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Principal Learning

Engineering

OCR Level 3 Principal Learning in Engineering **F563**

Mathematical techniques and applications for engineers

Mark Scheme for June 2010

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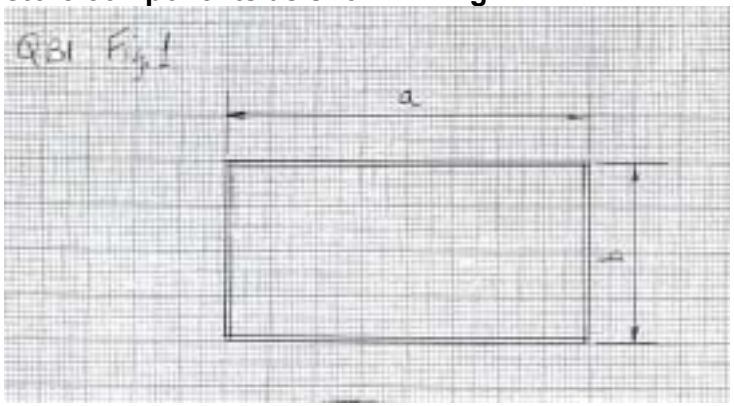
Section A

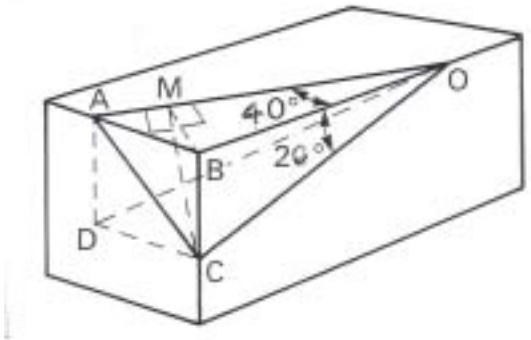
Question	Expected Answers	Marks	Rationale
1	$(2x + 6)(x - 3) = 2x^2 - 6x + 6x - 18$ [1] $= 2x^2 - 18 = 2(x^2 - 9)$ [1]	**[2]**	
2	$4x^2 - 14x + 6 = (4x - 2)(x - 3)$ $(4x - 2)$ [1] $(x - 3)$ [1]	**[2]**	**Question is incorrectly printed. Accept any reasonable answer. BOD**
3	$(2x + 4)/3 - (x + 5)/6 = (4x + 8 - x - 5)/6$ [1] $= (3x + 3)/6 = (x + 1)/2$ [1]	**[2]**	**Accept (3x+3)/6 for two marks**
4	$2(3x - 4) = 5x + 6$ $6x - 8 = 5x + 6$ [1] $6x - 5x = 6 + 8, \quad x = 14$ [1]	**[2]**	
5	Length of arc (s) = $(\pi x^0 r)/180$ [1] $= (100 \times \pi \times 40)/180 = 69.8$ mm correct to 1dp [1]	**[2]**	**Accept correct answer with no workings for 2 marks**
6	$\tan 46^o = c/125$ [1] therefore $c = 125 \tan 46^o = 129.4$ mm correct to 1dp [1]	**[2]**	**Accept use of Sine rule**
7	$i = 10 \sin \theta$ $= 10 \sin 60^o$ [1] $= 8.66$ A correct to 2dp [1]	**[2]**	
8	Area = ½ bc sin A [1] $= ½ \times 60 \times 100 \times \sin 30^o = 1500$ mm^2 [1]	**[2]**	

