OCR
Oxford Cambridge and RSA

Monday 25 June 2018 – Morning

A2 GCE MATHEMATICS (MEI)

4756/01 Further Methods for Advanced Mathematics (FP2)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4756/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (54 marks)

1 **(a)** The polar equation of a curve is $r = a\sin^2\theta\cos\theta$ for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$.

(i) Find the value of θ for which the curve has the maximum x-coordinate. **[3]**

(ii) Prove that the maximum y-coordinate on the curve is $\frac{3\sqrt{3}}{16}a$ and state the value of θ at which this is attained. **[4]**

(b) **(i)** Sketch the graph of $y = \arcsin x$ for $-1 \leqslant x \leqslant 1$. **[1]**

(ii) Prove that $\frac{\mathrm{d}}{\mathrm{d}x}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$. **[4]**

(iii) Using integration by parts and a suitable substitution, show that

$$\int_0^1 x^2 \arcsin x \,\mathrm{d}x = \frac{3\pi - 4}{18}.$$ **[6]**

2 **(a)** **(i)** Use de Moivre's theorem to prove that

$$\cot 4\theta = \frac{1 - 6\tan^2\theta + \tan^4\theta}{4\tan\theta(1 - \tan^2\theta)}.$$ **[5]**

(ii) Hence express the roots of the equation

$$x^4 + 4x^3 - 6x^2 - 4x + 1 = 0$$

in exact trigonometrical form. **[4]**

(b) The vertices of a square with sides of length 1 unit lie on the axes of an Argand diagram. The vertices represent the complex numbers z_1, z_2, z_3 and z_4 and the midpoints of the sides of the square represent the complex numbers z_5, z_6, z_7 and z_8.

(i) Express z_5, z_6, z_7 and z_8 in modulus-argument form, and hence determine a polynomial equation of degree 4, with integer coefficients, whose roots are z_5, z_6, z_7 and z_8. **[4]**

Let $\mathrm{P}(z) = 0$ be a polynomial equation of degree 8, with integer coefficients, whose roots are z_1, z_2, z_3, z_4, z_5, z_6, z_7 and z_8.

(ii) Explain why $\mathrm{P}(z)$ cannot be of the form $az^8 + b$ where a and b are integers. **[1]**

(iii) Find $\mathrm{P}(z)$. **[4]**

3 **(i)** Find the inverse of the matrix $\begin{pmatrix} 1 & 3 & 2 \\ -1 & 2 & k \\ k & 1 & 6 \end{pmatrix}$. **[5]**

The matrix $\mathbf{M}$ has eigenvalues 1, 2 and 3. The corresponding eigenvectors are $\begin{pmatrix} 2 \\ 0 \\ 6 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ respectively.

(ii) Write down the matrix $\mathbf{P}$ such that $\mathbf{M} = \mathbf{PDP}^{-1}$ where $\mathbf{D} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. **[2]**

(iii) Hence find $\mathbf{M}$. **[5]**

(iv) Find constants a, b and c such that $\mathbf{M}^{-1} = a\mathbf{M}^2 + b\mathbf{M} + c\mathbf{I}$. **[6]**

Section B (18 marks)

4 **(i)** Prove, using definitions in terms of exponential functions, that

$$\cosh 2A = 1 + 2\sinh^2 A.$$ **[3]**

(ii) Find $\int \sinh^2 x \, dx$. **[3]**

(iii) Let $z = \operatorname{arsinh}(1)$. Form an equation involving z and solve it to find the exact value of arsinh(1) in logarithmic form. **[4]**

(iv) Using a substitution of the form $ax = b\sinh u$, find the exact value of

$$\int_0^{\frac{2}{3}} \frac{x^2}{\sqrt{4+9x^2}} \, dx,$$

giving your answer in the form $p(q - \ln r)$, where p, q and r are constants. **[8]**

END OF QUESTION PAPER