# AS Level Further Mathematics B (MEI) <br> Y410/01 Core Pure <br> Question Paper 

## Monday 14 May 2018 - Afternoon <br> Time allowed: 1 hour 15 minutes

You must have:

- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:

- a scientific or graphical calculator


## INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer all the questions.
- Write your answer to each question in the space provided in the Printed Answer Booklet. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do not write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.


## INFORMATION

- The total number of marks for this paper is $\mathbf{6 0}$.
- The marks for each question are shown in brackets [ ].
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of 12 pages. The Question Paper consists of 4 pages.

Answer all the questions.

1 The matrices $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ are defined as follows:

$$
\mathbf{A}=\left(\begin{array}{l}
1 \\
2 \\
3
\end{array}\right), \quad \mathbf{B}=\left(\begin{array}{rrr}
2 & 0 & 3 \\
1 & -1 & 3
\end{array}\right), \quad \mathbf{C}=\left(\begin{array}{ll}
1 & 3
\end{array}\right)
$$

Calculate all possible products formed from two of these three matrices.
2 Find, to the nearest degree, the angle between the vectors $\left(\begin{array}{r}1 \\ 0 \\ -2\end{array}\right)$ and $\left(\begin{array}{r}-2 \\ 3 \\ -3\end{array}\right)$.

3 Find real numbers $a$ and $b$ such that $(a-3 i)(5-i)=b-17 i$.

4 Find a cubic equation with real coefficients, two of whose roots are $2-\mathrm{i}$ and 3 .

5 A transformation of the $x-y$ plane is represented by the matrix $\left(\begin{array}{rr}\cos \theta & 2 \sin \theta \\ 2 \sin \theta & -\cos \theta\end{array}\right)$, where $\theta$ is a positive acute angle.
(i) Write down the image of the point $(2,3)$ under this transformation.
(ii) You are given that this image is the point $(a, 0)$. Find the value of $a$.

6 Find the invariant line of the transformation of the $x-y$ plane represented by the matrix $\left(\begin{array}{rr}2 & 0 \\ 4 & -1\end{array}\right)$.

7
(i) Express $\frac{1}{2 r-1}-\frac{1}{2 r+1}$ as a single fraction.
(ii) Find how many terms of the series

$$
\frac{2}{1 \times 3}+\frac{2}{3 \times 5}+\frac{2}{5 \times 7}+\ldots+\frac{2}{(2 r-1)(2 r+1)}+\ldots
$$

are needed for the sum to exceed 0.999999 .

8 Prove by induction that $\left(\begin{array}{ll}1 & 1 \\ 0 & 2\end{array}\right)^{n}=\left(\begin{array}{cc}1 & 2^{n}-1 \\ 0 & 2^{n}\end{array}\right)$ for all positive integers $n$.

9 Fig. 9 shows a sketch of the region OPQ of the Argand diagram defined by

$$
\{z:|z| \leqslant 4 \sqrt{2}\} \cap\left\{z: \frac{1}{4} \pi \leqslant \arg z \leqslant \frac{1}{3} \pi\right\} .
$$



Fig. 9
(i) Find, in modulus-argument form, the complex number represented by the point P .
(ii) Find, in the form $a+\mathrm{i} b$, where $a$ and $b$ are exact real numbers, the complex number represented by the point Q .

## (iii) In this question you must show detailed reasoning.

Determine whether the points representing the complex numbers

- $3+5 \mathrm{i}$
- $5.5(\cos 0.8+\mathrm{i} \sin 0.8)$
lie within this region.

10 Three planes have equations

$$
\begin{aligned}
-x+2 y+z & =0 \\
2 x-y-z & =0 \\
x+y & =a
\end{aligned}
$$

where $a$ is a constant.
(i) Investigate the arrangement of the planes:

- when $a=0$;
- when $a \neq 0$.
(ii) Chris claims that the position vectors $-\mathbf{i}+2 \mathbf{j}+\mathbf{k}, 2 \mathbf{i}-\mathbf{j}-\mathbf{k}$ and $\mathbf{i}+\mathbf{j}$ lie in a plane. Determine whether or not Chris is correct.


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