Question	Expected Answers	Marks	Rationale
9	$s = 6t^4 - 4t^3 + 2t^2$ $ds/dt = 24t^3 - 12t^2 + 4t$ $24t^3$ [1] $- 12t^2 + 4t$ [1]	**[2]**	
10	$y = 2 \sin 3x + 4 \cos 5x$ $dy/dx = 6 \cos 3x - 20 \sin 5x$ $6 \cos 3x$ [1] $- 20 \sin 5x$ [1]	**[2]**	
11	$\int (8x^3 + 9x^2 + 12x)\, dx = 2x^4 + 3x^3 + 6x^2 + C$ $2x^4 + 3x^3 + 6x^2$ [1] $+ C$ [1]	**[2]**	
12	..... 20..... 20 Area $= \int_0 x^2.\, dx = [x^3/3]_0$ [1] $= 20^3/3 = 8000/3 = 2666.7$ square units correct to 1dp [1]	**[2]**	
13	Number of scores = 6 which is an even number Scores rearranged: 8, 9, 11, 12, 14, 15 By observation the median lies between 11 and 12 [1] Median = (11 + 12)/2 = 11.5 [1]	**[2]**	
14	Mean = (15 + 15.3 + 15.6 + 15.7 + 15.9)5 [1] = 77.5/5 = 15.5 ohms [1]	**[2]**	**Units (ohms) are not required to gain full marks**
15	4/9 4 [1] 9 [1]	**[2]**	

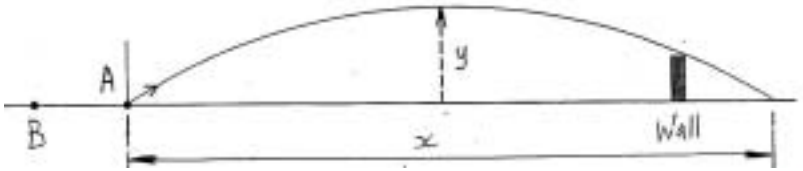
Section B

Question	Expected Answers	Marks	Rationale
1 (a)	**Newton's law of cooling is given by $x = x_o\ e^{-kt}$, where the temperature at zero time is x_o degrees Celsius and at t seconds is x degrees Celsius.**		
(i)	**Give the formula for the rate of change of temperature.** $x = x_o e^{-kt}$ Rate of change = $dx/dt = -k\ x_o e^{-kt}$ [1]		
(ii)	**Calculate, correct to 3 decimal places, the rate of change of temperature after 40s, given that x_o is 20 C and k = 0.04.** Rate of change = $-k\ x_o e^{-kt}$ $= - 0.04 \times 20 \times e^{-0.04 \times 40}$ [1] $= - 0.162$ deg s^{-1} correct to3 dp [1]	**[3]**	
(b)	**A container of rectangular cross-section is being designed to store components as shown in Fig. 1.**  **Determine an expression for the cross-sectional area of the container in terms of dimension b when all four sides add up to an overall length of 240 mm.**		

Question	Expected Answers	Marks	Rationale
	Perimeter = 240 mm Let a = length of one side and let b = length of the other side Then 240 = a + b + a + b [1] So 2(a + b) = 240 a+ b =120 so a = 120 – b or b = 120 - a [1] Area = ab = b(120 – b) = 120b – b^2 [1]	**[3]**	
(c)	**Calculate the maximum area of the container that can be enclosed by sides of overall length 240 mm.** dA/db = 120 – 2b d^2A/db^2 = -2 giving a maximum [1] For maximum value dA/db = 0 [1] So 0 = 120 – 2b 2b = 120 b = 60 [1] Substitute b = 60 into a + b =120 then a = 120 – b = 120 – 60 = 60 mm The dimensions of the box are 60 mm x 60 mm giving a maximum [1] area of 3600 mm^2	**[4]**	
			[Total: 10]
2 (a)	**The gradient of a curve is given by the expression** **dy/dx = 4x –3** **Determine the equation of the curve** dy/dx = 4x – 3 Equation of the curve = ∫ 4x – 3 [1] = $(4x^2)/2$ - 3x + C [1] = $2x^2$ – 3x + C [1]	**[3]**	

