Oxford Cambridge and RSA

## GCE

## Mathematics

Unit 4721: Core Mathematics 1
Advanced Subsidiary GCE

## Mark Scheme for June 2018

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.
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## Annotations and abbreviations

These are the annotations, (including abbreviations), including those used in scoris, which are used when marking

| Annotation in scoris | Meaning |
| :--- | :--- |
| $\checkmark$ and $\mathbf{x}$ |  |
| BOD | Benefit of doubt |
| FT | Follow through |
| ISW | Ignore subsequent working |
| M0, M1 | Method mark awarded 0, 1 |
| A0, A1 | Accuracy mark awarded 0, 1 |
| B0, B1 | Independent mark awarded 0, 1 |
| SC | Special case |
| ^ | Omission sign |
| MR | Misread |
| Highlighting |  |
|  |  |
| Other abbreviations in <br> mark scheme | Meaning |
| E1 | Mark for explaining |
| U1 | Mark for correct units |
| G1 | Mark for a correct feature on a graph |
| M1 dep* | Method mark dependent on a previous mark, indicated by * |
| cao | Correct answer only |
| oe | Or equivalent |
| rot | Rounded or truncated |
| soi | Seen or implied |
| www | Without wrong working |
|  |  |
|  |  |

## Subject-specific Marking Instructions for GCE Mathematics Pure strand

Annotations should be used whenever appropriate during your marking.
The $A, M$ and $B$ annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded
An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader

The following types of marks are available.

## M

A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only - differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
$\mathrm{f} \quad$ Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

Rules for replaced work
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.
$\mathrm{h} \quad$ For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

| Question |  | Answer |  | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  | $\begin{aligned} & \frac{6-\sqrt{5}}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}} \\ & \frac{17-8 \sqrt{5}}{4-5} \\ & -17+8 \sqrt{5} \end{aligned}$ | M1 <br> A1 <br> A1 <br> A1 <br> [4] | Attempt to rearrange and rationalise the denominator <br> Numerator correct (allow 4 terms) Denominator correct (allow 2 terms) <br> Fully correct and simplified, accept $8 \sqrt{5}-17$ <br> Do not ISW if answer is multiplied by -1 to give $17-8 \sqrt{5}$ | Alt 1 <br> Multiply both sides by $2-\sqrt{5}$ M1 <br> LHS correct A1 <br> RHS correct A1 <br> Fully correct A1 <br> Alt 2 <br> Correct method to solve simultaneous equations formed by putting $x=a+b \sqrt{5}$ M1 <br> Correct equations obtained A1 <br> Either $a$ or $b$ correct A1 <br> Both correct A1 |
| 2 | (i) | $\frac{d v}{d t}=20-8 t$ <br> When $t=3, \frac{d v}{d t}=20-24=-4$ <br> Rate of change of velocity $=-4 \mathrm{~ms}^{-2}$ | $\begin{aligned} & \hline \text { B1 } \\ & \text { M1 } \\ & \text { A1 } \\ & {[3]} \end{aligned}$ | Fully correct differentiation <br> Substitutes 3 into their derivative Correct answer (ignore units) | No "+ C" <br> M0 for substitution after integrating |
|  | (ii) | $\frac{d v}{d t}<0$, so decreasing | B1 ft [1] | Follow through sign of their (i). Allow any sensible explanation e.g. <br> - Acceleration negative so slowing down <br> - Gradient of $v / t$ graph is negative so velocity is decreasing | Statements must be fully correct and relate to their derivative e.g. do not accept <br> - It's a negative function so it's decreasing <br> - It is decreasing as the velocity of the object is negative |
| 3 |  | $\begin{aligned} & \text { Gradient }=\frac{4-6}{3--1}=-\frac{1}{2} \\ & y-4=-\frac{1}{2}(x-3) \end{aligned}$ $x+2 y-11=0$ | M1 <br> A1 <br> M1 <br> Alft <br> A1 <br> [5] | Attempt to find gradient <br> Correct gradient found <br> Correct method to find equation of straight <br> line through either point, any non-zero <br> gradient <br> Correct answer for their gradient (any form) <br> cao Correct answer in required form i.e. <br> $k(x+2 y-11)=0$ for any integer $k$ | At least 3 out of 4 values correct May be unsimplified Allow through $(1,5)$ for full marks <br> May be unsimplified Terms may be in any order for final A1 |



