

Principal Learning

Engineering

OCR Level 3 Principal Learning

Unit **F563**: Mathematical techniques and applications for engineers

Mark Scheme for June 2011

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Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

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Question	Expected Answer	Mark	Rationale/Additional Guidance
Section A			
1	$2a(3b - 4c) = 6ab - 8ac$ $6ab$ $-8ac$	[1] [1]	
2	$x^2 - 10x + 25 = (x - 5)(x - 5)$ $(x - 5)$ $(x - 5)$	[1] [1]	
3	$a/(b + c) + d(c + b) = a/(b + c) + d(b + c) = (a + d)/(b + c)$ $(a + d)$ $(b + c)$	[1] [1]	
4	$4x + 8 = 2(5x + 6)$ $4x - 10x = 12 - 8$ $-6x = 4$ $x = -4/6 = -0.67$	[1] [1]	
5	Length of minor arc (s) = $(\pi x^{\circ} r)/180$ $x^{\circ} = 180 s / \pi r$ $= (180 \times 40) / (\pi \times 100)$ $= 23$ correct to the nearest degree	[1] [1]	Accept 22.9 degrees

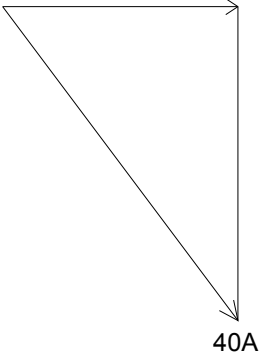
Question	Expected Answer	Mark	Rationale/Additional Guidance
Section A			
6	$\sin A = (7.82 \sin 33^\circ)/6.93$ Angle $A = \sin^{-1} (7.82 \sin 33^\circ)/6.93$ $= 38^\circ$ correct to the nearest degree	[1] [1]	Accept 37.92 degrees
7	Consider the triangle Fig.1 $b = \sqrt{1^2 + 1^2} = \sqrt{2}$ So $\sin 45^\circ = 1/\sqrt{2}$	[1] [1]	
8	$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$ $s = (a+b+c)/2 = (40+50+60)/2 = 75$ $\text{Area} = \sqrt{75(75-40)(75-50)(75-60)}$ $= 992.2 \text{ mm}^2$ correct to 1dp	[1] [1]	Accept answers between 992.1 and 992.5 mm ²
9	$y = \cos x + \sin x$ $dy/dx = -\sin x + \cos x$ $-\sin x$ $\cos x$	[1] [1]	
10	$y = 3x^2 + \ln(4x)$ $dy/dx = 6x + 1/x$ $6x$ $+ 1/x$	[1] [1]	

Question	Expected Answer	Mark	Rationale/Additional Guidance										
Section A													
11	$\int (12x^5 + 5x^4) dx = 2x^6 + x^5 + C$	[1] [1]											
12	$\int_2^4 (3x + 4) dx = [1.5x^2 + 4x]_2^4$ $= (24 + 16) - (6 + 8) = 26$	[1] [1]											
13	Relative frequency of an event = frequency of the event/total frequency which gives an estimate of probability	[1] [1]											
14 (a)	Probability of selecting a brass washer $P_b = 40/(40 + 60) = 0.4$ to 1	[1]	Accept 40/100 or 40%										
14 (b)	Probability of selecting a steel washer $P_s = 60/(60 + 40) = 0.6$ to 1	[1]	Accept 60/100 or 60%										
15	Mean value = $(3 + 8 + 10)/3 = 7$ <table style="margin-left: 40px;"> <tr> <td>x</td> <td>x-mean</td> </tr> <tr> <td>3</td> <td>-4</td> </tr> <tr> <td>8</td> <td>1</td> </tr> <tr> <td>10</td> <td>3</td> </tr> <tr> <td>Total</td> <td>0</td> </tr> </table>	x	x-mean	3	-4	8	1	10	3	Total	0	[1] [1]	Accept total 21 and x - mean of 14
x	x-mean												
3	-4												
8	1												
10	3												
Total	0												
	Total	[30]											