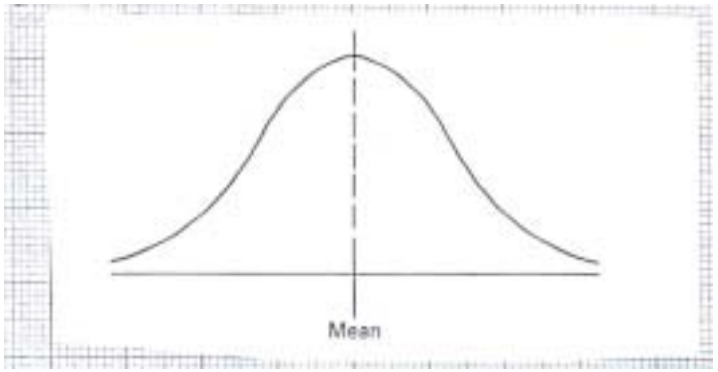
Question	Expected Answers	Marks	Rationale
(b)	**Integrate with respect to x: sin 4x + 2 cos 3x + √ x** $\int \sin 4x + 2\cos 3x + \sqrt{x}.\, dx = \int \sin 4x + 2\cos 3x + x^{0.5}\, dx$ [1] $= -\frac{1}{4}\cos 4x + \frac{2}{3}\sin 3x + \{(x^{1.5})/1.5\} + C$ [1] $= -\frac{1}{4}\cos 4x + \frac{2}{3}\sin 3x + \sqrt{(x^3)}/1.5 + C$ [1]	**[3]**	
(c)	**A body moves in a straight line so that its velocity v metres per second after time t seconds is given by $v = 16t - 6t^2$. Its initial distance from the origin is 30m. Calculate its distance from the origin after 3s.** Velocity $v = 16t - 6t^2$ Distance $s = \int 16t - 6t^2\, dt$ $s = 8t^2 - 2t^3 + C$ [1] When t = 0 s = 30 so substitute in equation to solve for C $30 = 0 - 0 + C \quad C = 30$ [1] So $s = 8t^2 - 2t^3 + C = s = 8t^2 - 2t^3 + 30$ [1] When t = 3 s Then $s = 8(3^2) - 2(3^3) + 30 = 72 - 54 + 30 = 48$ m [1]	**[4]**	
			[Total: 10]
(3)	**A triangular prism OABC is being cut from a rectangular block by an oblique plane OAC as shown in Fig.2**  		

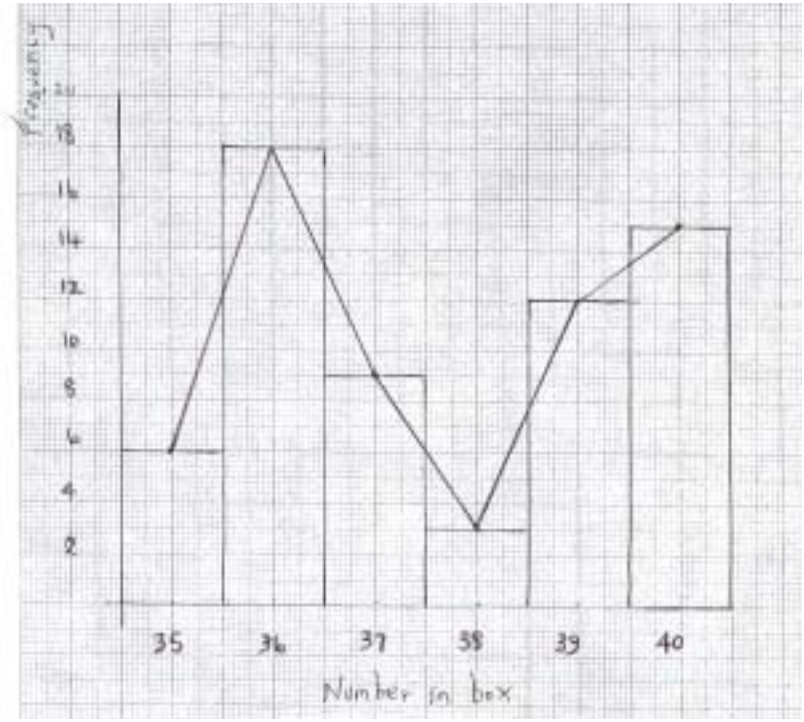
Question	Expected Answers	Marks	Rationale
(a) (i)	**Give the formula for the length BM from the triangle OBM.** $\sin 40^o = BM/OB$ then $BM = OB \sin 40^o$ [1]	**[1]**	
(ii)	**Give the formula for the length BC from the triangle OBC.** $\tan 20^o = BC/OB$ then $BC = OB \tan 20^o$ [1]	**[1]**	
(b)	**Calculate, correct to 2 decimal places, the angle between the oblique plane OAC and the top surface of the block** Draw BM perpendicular to AO. Join CM. CM is also be perpendicular to AO. Since both these lines are perpendicular to the line of intersection AO of the plane OAC and the top face of the block, the required angle is CMB $\tan CMB = BC/BM$ $= OB \tan 20^o / OB \sin 40^o$ [1] $= \tan 20^o / \sin 40^o$ $= 0.566$ [1] Angle CMB $= \tan^{-1} 0.566 = 29.52^o$ correct to 2dp [1]	**[3]**	
(c) (i)	**Give the formula for the length AB from the triangle OAB** $\tan 40^o = AB/OB$ then $AB = OB \tan 40^o$ [1]	**[1]**	