| Question |  | Answer | Marks <br> B1 | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 6 | (i) |  | B1 <br> B1 <br> [2] | Correct shape in both quadrants and no others <br> Excellent sketch: <br> - correct shape, symmetrical, not touching axes <br> - asymptotes clearly the axes <br> - not finite <br> - allow slight movement away from asymptote at one end but not more. | SC B1 Excellent sketch in the first quadrant only |
|  | (ii) | $y=\frac{3}{x-4}$ | B1 <br> B1 <br> [2] | $\frac{3}{x-4} \text { or } \frac{3}{x+4}$ <br> Fully correct, must have " $y=$ " |  |
|  | (iii) | Stretch <br> Scale factor $\frac{2}{3}$ parallel to the $y$-axis | B1 <br> B1 <br> [2] | Must be "stretch" <br> Allow "factor" or "SF" for "scale factor" For "parallel to the $y$ axis" allow "vertically", "in the $y$ direction". Do not accept "in/on/across/up/along the $y$ axis" "in the positive y direction" "SF $\frac{2}{3}$ units" $\mathbf{N B}$ - Scale factor $\frac{2}{3}$ parallel to the $x$-axis also correct. | B0B1 is possible for e.g. "enlarge by scale factor..." etc. but not for (e.g.) "translate by scale factor" etc. <br> Do not ignore extra incorrect statements |


| Question |  | Answer | Marks | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | (i) | $\begin{aligned} & -2\left(x^{2}+8 x\right)-9 \\ & -2\left[(x+4)^{2}-16\right]-9 \\ & -2(x+4)^{2}+23 \end{aligned}$ | B1 <br> B1 <br> M1 <br> A1 <br> [4] | $\begin{aligned} & a=-2 \\ & b=4 \\ & -9+2 b^{2} \text { or their } a \times\left(4.5-b^{2}\right) \\ & c=23 \end{aligned}$ <br> If signs of all terms changed at start, can only score SC B1 for fully correct working to obtain $2(x+4)^{2}-23$ <br> If done correctly and then signs changed at end, do not ISW, award B1B1M1A0 | $\begin{aligned} & -2(x-4)^{2}+23 \text { B1 B0 M1 A0 } \\ & -2(x+4)^{2}-23 \text { B1 B1 M0 A0 } \\ & -2(x+4)+23 \text { BOD 4/4 } \\ & -2(x+4 x)^{2}+23 \text { B1 B0 M1 A0 } \\ & -2\left(x^{2}+4\right)^{2}+23 \text { B1 B0 M1 A0 } \\ & -2 x(x+4)^{2}+23 \text { B0 B1 M0 A0 } \\ & -2\left(x^{2}+4\right)+23 \text { B1 B0 M1 A0 } \end{aligned}$ |
|  | (ii) | 23 | $\begin{gathered} \text { B1 ft } \\ {[1]} \end{gathered}$ | Follow through their $c$ provided $a$ is negative. Must be consistent with (i) unless restarted e.g. from differentiation. | Allow $y=23$ or $f(x)=23$, but $(-4,23)$ only scores B0 |
|  | (iii) | $x=-4$ | $\begin{gathered} \hline \text { B1 ft } \\ {[1]} \\ \hline \end{gathered}$ | $x=-$ (their $b$ ). Must be consistent with (i) unless restarted e.g. from differentiation. |  |
| 8 |  | $\begin{aligned} & x^{2}+(1-2 x)^{2}=13 \\ & 5 x^{2}-4 x-12=0 \\ & (5 x+6)(x-2)=0 \\ & x=-\frac{6}{5}, x=2 \\ & y=\frac{17}{5}, y=-3 \\ & \left(\left(-\frac{6}{5}+2\right) \div 2,\left(\frac{17}{5}-3\right) \div 2\right) \\ & \left(\frac{2}{5}, \frac{1}{5}\right) \end{aligned}$ | M1* <br> A1 M1 dep* <br> A1 <br> A1 <br> M1 <br> A1 <br> [7] | Substitute for $x / y$ or attempt to eliminate one of the variables <br> Correct three-term quadratic found Correct method to solve their three-term quadratic See appendix 1. <br> $x$ values correct <br> $y$ values correct <br> Award A1 A0 for one pair correctly found from correctly factorised quadratic <br> Correct method to find midpoint oe Correct answer | If $x$ eliminated: $\begin{aligned} & \left(\frac{1-y}{2}\right)^{2}+y^{2}=13 \\ & 5 y^{2}-2 y-51=0 \\ & (5 y-17)(y+3)=0 \end{aligned}$ <br> Spotted solutions: <br> SC B1 One correct pair www SC B1 Second correct pair www Must show on both line and curve (Can then get all of first five marks if both found www and exactly two solutions justified) |
| 9 | (i) | $\begin{aligned} & (-4)^{2}-4 \times k \times(3 k-1)<0 \\ & 16-12 k^{2}+4 k<0 \\ & 3 k^{2}-k-4>0 \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \\ & \text { A1 } \\ & \text { [3] } \end{aligned}$ | Uses $b^{2}-4 a c$, must involve $3 k-1$ <br> Fully correct substitution to $b^{2}-4 a c<0$ <br> Fully correct working to correct answer AG | Allow $\sqrt{b^{2}-4 a c}$ for M1 only <br> If formula not quoted then $0 / 3$ if sign errors |