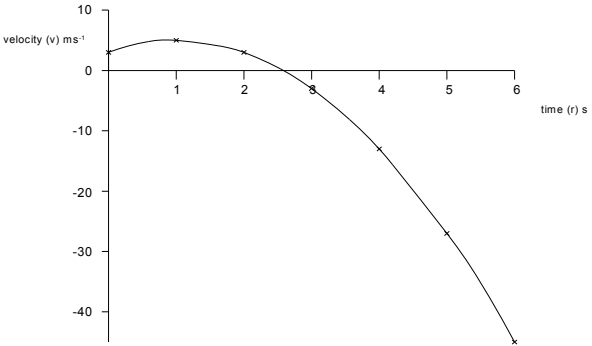
Question		Expected Answer	Mark	Rationale/Additional Guidance
Section B				
1	(a) (i)	Given $\frac{1}{2}mv^2 = mgh + \frac{1}{2}mu^2$ Divide both sides by m then $\frac{1}{2}v^2 = gh + \frac{1}{2}u^2$ Subtract both sides by gh then $\frac{1}{2}u^2 = \frac{1}{2}v^2 - gh$ Divide through by $\frac{1}{2}$ then $u^2 = v^2 - 2gh$ Square root both sides then $u = \sqrt{v^2 - 2gh}$	 [1] [1] [1] [1]	
	(a) (ii)	Given $u = \sqrt{v^2 - 2gh}$ Substitute $g = 9.8$, $v = 30$ and $h = 25$ Then $u = \sqrt{30^2 - 2 \times 9.8 \times 25}$ $u = 20.3$ m s correct to 1dp	 [1] [1]	Accept answers between 20.2 and 20.3
	(b)	Given $1/R = 1/R_1 + 1/R_2$ Subtract both sides by $1/R_1$ then $1/R_2 = 1/R - 1/R_1$ Common denominator RR_1 for right hand expression gives $1/R_2 = (R_1 - R)/RR_1$ Then $R_2 = RR_1/(R_1 - R)$	 [1] [1] [1] [1]	
		Total	[10]	

Question	Expected Answer	Mark	Rationale/Additional Guidance
2 (a)	<p>Given $P + Q = 36$ Eqn. 1</p> <p>$15P = 3Q$ Eqn. 2</p> <p>From Eqn. 2 $P = 3Q/15 = 0.2 Q$</p> <p>Substitute $P = 0.2 Q$ into Eqn. 1</p> <p>So $0.2 Q + Q = 36$</p> <p>Then $1.2 Q = 36$</p> <p>So $Q = 36/1.2 = 30$ N</p> <p>Substitute $Q = 30$ into Eqn. 2</p> <p>$15 P = 3 Q$</p> <p>So $15 P = 3 \times 30 = 90$</p> <p>So $P = 90/15 = 6$ N</p> <p>Accept a correct alternative method of solution by substitution</p>	<p>[1]</p> <p>[1]</p> <p>[1]</p> <p>[1]</p>	
(b) (i)	<p>Given equations $4F_1 - 2F_2 = 18$ Eqn. 1</p> <p>$6F_1 - 14F_2 = 38$ Eqn. 2</p> <p>Multiply Eqn. 1 x 7 $28F_1 - 14F_2 = 126$ Eqn. 3</p> <p>Multiply Eqn. 2 x 1 $6F_1 - 14F_2 = 38$ Eqn. 4</p> <p>Subtract Eqn. 4 from Eqn. 3.</p>	[1]	