Question	Expected Answers	Marks	Rationale
(ii)	**Calculate, correct to 2 decimal places, the angle COD where AD is perpendicular to the top face and CD is perpendicular to the front face of the block.** ABCD is a rectangle, so DC = AB so DC = OB tan 40^o [1] cos 20^o = OB/OC then OC = OB/cos 20^o [1] tan COD = DC/OC [1] = OB tan 40^o/ (OB/cos 20^o) = cos 20^o tan 40^o = 0.7885 Angle COD = $\tan^{-1}$ 0.7885 = 38.26^0 correct to 2dp [1]	**[4]**	
			[Total: 10]
(4)	**The path of a jet of water shown in Fig.3 from a hose pipe at point A describes a parabola with an equation $y = 0.8x – 0.02x^2$, where y metres is the height of the water and x metres is the horizontal distance travelled.** 		
(a)	**Show that the water reaches a distance of 40 m along the ground.** Given equation $y = 0.8x – 0.02x^2$ Given x = 40 m then y = (0.8 x 40) – (0.02 x 40^2) [1] = 32 – 32 = 0 QED [1]	**[2]**	

Question	Expected Answers	Marks	Rationale
(b)	**Calculate the greatest height that the water achieves when it has travelled for half its horizontal distance.** In this case x = 20 m Height $y = 0.8x - 0.02x^2$ $= (0.8 \times 20) - (0.02 \times 20^2)$ [1] $= 16 - 8$ $= 8$ m [1]	**[2]**	
(c)	**The water just goes over a 3 m high wall. Substitute y = 3 m in the equation, and show that this can be arranged as $2x^2 - 80x + 300 = 0$** Given $y = 0.8x - 0.02x^2$ In this case y = 3 m So $3 = 0.8x - 0.02x^2$ [1] Transpose so $0.02x^2 - 0.8x + 3 = 0$ Multiply through by 100 then $2x^2 - 80x + 300 = 0$ QED [1]	**[2]**	

Question	Expected Answers	Marks	Rationale
(d)	**Calculate, correct to 2 decimal places, the distance of the wall from point A** Solution of quadratic equation by formulae [1] $x = [-b \pm \sqrt{(b^2 - 4ac)}]/2a$ where a =1, b = - 40 and c = 150 so $x = [40 \pm \sqrt{(40^2 - 4 \times 1 \times 150)}]/(2 \times 1)$ [1] $x = [40 \pm \sqrt{(1600 - 600]}/2$ $x = [40 \pm \sqrt{1000}]/2$ $x = [40 \pm 31.62]/2$ x = 35.81 correct to 2dp or x = 4.19 correct to 2dp By inspection of Fig.3 the wall is in the second half of the parabola so x = 35.81 m [1]	**[3]**	

