## Monday 25 June 2018 - Morning

## A2 GCE MATHEMATICS (MEI)

4756/01 Further Methods for Advanced Mathematics (FP2)

## QUESTION PAPER

Candidates answer on the Printed Answer Book.
OCR supplied materials:
Duration: 1 hour 30 minutes

- Printed Answer Book 4756/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator


## INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the Printed Answer Book. If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer all the questions.
- Do not write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.


## INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.
- The Printed Answer Book consists of $\mathbf{1 2}$ pages. The Question Paper consists of 4 pages. Any blank pages are indicated.


## INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.


## Section A (54 marks)

1 (a) The polar equation of a curve is $r=a \sin ^{2} \theta \cos \theta$ for $0 \leqslant \theta \leqslant \frac{1}{2} \pi$.
(i) Find the value of $\theta$ for which the curve has the maximum $x$-coordinate.
(ii) Prove that the maximum $y$-coordinate on the curve is $\frac{3 \sqrt{3}}{16} a$ and state the value of $\theta$ at which this is attained.
(b) (i) Sketch the graph of $y=\arcsin x$ for $-1 \leqslant x \leqslant 1$.
(ii) Prove that $\frac{\mathrm{d}}{\mathrm{d} x}(\arcsin x)=\frac{1}{\sqrt{1-x^{2}}}$.
(iii) Using integration by parts and a suitable substitution, show that

$$
\begin{equation*}
\int_{0}^{1} x^{2} \arcsin x \mathrm{~d} x=\frac{3 \pi-4}{18} \tag{6}
\end{equation*}
$$

2 (a) (i) Use de Moivre's theorem to prove that

$$
\begin{equation*}
\cot 4 \theta=\frac{1-6 \tan ^{2} \theta+\tan ^{4} \theta}{4 \tan \theta\left(1-\tan ^{2} \theta\right)} \tag{5}
\end{equation*}
$$

(ii) Hence express the roots of the equation

$$
x^{4}+4 x^{3}-6 x^{2}-4 x+1=0
$$

in exact trigonometrical form.
(b) The vertices of a square with sides of length 1 unit lie on the axes of an Argand diagram. The vertices represent the complex numbers $z_{1}, z_{2}, z_{3}$ and $z_{4}$ and the midpoints of the sides of the square represent the complex numbers $z_{5}, z_{6}, z_{7}$ and $z_{8}$.
(i) Express $z_{5}, z_{6}, z_{7}$ and $z_{8}$ in modulus-argument form, and hence determine a polynomial equation of degree 4 , with integer coefficients, whose roots are $z_{5}, z_{6}, z_{7}$ and $z_{8}$.

Let $\mathrm{P}(z)=0$ be a polynomial equation of degree 8 , with integer coefficients, whose roots are $z_{1}, z_{2}, z_{3}, z_{4}, z_{5}, z_{6}, z_{7}$ and $z_{8}$.
(ii) Explain why $\mathrm{P}(z)$ cannot be of the form $a z^{8}+b$ where $a$ and $b$ are integers.
(iii) Find $\mathrm{P}(z)$.
(i) Find the inverse of the matrix $\left(\begin{array}{rrr}1 & 3 & 2 \\ -1 & 2 & k \\ k & 1 & 6\end{array}\right)$.

The matrix $\mathbf{M}$ has eigenvalues 1, 2 and 3. The corresponding eigenvectors are $\left(\begin{array}{l}2 \\ 0 \\ 6\end{array}\right),\left(\begin{array}{l}3 \\ 2 \\ 1\end{array}\right)$ and $\left(\begin{array}{r}1 \\ -1 \\ 0\end{array}\right)$ respectively.
(ii) Write down the matrix $\mathbf{P}$ such that $\mathbf{M}=\mathbf{P D P}^{-1}$ where $\mathbf{D}=\left(\begin{array}{lll}3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1\end{array}\right)$.
(iii) Hence find $\mathbf{M}$.
(iv) Find constants $a, b$ and $c$ such that $\mathbf{M}^{-1}=a \mathbf{M}^{2}+b \mathbf{M}+c \mathbf{I}$.

## Section B (18 marks)

4 (i) Prove, using definitions in terms of exponential functions, that

$$
\begin{equation*}
\cosh 2 A=1+2 \sinh ^{2} A . \tag{3}
\end{equation*}
$$

(ii) Find $\int \sinh ^{2} x \mathrm{~d} x$.
(iii) Let $z=\operatorname{arsinh}(1)$. Form an equation involving $z$ and solve it to find the exact value of $\operatorname{arsinh}(1)$ in logarithmic form.
(iv) Using a substitution of the form $a x=b \sinh u$, find the exact value of

$$
\int_{0}^{\frac{2}{3}} \frac{x^{2}}{\sqrt{4+9 x^{2}}} \mathrm{~d} x
$$

giving your answer in the form $p(q-\ln r)$, where $p, q$ and $r$ are constants.

Oxford Cambridge and RS

## Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series. If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.
For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.
OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