Question			Expected Answer	Mark	Rationale/Additional Guidance
			$22F_1 = 88$ <p style="text-align: center;">So $F_1 = 88/22 = 4$</p> <p>N</p> <p>Substitute $F_1 = 4$ N into Eqn 1.</p> <p>So $(4 \times 4) - 2F_2 = 18$</p> <p style="text-align: center;">$16 - 2F_2 = 18$</p> <p>Subtract both sides by 16 so $-2F_2 = 18 - 16 = 2$</p> <p>So $F_2 = 2/-2 = -1$ N</p>	[1]	Accept error carried forward for F_1 to calculate F_2
			[1]		
			[1]		
	(b)	(ii)	<p>Given $F_1 = 4$ N and $F_2 = -1$ N</p> <p style="text-align: center;">Substitute into Eqn. 1 $4F_1 - 2F_2 = 18$</p> <p style="text-align: center;">$(4 \times 4) - (2 \times -1) = 18$ QED</p> <p>Substitute into Eqn. 2 $6F_1 - 14F_2 = 38$</p> <p>So $(6 \times 4) - (14 \times -1) = 38$</p> <p style="text-align: center;">$24 + 14 = 38$ QED</p>	[1]	Accept errors carried forward for the check
			[1]		
			Total	[10]	

Question		Expected Answer	Mark	Rationale/Additional Guidance
3	(a)	Fig. 2 	[1]	
	(b)	$\cos \theta = 0.6$ So $\theta = \sin^{-1} 0.6 = 53.13^\circ$	[1]	
	(c)	From the phasor diagram $I_c = I_L \sin \theta$ So $I_c = 40 \sin 53.13^\circ$ $I_c = 32 \text{ A}$	[1] [1] [1]	
	(d)	Capacitive reactance $X_c = V / I_c$ So $X_c = 230 / 32 = 7.1875 \Omega$	[1] [1]	
	(e)	Capacitance $C = 1 / (2\pi f X_c)$ So $C = 1 / (2\pi \times 50 \times 7.1875)$ $C = 4.4286 \times 10^{-4} \text{ F} = 443 \mu\text{F}$ to the nearest whole μF	[1] [1] [1]	Accept 442 to 444
		Total	[10]	

Question		Expected Answer	Mark	Rationale/Additional Guidance
4	(a)	$\tan P = 105/r$ So $r = 105/\tan 36^\circ$ $r = 144.5$ m correct to 1dp	[1] [1] [1]	When $Q = 90^\circ$ accept responses for r between 144.4 and 144.6 If angle Q is not taken as 90° Accept an alternative response for 2 marks
	(b) (i)	Angle $ABC = 90^\circ + 15^\circ = 105^\circ$	[1]	
	(ii)	$AC^2 = BC^2 + AB^2 - 2 \cdot AC \cdot AB \cdot \cos ABC$ $= 5^2 + 4^2 - 2 \times 5 \times 4 \cos 105^\circ$ $= 51.4$ So $AC = \sqrt{51.4} = 7.2$ m correct to 1dp	[1] [1] [1]	Accept ECF from (b)(i) Accept responses between 7.1 and 7.2 meters
	(iii)	$4/\sin C = b/\sin 105$ So $\sin C = (4 \times \sin 105^\circ)/7.2$ $= 0.5366$ So Angle $ACB = \sin^{-1} 0.5366 = 33^\circ$ correct to the nearest degree	[1] [1] [1]	Accept ECF from (b)(i) and (b) (ii) Accept responses between 32 and 33 degrees
		Total	[10]	

Question	Expected Answer	Mark	Rationale/Additional Guidance
<p>5 (a)</p>	<p>Time (t) s 0 1 2 3 4 5 6</p> <p>Velocity (v) ms⁻¹ 3 5 3 -3 -13 -27 -45</p> 	<p>[2]</p>	<p>Award one mark for correct completion of the table</p> <p>Award one mark for correctly plotting the graph</p>
<p>(b)</p>	<p>The time when the graph crosses the horizontal axis at the point of zero velocity is 2.6 s.</p>	<p>[1]</p>	<p>Accept 2.5 – 2.7 seconds</p> <p>Accept ECF from 5 (a)</p>
<p>(c)</p>	<p>Area below the horizontal axis = $\int_{2.6}^6 -2t^2 + 4t + 3 \, dt$</p> <p>$= \left[-\frac{2}{3}t^3 + 2t^2 + 3t \right]_{2.6}^6$</p> <p>$= (-144 + 72 + 18) - (-11.717 + 13.52 + 7.8)$</p> <p>$= 63.6$</p>	<p>[1]</p> <p>[2]</p> <p>[1]</p>	<p>Accept ECF from 5 (a) and (b)</p>