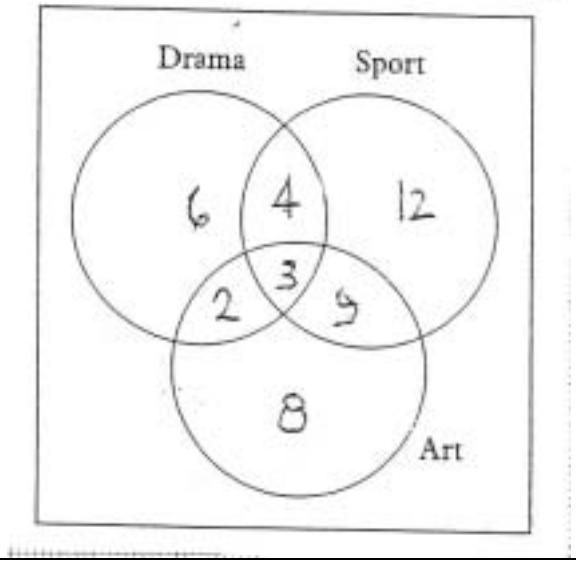
Question	Expected Answers	Marks	Rationale
(e)	**The hose pipe is now moved from point A to point B. Calculate the distance that the hose pipe is moved** distance that the hose pipe is moved = 40 – 35.81 = 4.19 m [1]	**[1]**	
			[Total: 10]
5 (a)	**Draw a normal distribution curve showing the position of the mean value.**  MS Fig. 1 Bell shape [1] Symmetry either side of the centre line [1]	**[2]**	
(b)	**State the percentage area under the normal curve bounded by**		
(i)	**one standard deviation on either side of the mean**		
	68.2% [1]	**[1]**	

Question	Expected Answers	Marks	Rationale
(ii)	**two standard deviations on either side of the mean**		
	95.44 % [1]	**[1]**	
(c)	**Draw a histogram and a frequency polygon using the information provided in the table**		
	 MS Fig. 2 Correct horizontal and vertical axis [1] Correct six rectangles for histogram [1] Correct six dots for frequency polygon [1] Correct five lines joining up the dots [1]	**[4]**	

Question	Expected Answers	Marks	Rationale
(d)	**Explain how a frequency distribution can be changed to a cumulative frequency distribution** Add each frequency [1] to the total of its predecessors [1]	**[2]**	
			[Total: 10]
(6) (a)	**The equation $S\ T^a\ f^b = c$ is used to find the expected life of a grinding tool where S is the cutting speed, T is the tool life in hours, f is the rate of feed and a, b and c are constants.**		
(i)	**Transpose the equation to make T^a the subject** Given equation $S\ T^a\ f^b = c$ Divide both sides by $S\ f^b$ [1] Gives $S\ T^a\ f^b / S\ f^b = c / S\ f^b$ Cancel leaving $T^a = c / Sf^b$ [1]	**[2]**	
(ii)	**Calculate, the tool life correct to 1 decimal place when S = 3, a = 2, b = 0.3, c = 1900 and f = 0.01** Substitute in formula S = 3, b = 0.3, c = 1900 and f = 0.01 So $T^a = 1900/(0.01^{0.3} \times 3)$ [1] $T^2 = 2521$ [1] So $T = \sqrt{2521}$ T = 50.2 hours correct to 1dp [1]	**[3]**	

