{3}$</p>

Question			Answer	Marks	Guidance
1	(b)	(iii)	$u = 30 \text{ (ms}^{-1}\text{)}$	B1 [1]	From $s = \frac{1}{2}(u + v)t$ with $v = 0$ and $t = 3$; $u = \frac{2 \times 45}{3} = 30$ OR using $u = \frac{ds}{dt} = 2At + B$; $u(0) = B = 30 \text{ m s}^{-1}$
1	(b)	(iv)	Deceleration = $10 \text{ (ms}^{-2}\text{)}$	B1 [1]	Accept -10ms^{-2} From $v^2 = u^2 + 2as$; $0 = 30^2 + 2a \times 45$; $a = \frac{-900}{90} = -10$ OR Compare $s = At^2 + Bt$ with standard formula $s = ut + \frac{1}{2}at^2$; $\frac{1}{2}a = -5$, $a = -10 \text{ m s}^{-2}$ OR $a = \frac{du}{dt} = 2A = 2 \times (-5) = -10 \text{ m s}^{-2}$ Accept any correct alternative solutions

Question	Answer	Marks	Guidance
2 (a)	$\tan(\theta/2) = \frac{W/2}{D}$ $W = 2D \tan(\theta/2)$ $A = \frac{2D \tan(\theta/2) D}{2} = D^2 \tan(\theta/2)$	M1 A1 A1 [3]	Uses $\tan = \text{op/adj}$ in right angled triangle Correct expression for width; or half-width if subsequently used appropriately for area AG; necessary detail needed 
2 (b) (i)	$5d^2 \tan(\theta/2)$ <p>[Mass of water displaced = mass of hull]</p> $5d^2 \tan(\theta/2) \times 1000 = 4000$ $d^2 = \frac{4}{5 \tan(\theta/2)}$ $d = 2 \sqrt{\frac{1}{5 \tan(\theta/2)}} \text{ AG}$	B1 M1 M1 A1 [4]	Correct expression for volume of submerged part of hull soi Form equation; allow $5d^2 \tan(\theta/2) = 4$ with 4 justified as volume of displaced water Rearrange to make d^2 the subject of formula AG; sufficient detail needed;
2 (b) (ii)	$1.7 = 2 \sqrt{\frac{1}{5 \tan(\theta/2)}}$ $(0.85)^2 = \frac{1}{5 \tan(\theta/2)}$ $\tan(\theta/2) = \frac{1}{5 \times 0.85^2} = 0.2768(17..)$ $\theta/2 = \tan^{-1}(0.2768) = 15.4729(72..)$ $\theta = 30.9(45944.)$	B1 M1 M1 A1 [4]	Substitution of 1.7 soi Attempt solution by squaring both sides Transpose to make angle the subject of formula Evaluate using inverse tan

Question	Answer	Marks	Guidance
3 (a)	$\bar{T}_1 = (T_1 \cos 2\alpha, T_1 \sin 2\alpha)$ $\bar{T}_2 = (T_2 \cos \alpha, T_2 \sin \alpha)$	B1B1 B1B1 [4]	Or $\bar{T}_1 = (-T_1 \cos(180 - 2\alpha), T_1 \sin(180 - 2\alpha))$ Two marks available for each tension given in horizontal & vertical component form; other notations acceptable eg $\mathbf{ai+bj}$ and column vectors. SC B2 all 4 components seen separately eg in correct equilibrium statements.
3 (b)		B1 [2]	3 correct vectors; with directions shown Correct configuration (soi from triangle of forces/parallelogram of forces) with or without angles; no need for R to be vertical
3 (c)	$\bar{R} = (T_1 \cos 2\alpha + T_2 \cos \alpha, T_1 \sin 2\alpha + T_2 \sin \alpha)$	M1 A1 [2]	Vector addition method for either component Correct answer comprising both components Or $\bar{R} = (-T_1 \cos(180 - 2\alpha) + T_2 \cos \alpha, T_1 \sin(180 - 2\alpha) + T_2 \sin \alpha)$
3 (d) (i)	$\cos 2\theta = 2\cos^2 \theta - 1$ $T_1 \cos 2\alpha + T_2 \cos \alpha = 0$ $T_1(2\cos^2 \alpha - 1) + T_2 \cos \alpha = 0$ $2T_1 \cos^2 \alpha + T_2 \cos \alpha - T_1 = 0$ $\cos \alpha = \frac{-T_2 \pm \sqrt{T_2^2 - 4 \times 2T_1 \times (-T_1)}}{4T_1}$	B1 [4]	Correct double angle identity soi For equating their horizontal component to zero Use of appropriate identity and simplification to obtain 3 term quadratic in $\cos \alpha$ Obtain expression for $\cos \alpha$; need not be simplified
	(ii) $\cos \alpha = \frac{-T_1 \pm \sqrt{T_1^2 - 4 \times 2T_1 \times (-T_1)}}{4T_1} (= 0.5)$ $\alpha = 60^\circ$	M1 [2]	Substitute $T_1 = T_2$ into their answer from (i) and attempt simplification Alternative; equates horizontal components when $T_1 = T_2$ $[T_1] \cos(180 - 2\alpha) = [T_2] \cos \alpha$ $180 - 2\alpha = \alpha$; M1 www SC answer only B1