| Question |  | Answer | Marks | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 9 | (ii) | $\begin{aligned} & (3 k-4)(k+1) \\ & \frac{4}{3},-1 \\ & k<-1, k>\frac{4}{3} \end{aligned}$ | M1 <br> A1 <br> M1 <br> A1 <br> [4] | Correct method to find roots. See appendix <br> 1. <br> Correct roots found <br> Chooses the "outside region" for their roots <br> Allow " $k<-1, k>\frac{4}{3}$ ", " $k<-1$ or $k>\frac{4}{3}$ " but do not allow $" k<-1 \text { and } k>\frac{4}{3} \text { " }$ | NB e.g. $-1>k>\frac{4}{3}$ scores M1A0 if correct answer not previously seen. Must be strict inequalities for final $\mathbf{A}$ mark. |
| 10 |  | $\begin{aligned} & (y=)(x+1)^{2}(x-3) \\ & (y=)\left(x^{2}+2 x+1\right)(x-3) \\ & (y=) x^{3}-x^{2}-5 x-3 \\ & \frac{d y}{d x}=3 x^{2}-2 x-5 \\ & 3 x^{2}-2 x-5=0 \\ & (3 x-5)(x+1)=0 \\ & x=\frac{5}{3} \\ & y=-\frac{256}{27} \end{aligned}$ | B1 B1 M1 A1 B1 ft M1 M1 A1 A1 $[9]$ | $(x-3)$ seen $(x+1)$ or $(x+1)^{2}$ seen <br> Multiply repeated root by linear factor to obtain at least five terms Correct values of $p, q, r$ obtained Correct differentiation of their cubic <br> Sets derivative equal to zero <br> Correct method to find roots. See appendix1 <br> Correct value of $x$ obtained <br> Correct value of $y$ obtained | Alt - using simultaneous equations $-1+p-q+r=0 \mathbf{B} 1$ $27+9 p+3 q+r=0 \mathbf{B} 1$ $\frac{d y}{d x}=3 x^{2}+2 p x+q \mathbf{B} 1$ <br> Sets gradient to zero at $x=-1 \mathbf{M 1}$ $3-2 p+q=0$ <br> Correct method to solve three simultaneous equations in three variables; do not allow incorrectly obtained equations e.g. from $\frac{d y}{d x}=0$ at $x=3$. Must get as far as finding at least one of $p, q, r \mathbf{M 1}$ Correct values of $p, q, r$ obtained A1 Correct method to find $x$ M1 <br> Correct x A1 <br> Correct y A1 |
| 11 | (i) | $\begin{aligned} & (x-6)^{2}+(y+3)^{2}=10 \\ & x^{2}-12 x+36+y^{2}+6 y+9=10 \\ & x^{2}+y^{2}-12 x+6 y+35=0 \end{aligned}$ | B1 <br> M1 <br> A1 [3] | $\begin{aligned} & (x-6)^{2} \text { and }(y+3)^{2} \text { seen } \\ & (x \pm 6)^{2}+(y \pm 3)^{2}=10 \end{aligned}$ <br> Correct equation in correct form, terms may be in any order, but must have " $=0$ " | For $f, g$, $c$ method: $\begin{aligned} & x^{2}-12 x+y^{2}+6 y \mathbf{B} 1 \\ & c=( \pm 6)^{2}+3^{2}-10 \mathbf{M 1} \end{aligned}$ <br> Correct equation in correct form A1 |


| Question |  | Answer ${ }^{\text {a }}$ Marks |  | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | (ii) | $\begin{aligned} & \text { Gradient of radius }=-\frac{1}{3} \\ & \text { Gradient of tangent }=3 \\ & y--2=3(x-3) \\ & y=3 x-11 \end{aligned}$ | B1 <br> B1ft <br> M1 <br> A1 <br> A1 <br> [5] | Gradient of radius <br> Gradient of tangent <br> Correct method to find equation of straight line through $(3,-2)$, any non-zero gradient Correct answer in any form Correct answer in any three-term form | Alternative for first two marks: <br> M1 Attempt at implicit differentiation as evidenced by $2 y \frac{d y}{d x}$ term <br> A1 $2 x+2 y \frac{d y}{d x}-12+6 \frac{d y}{d x}=0$ and substitutes $(3,-2)$ to obtain $\frac{d y}{d x}=3$ |
| 11 | (iii) | $\begin{aligned} & \sqrt{(10-6)^{2}+(-3-1)^{2}} \\ & 4 \sqrt{2} \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \\ & {[2]} \\ & \hline \end{aligned}$ | Uses Pythagoras' theorem Correct answer in correct form - do not accept $\sqrt{32}, 2 \sqrt{8}$ or other unsimplified surds |  |
|  | (iv) | $\begin{aligned} & C T^{2}+Q T^{2}=Q C^{2} \\ & 10+Q T^{2}=32 \\ & Q T=\sqrt{22} \end{aligned}$ | M1 <br> A1ft <br> A1 <br> [3] | Uses tangent perpendicular to radius and Pythagoras' theorem $10+\mathrm{QT}^{2}=$ their 32 from (iii) provided this is greater than 10 cao | Must have QC as the hypotenuse $\mathrm{NB} \sqrt{32}-\sqrt{10}=\sqrt{22} \mathrm{M} 0$ |

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