OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4758/01

Differential Equations

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Wednesday 21 January 2009
Afternoon

Duration: 1 hour 30 minutes

475801

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 The differential equation

$$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 2$$

is to be solved.

(i) Write down the auxiliary equation. Show that -2 is a root of this equation and find the other two roots. Hence write down the complementary function. **[6]**

(ii) Find the general solution. **[3]**

When $x = 0$, $y = 0$ and when $x = \ln 2$, $y = 0$. As $x \to \infty$, y tends to a finite limit.

(iii) Show that $y = -2e^{-2x} + 3e^{-x} - 1$. **[6]**

(iv) Show that $y = 0$ only when $x = 0$ or $\ln 2$. Show also that the graph of y against x has only one stationary point, and determine its coordinates. **[5]**

(v) Sketch the graph of the solution for $x \geqslant 0$. **[4]**

2 The differential equation

$$\frac{dy}{dx}\cos x + y\sin x = x\cos^2 x$$

is to be solved for $|x| < \frac{1}{2}\pi$ subject to the condition that $y = 1$ when $x = 0$.

(i) Find the solution. **[10]**

(ii) Sketch the solution curve. **[2]**

Now consider the differential equation

$$\frac{dy}{dx}\cos x + y\sin x = x\cos x\sin x$$

for $|x| < \frac{1}{2}\pi$, subject to the condition that $y = 1$ when $x = 0$.

(iii) Use Euler's method with a step length of 0.1 to estimate y when $x = 0.2$. The algorithm is given by $x_{r+1} = x_r + h$, $y_{r+1} = y_r + hy'_r$. **[6]**

(iv) Use the integrating factor method and the numerical approximation

$$\int_0^{0.2} x\tan x\,dx \approx 0.002\,688$$

to estimate y when $x = 0.2$. **[6]**

3 An oil drum of mass 60 kg is dropped from rest from a point A which is at a height of 10 m above a lake. The oil drum is modelled as a particle that moves vertically. When it is x m below A, its speed is $v\,\mathrm{m\,s^{-1}}$. Before it enters the water, the forces acting on it are its weight and a resistance force of magnitude $\frac{1}{4}v^2$ N.

(i) Show that

$$\frac{v}{240g - v^2}\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{1}{240}$$

and hence find v^2 in terms of x. **[9]**

(ii) Show that the speed of the oil drum as it reaches the water is $13.71\,\mathrm{m\,s^{-1}}$, correct to two decimal places. **[1]**

After it enters the water, the forces acting on the oil drum are its weight, a resistance force of magnitude $60v$ N and a buoyancy force of $90g$ N vertically upwards.

Assume that the initial speed in the water is $13.71\,\mathrm{m\,s^{-1}}$ and that the oil drum moves vertically.

(iii) Show that t seconds after entering the water its speed is given by $v = 18.61\mathrm{e}^{-t} - 4.9$. **[8]**

(iv) Calculate the greatest depth below the surface of the water that the oil drum reaches. **[6]**

4 The simultaneous differential equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -3x - y + 7$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = 2x - y + 2$$

are to be solved for $t \geqslant 0$.

(i) Find the values of x and y for which $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}t} = 0$. **[2]**

(ii) Show that

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4\frac{\mathrm{d}x}{\mathrm{d}t} + 5x = 5.$$ **[5]**

(iii) Find the general solution for x. **[6]**

(iv) Find the corresponding general solution for y. **[3]**

When $t = 0$, $x = 4$ and $y = 0$.

(v) Find the solutions for x and y. **[3]**

(vi) Sketch the graphs of x against t and y against t, for $t \geqslant 0$. Explain how your solution to part **(i)** relates to your graphs. **[5]**