OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI)

Decision Mathematics Computation

4773

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator
- Computer with appropriate software and printing facilities

Friday 24 June 2011
Afternoon

Duration: 2 hours 30 minutes

4771389061

4773

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Additional sheets, including computer print-outs, should be fastened securely to the Answer Booklet.
- Do **not** write in the bar codes.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- In each of the questions you are required to write spreadsheet or other routines to carry out various processes.
- For each question you attempt, you should submit print-outs showing the routine you have written and the output it generates.
- You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program, a linear programming package and suitable printing facilities throughout the examination.

1 Five friends on a skiing holiday arrive late for lunch at an alpine refuge. Only five portions of food are left:

- one portion of macaroni cheese (a pasta dish with cheese sauce − suitable for vegetarians)
- one portion of crespelle (a pancake with ham and cheese)
- one portion of vegetable lasagne (pasta − suitable for vegetarians)
- two portions of spaghetti carbonara (spaghetti with a creamy meat sauce)

Arthur and Bertie are both vegetarians. Charles will eat anything that is not vegetarian. David would like the lasagne. Edward wants spaghetti.

(i) Represent this information on a bipartite graph, and give the number of maximal matchings that are available (counting the two portions of spaghetti as different). **[4]**

(ii) Draw a separate bipartite graph showing a maximal matching in which Bertie gets no lunch. **[1]**

(iii) By considering all possible alternating paths starting with the vertex representing Bertie, prove that there is not a complete matching. **[3]**

(iv) Formulate the problem as an LP using variables such as Cc, which is to take the value 1 if Charles is allocated the crespelle, and 0 if not. Run your LP and interpret the results. **[8]**

(v) By drawing an appropriate network, show how the problem could be set up as a network flow problem. You are not required to solve your network problem. **[2]**

2 A number of investments are available to an investor. Their starting times, durations and returns are listed in the table. Thus, for example, investment number 4 requires money to be deposited in one year's time (time = 1) and will return £109 in three years' time (time = 3) for every £100 invested.

Investment number	1	2	3	4	5	6	7	8
Start	0	0	1	1	2	2	3	4
End	1	2	2	3	3	4	5	5
Return	4%	10%	4%	9%	4%	10%	12%	4%

The investor has £10 000 to invest over the next five years.

A financial analyst formulates this investment problem as an LP.

(i) Complete the following set of inequalities in the analyst's formulation (all variables are non-negative):

$x1 + x2 \leqslant 10\,000$
$x3 + x4 \leqslant 1.04x1$
$x5 + x6 \leqslant 1.10x2 + 1.04x3$
…
… **[5]**

(ii) Convert the inequalities into a form suitable for submission to LINDO. **[1]**

(iii) Give a suitable objective function. **[3]**

(iv) Run your LP and interpret the results. **[4]**

(v) Another analyst observes that, for this problem, there are five possible investment strategies. Identify them and pick the best. **[4]**

(vi) Why use LP to solve problems of this type? **[1]**

3 Keith has had a tooth removed and has been told that he must take an antibiotic. Each tablet contains 250 mg of antibiotic, and he should take 4 tablets a day.

The instructions supplied with the packet indicate the proportion of the antibiotic in the body which will be excreted over a given period. Keith knows that the rate of excretion will be proportional to the amount of drug in the body. From the information he deduces that, if $x(t)$ mg is the amount of antibiotic in his body at time t hours, then an appropriate model is $x(t) = 0.89^t x(0)$, provided that no tablet is taken between time 0 and time t.

Let u_n be the amount of antibiotic in mg in Keith's body immediately after he has taken the nth tablet, so that $u_1 = 250$.

(i) Produce a recurrence relation for u_{n+1} in terms of u_n, given that Keith takes a tablet every 6 hours. **[2]**

(ii) Solve your recurrence relation and find the limiting value of u_n as n increases. **[4]**

(iii) Draw a sketch graph showing the amount of antibiotic in Keith's body over a 24-hour period after he has been taking his tablets for a long time, given that he takes a tablet every 6 hours. You should accurately label relevant amounts of antibiotic on your graph. **[3]**

In fact, Keith does not disturb his sleep to take a tablet, so the intervals between him taking tablets during a day are 5 hours, 5 hours, 5 hours and 9 hours.

(iv) Construct a spreadsheet to show the amount of antibiotic in Keith's body immediately after he takes each of his 28 tablets. **[4]**

(v) Draw a sketch graph showing the amount of antibiotic in Keith's body over a 24-hour period after he has been taking his tablets for a long time, given that he takes tablets at intervals of 5, 5, 5 and 9 hours each day. You should accurately label relevant amounts of antibiotic on your graph. **[3]**

(vi) After he has been taking the tablets for a long time, Keith forgets to take his bedtime tablet. Find the amount of antibiotic in his body when his next tablet is due.
Should he take two tablets at this time? **[2]**

4 The management of an alpine refuge has a difficult task in planning how many meals of different types to provide at lunchtime. The number of customers is dependent on the weather. The approximate distribution of the number of customers is given in the table.

Number of customers	5	10	15	20
Probability	0.35	0.15	0.15	0.35

The refuge provides four different lunch dishes:

- macaroni cheese
- crespelle
- vegetable lasagne
- spaghetti carbonara

Customers choose these dishes in the following proportions:

Dish	macaroni	crespelle	lasagne	spaghetti
Probability	0.3	0.1	0.2	0.4

(i) Construct a spreadsheet to simulate the demand for each dish during one lunchtime.
You need to simulate the number of customers, and then for each customer you need to simulate the dish chosen by that customer. You are advised to try to arrange your simulation on one line of your spreadsheet. **[9]**

(ii) Repeat your simulation 100 times and record the number of times each dish is chosen on each day. For each dish find the mean and standard deviation of the number chosen on each day. **[5]**

(iii) Explain why it is that standard deviations are relevant here rather than standard errors. **[2]**

(iv) How would you advise the management on planning the provision? **[2]**

BLANK PAGE

BLANK PAGE