Question	Answer	Marks	Guidance
4 (a)	$\frac{d\eta}{dN} = -0.02\left(\frac{N}{100}\right) + \frac{0.06}{10}$ $0.02\left(\frac{N}{100}\right) = \frac{0.06}{10}$ Obtain 30 (rps)	*M1 DM1 A1 [3]	Differentiate to obtain form $k_1N + k_2$ Dep *M1; equate to zero and attempt to solve www
4 (b)	$\eta = -0.01\left(\frac{30}{10}\right)^2 + 0.06\left(\frac{30}{10}\right) + 0.2 = 0.29$ Obtain ($P = 10000 \times 0.29 \times 30 =$) 87000(W)/87 (kW)	B1 FT B1 [2]	Substitute their N to correctly evaluate η ; allow ECF from (a) From 30 obtained in (a)
4 (c)	$P = 10000 \left(-0.01\left(\frac{N}{10}\right)^2 + 0.06\left(\frac{N}{10}\right) + 0.2 \right) N$ $= 10000 \left(-0.01\frac{N^3}{100} + 0.06\frac{N^2}{10} + 0.2N \right)$ $= -N^3 + 60N^2 + 2000N$ $\frac{dP}{dN} = -3N^2 + 120N + 2000 = 0$ $N = \frac{-120 \pm \sqrt{120^2 - 4 \times (-3) \times 2000}}{-6}$ Obtain N= 52.6598(6...)	B1 M1 M1 M1 A1 [5]	Correct equation for P; soi Simplify to obtain a cubic polynomial in N Differentiate to obtain a quadratic and equate to zero Solve a 3-term quadratic
4 (d)	Compare earlier answers to draw appropriate conclusion The claim is NOT true.	M1 A1 [2]	Alternatives; eg calculates power at speed corresponding to their maximum efficiency CAO; reference to the student's claim

Question	Answer	Marks	Guidance
5 (a)	$\frac{dP}{dh} = 4\left(-\frac{he^{-\frac{h}{2}}}{2} + e^{-\frac{h}{2}}\right)$ $= 4e^{-\frac{h}{2}}\left(1 - \frac{h}{2}\right) = 0$ $\Rightarrow h = 2$	<p>*M1</p> <p>A1</p> <p>DM1</p> <p>A1</p> <p>[4]</p>	<p>Differentiate using product rule to obtain two terms</p> <p>Correct derivative</p> <p>Dep *M1; equate to zero and attempt to solve www; and no other solutions</p>
5 (b)	<p>Identifies u and v' correctly</p> $\int e^{-\frac{h}{2}} = -2e^{-\frac{h}{2}}$ <p>Constructs correct form for integration by parts</p> <p>Obtains $\frac{4}{10} \left\{ -2he^{-\frac{h}{2}} - \int -2e^{-\frac{h}{2}} \right\}_0^{10}$</p> $= \frac{4}{10} \left\{ -2he^{-\frac{h}{2}} - 4e^{-\frac{h}{2}} \right\}_0^{10}$ $= \frac{4}{10} \{ (-20e^{-5} - 4e^{-5}) - (-4) \}$ $\frac{4}{10} \{ -24e^{-5} + 4 \} = 1.535(316.) \text{ (kW)}$	<p>M1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[7]</p>	<p>Correct direction chosen for integration by parts; soi</p> <p>Integration by parts as far as $f(h) +/ - \int g(h)dx$</p> <p>Condone absence of limits here</p> <p>Attempt to integrate their remaining term</p> <p>Substitution of limits soi</p>

