# Section Check In 4.10 –

# Core Pure Differential Equations

## Questions

1.\* (i) The resistance to motion for a person falling through the air with a parachute is modelled

as being proportional to the square of the velocity, *v*. Show that the motion of a particular person falling through the air with a parachute may be modelled by the differential equation  where *x* is the displacement of the person, g is the acceleration due to gravity and *k* is a constant.

(ii) Find the general solution of the differential equation as an expression for *x* in terms of *v*.

2.\* Choose an appropriate method to find the general solution to .

3.\* (i) Write down the auxiliary equation for  and hence find the   
 general solution.

(ii) We require thatas and when. Using this information and your answer to i), find the particular solution.

4.\* (i) Find the general solution to .

(ii) Find the particular integral of  and hence write down the full general solution.

5.\* Find the solution to given that  when  and  when .

6.\* The equation  is studied, where and are constants. Make a sketch of the solutions in the following cases:

(i) ,

(ii) , , 

7.\* The equation for damped Simple Harmonic Motion of a bridge is given by



(i) Explain the physical significance of the constantsand.

(ii) Write down the conditions onand for *under-damping*.

8.\* *Although not part of the syllabus. This second-order ODE with simple polynomial coefficients can be solved simply using the method taught in this module and provides a nice test of understanding in verifying solution to differential equations.*

By direct substitution, verify that  for constant  is a solution to . State the possible values of .

9.\* A model is proposed to investigate the population of foxesand chickensin the form of a system of differential equations:



for constants,  . Initially the population of foxes is and chickens is.

(i) Solve the system of equations for  in terms of  and .

(ii) Describe the long-term behaviour of the fox and chicken populations.

(iii) Give a possible criticism of the suitability of this model.

10.\* An object is dropped into the sea from an oil rig platform. When it hits the surface, it is travelling at a speed of ms-1. The time is measured from when the object hits the surface at a depth . The resistance force  is modelled by where is the distance travelled from the oil rig platform. The only other force acting on the object as it falls is gravity. The resistance force is thought to be double the speed of the object and the object has a mass of 10kg. (take ms-2).

(i) Write down an equation of motion using Newton’s second law.

(ii) Solve this equation using the initial conditions.

**Extension**

Investigate the solutions to



for different 

Can you find conditions on so that (provided the arbitrary constants are chosen carefully)

(i) *x* and as 

(ii) *x* and as 

(iii) *x* and *y* are oscillatory as 

## Worked solutions

1. (i) Using Newton’s second law,  where *m* is the mass of the person and

*K* is a constant

For a particular person, *m* is a constant so dividing through by *m* gives

 so 

(ii) Separating variables



 where *C* is a constant.

2. This is solved using the method of integrating factor. Comparing to the general form of a linear ODE: 

The integrating factor is 

Multiplying the ODE by: 

The LHS can be written as a perfect derivative: 

Integrating with respect to: , where  is a constant of integration.

Finally, we have the general solution:

3. i) The auxiliary equation is found by substitutinginto the ODE:   
 

The exponentials cancel and we are left with the quadratic: 

This has roots 

Therefore, the general solution is 

Where  and . [the choice of which is which is arbitrary]

ii) It is easy to see that and . Therefore, for solutions to tend to zero as, we must have the coefficient of  be zero. This means  and hence  using the initial conditions:  and therefore, the particular solution is .

4. i) The auxiliary equation is  which has roots at and hence the general   
 solution is .

ii) The particular integral is found by substituting into the ODE:



and hence substituting: 

Rearranging the LHS: 

and hence equating coefficients: , , 

which are solved byand and hence the general solution is: 



5. The auxiliary equation is  which has roots at and hence the general   
 solution is 

The particular integral is found by substituting into the ODE:



and hence substituting: 

Rearranging the LHS: 

and hence equating coefficients: , , 

which are solved byand and hence the general solution is: 



 when 

 so 

 when 





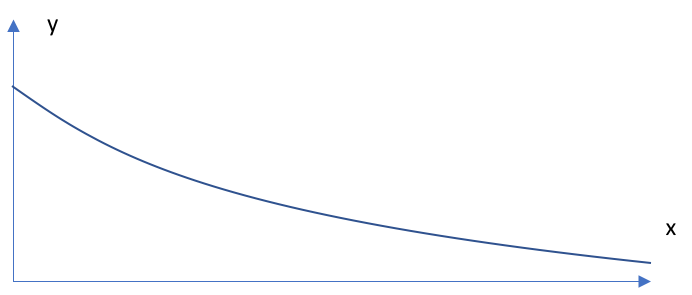
so 



6. The auxiliary equation is  which has solution .

i) For this case, there are 2 distinct real roots of the auxiliary equation. Additionally, we know that because and are both positive then the square root term in the formula has a magnitude smaller than, i.e. 

and hence the two real roots are both negative. Therefore, the graph will look like:



ii) In this case, the roots are complex. The real part of the roots is given by: 

Because and . Therefore, the solution will oscillate in a decaying manner around as :

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Description automatically generated

7. i) is the damping coefficient – a measure of the bridge’s resistance to motion via air-  
 resistance/dashpot.

The constant is the natural frequency of the oscillations of the un-damped system.

ii) For under-damping the solution is oscillatory. Hence the auxiliary equation:  must have complex roots and hence .

8. To verify a solution first we differentiate the possible form of the solution:



Substituting these expressions into the ODE gives 

which can be simplified to .

We have therefore that the term in brackets has to be zero. Hence 

Simplifying:  which has two solutions 

and hence the general solution is 

9. i) The system becomes:



The first equation is rearranged for:



 It is then differentiated:

These two expressions are substituted

into the second equation to get:



which is simplified to

This has auxiliary equation: 

which has roots and

and hence the complementary function is

and hence using the expression for above we have

which is simplified to

Now using the initial conditions we have that at, , and hence

Using the initial conditions for  we have

and solving these simultaneously gives

and the general solution is

ii) The chickens for any will die out as the solution is decaying exponential. The foxes population will increase provided that .

If this is not satisfied then the foxes will also die out as there will only be a decaying exponential.

iii) If the initial chicken population is then the fox numbers will evolve like 

i.e. they will experience exponential growth; unrealistic in the long term.

10. i) Using Newton’s second law we have 

The constant of proportionality for the air resistance force is 2 and the mass is 10 kg hence



ii) We solve the homogenous equation first.

This could be done by using an auxiliary equation or, alternatively, as follows.

We write this as a first-order ODE: 

where is the speed. Therefore, separating the variables: 

Evaluating the integrals:  and hence 

Using the initial conditions we have that  and hence 

Now the equation:  is solved directly by integration to get 

Using the initial conditions that at , we have  and hence.

The homogenous solution is therefore .

The particular integral is found by substitution of  as the RHS of the ODE has a similar form to one of the homogenous solutions; ie. A constant. Substituting  into the ODE gives  and hence and the full solution is 

**Extension**

i) 

ii) 

iii) 

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