Question	Expected Answers	Marks	Rationale
(b)	**Solve the equation [(2x + 5)/(x – 3)] = [(6x + 4)/(3x – 1)]** Cross multiply (2x + 5)(3x – 1) = (x – 3)(6x + 4) [1] Open brackets $6x^2$ - 2x + 15x -5 = $6x^2$ + 4x – 18x – 12 [1] 13x – 5 = -14x - 12 [1] 27x = -7 [1] x = -7/27 x = - 0.26 correct to 2dp [1]	**[5]**	
			[Total: 10]
7 (a)	**The three sides of a triangle ABC have dimensions a = 125 mm, b = 140 mm and c = 230 mm.**		
(i)	**State the cosine rule making Cos A the subject.** Given a = 125 mm, b = 140 mm and c = 230 mm. Cosine Rule: $a^2 = b^2 + c^2 – 2bc \cos A$ So cos A = $(b^2 + c^2 – a^2)/2bc$ [1]		
(ii)	**Determine, correct to 2 decimal places, the value of angle A.** Given a = 125 mm, b = 140 mm and c = 230 mm. cos A = $(b^2 + c^2 – a^2)/2bc$ = $(140^2 + 230^2 – 125^2)/(2 \times 140 \times 230)$ = 0.8832 Then angle A = $\cos^{-1}$ 0.8832 = 27.97° correct to 2dp [1]		

Question	Expected Answers	Marks	Rationale
(iii)	**Determine, correct to 2 decimal places, the value of angles B and C.**		
	Cosine Rule: $b^2 = a^2 + c^2 - 2ac \cos B$ So $\cos B = (a^2 + c^2 - b^2)/2ac$ $= (125^2 + 230^2 - 140^2)/(2 \times 125 \times 230)$ [1] $= 0.8509$ [1] Then angle B $= \cos^{-1} 0.8509 = 31.69^o$ correct to 2 dp So angle C = 180 – angle A – angle B = 180 - 27.97 – 31.69 = 120.34^o correct to 2dp [1]	**[3]**	
(b)	**In a triangle ABC, angle A = 75^o, angle C = 50^o and side b = 150 mm.**		
(i)	**Determine the value of angle B.**		
	Given A = 75^o, C = 50^o and b = 150 mm Angle B = 180 – 75 – 50 = 55^o [1]	**[1]**	
(ii)	**Calculate, correct to 2 decimal places, the length of side a.** Sine Rule: $a/\sin A = b/\sin B$ $a = (b \sin A)/\sin B$ [1] $a = (150 \times \sin 75^0)/\sin 55^0$ a = 176.88 mm correct to 2dp [1]	**[2]**	

Question	Expected Answers	Marks	Rationale
(iii)	**Calculate, correct to 2 decimal places, the length of side c.**		
	Sine rule: b/sin B = c/sin C c =(b sin C)/sin B [1] c = (150 x sin 50^0)/sin 55^0 c = 140.28 mm correct to 2dp [1]	**[2]**	
			[Total: 10]
8	**The diagram in Fig.6 shows the hobbies of a group of students.** 		

Question	Expected Answers	Marks	Rationale
(a)	**Calculate the total number of students.**		
	Total number of students = 6+4+12+2+3+9+8 = 44 [1]	**[1]**	
(b)	**State the number of students who study:**		
(i)	**Drama**		
	Drama = 6+4+3+2 = 15 [1]	**[1]**	
(ii)	**Sport only**		
	Sport only = 12 [1]	**[1]**	
(c)	**State the number of students who study**		
(i)	**Drama and Sport**		
	Both drama and sport = 3+4 = 7 [1]	**[1]**	
(ii)	**Sport and Art**		
	Both sport and art = 9+3 = 12 [1]	**[1]**	
(iii)	**Sport, Drama and Art** All three = 3 [1]	**[1]**	

Question	Expected Answers	Marks	Rationale
(d)	**A student is chosen at random.**		
(i)	**State the probability that the student studies both Drama and Art.** The probability that the student studies both drama and art is 5/44 [1]	**[1]**	
(ii)	**The student studies Drama. State the probability that the student also studies Art.** The probability that the student also studies art is 5/15 i.e. 1/3 [1]	**[1]**	
(iii)	**The student studies Sport. State the probability that the student also studies Drama.** The probability that the student also studies drama 7/28 i.e. ¼ [1]	**[1]**	
(iv)	**The student studies both Drama and Art. State the probability that the student studies Drama, Art and Sport.** The probability that the student studies all three hobbies is 3/5 [1]	**[1]**	
			[Total: 10]

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