Question	Answer	Marks	Guidance
6 (a)	$\frac{1}{Z_t} = \frac{1}{R} + \frac{1}{\omega L j} + \omega C j$ <p>use of $j^2 = -1$</p> $= \frac{1}{R} - \frac{j}{\omega L} + \omega C j = \frac{\omega L - Rj + \omega^2 RCLj}{R\omega L}$ $= \frac{\omega L + R(\omega^2 RCL - 1)j}{R\omega L}$ $Z_t = \frac{R\omega L}{\omega L + R(\omega^2 LC - 1)j} \quad \text{AG}$	<p>B1</p> <p>B1</p> <p>M1</p> <p>DM1</p> <p>A1</p> <p>[5]</p>	<p>Accept correct alternative workings throughout</p> <p>Insert component impedances</p> <p>Soi anywhere in solution</p> <p>Common denominator; denominator may still include j $\frac{\omega L j + R + R\omega^2 L C j^2}{R\omega L j}$</p> <p>Simplification with $R\omega L$ as denominator; dep on all earlier marks</p> <p>Invert to obtain given answer; CAO; sufficient detail needed</p>
6 (b) (i)	$Z_t = \frac{1 \times 10^4 \times 10^{-2}}{10^4 10^{-2} + 1(10^8 10^{-2} 10^{-5} - 1)j} = \frac{100}{100 + 9j}$ $= \frac{100(100 - 9j)}{10000 + 81}$ $= \frac{10000 - 900j}{10081}$ $= 0.9919(65.) - j0.08927(68.)$	<p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>Insert values and simplify; accept 10^2</p> <p>Use of complex conjugate to rationalise expression;</p> <p>Accept decimals or fractions</p>

Question			Answer	Marks	Guidance
6	(b)	(ii)	$r \approx \sqrt{0.992^2 + 0.0893^2}$ $= \sqrt{0.992^2 + 0.0893^2}$ $= 0.99(6..)$ $\theta \approx \tan^{-1}\left(\frac{-0.893}{0.992}\right)$ $= -5.1(4..)^\circ$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>Use of correct formula for r; soi</p> <p>Evaluation of r</p> <p>Appropriate use of inverse tan (\pm); soi</p> <p>Award full marks for correct values of r and θ seen.</p>
7	(a)		$\frac{(1+x) - x(1)}{(1+x)^2}$ $= \frac{1}{(1+x)^2} \text{ AG}$	<p>M1</p> <p>A1</p> <p>[2]</p>	$\frac{d}{dx}\left(\frac{x}{1+x}\right) = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$ <p>Application of quotient formula; product rule alternative</p> <p>AG; sufficient detail needed</p>

Question	Answer	Marks	Guidance
7 (b)	$\frac{d\omega}{dt} = \omega_0 \left(\frac{t}{(1+t)^2} - \frac{1}{(1+t)} \right)$ $= \omega_0 \left(\frac{t}{(1+t)^2} - \frac{(1+t)}{(1+t)^2} \right)$ $= \omega_0 \left(-\frac{1}{(1+t)^2} \right)$ $\int 1d\omega = -\omega_0 \int \frac{1}{(1+t)^2} dt$ $\omega = -\omega_0 \left(\frac{t}{1+t} \right) + c$ $\omega = \omega_0 \text{ when } t = 0 \Rightarrow c = \omega_0$ $\omega = \omega_0 \left(1 - \frac{t}{1+t} \right) \text{ AG}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>Division by $(1+t)$ to separate variables; accept unsimplified</p> <p>Simplify to obtain single fraction</p> <p>Integrate (both sides); condone absence of $+c$</p> <p>Apply initial conditions to evaluate c ; oe from use of limits</p> <p>CAO</p>
7 (c)	<p>Speed decreases with time as expected (so realistic)</p> <p>The flywheel never completely stops (so not realistic)</p>	<p>B1</p> <p>B1</p> <p>[2]</p>	<p>Soi</p> <p>Accept any 2 reasonable observations</p>

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