Question		Expected Answer	Mark	Rationale/Additional Guidance
	(d)	Distance travelled = $9.6 + 63.6 = 73.2$ m.	[3]	Accept ECF from 5 (a) and (b) and (c)
		Total	[10]	

Question		Expected Answer	Mark	Rationale/Additional Guidance
6	(a)	<p>Given equation $\theta = \sin 4t$ (Remember radian measure)</p> <p>Angular velocity = $d\theta/dt$</p> <p>So $d\theta/dt = 4 \cos 4t$</p> <p>When $t = 2.5$ s then $d\theta/dt = 4 \cos (4 \times 2.5)$</p> <p style="padding-left: 40px;">$= 4 \cos 10$ (Remember radian measure)</p> <p style="padding-left: 40px;">$= -3.36$ correct to 2dp.</p>	<p>[1]</p> <p>[1]</p> <p>[1]</p>	Accept responses between -3.35 to -3.36
	(b)	<p>Angular velocity = $d\theta/dt = 4 \cos 4t$</p> <p>Now $d\theta/dt = 3$</p> <p>So $3 = 4 \cos 4t$</p> <p>Then $\frac{3}{4} = \cos 4t$</p> <p>Then $4t = \cos^{-1} \frac{3}{4}$</p> <p style="padding-left: 40px;">$4t = 0.7227$</p> <p style="padding-left: 40px;">$t = 0.18$ s correct to 2 dp.</p>	<p>[1]</p> <p>[1]</p>	Accept responses between 0.18 to 0.185

Question		Expected Answer	Mark	Rationale/Additional Guidance
6	(c)	Angular velocity = $d\theta/dt = 4 \cos 4t$ Angular acceleration = $d^2\theta/dt^2$ So $d^2\theta/dt^2 = -16 \sin 4t$ When $t = 2s$ then $d^2\theta/dt^2 = -16 \sin 8$ $= -15.83 \text{ rad s}^{-2}$ correct to 2dp.	 [1] [1] [1]	Accept responses between 15.8 to 15.9
	(d)	Angular acceleration = $d^2\theta/dt^2 = -16 \sin 4t$ Now $d^2\theta/dt^2 = 8 \text{ rad s}^{-2}$ Then $8 = -16 \sin 4t$ Then $-0.5 = \sin 4t$ So $\sin 4t = -0.5236$ or $3.142 + 0.5236 = 3.6656$ Then $4t = 3.6656$ $t = 0.9164 \text{ s}$	 [1] [1]	Accept responses between 0.91 to 0.92
		Total	[10]	

Question	Expected Answer	Mark	Rationale/Additional Guidance
7 (a)	<p>The addition law of probability is recognised by the word “OR” joining the probabilities. The events are mutually exclusive.</p> <p>Let P_x be the probability of event x happening and let P_y be the probability of event y happening.</p> <p>Then the probability of event x or event y happening is given by $P_x + P_y$.</p> <p>The multiplication law of probability is recognised by the word “AND” joining the probabilities. The events are independent.</p> <p>Let p_x be the probability of event x happening and let P_y be the probability of event y happening</p> <p>Then the probability of event x and event y happening is given by $P_x \times P_y$.</p>	<p>[1]</p> <p>[1]</p> <p>[1]</p> <p>[1]</p>	

Question	Expected Answer	Mark	Rationale/Additional Guidance																																																
7 (b)	<p>Two round head screws (R) and one flat head screw (F) can be found in any of the following ways:</p> <table data-bbox="427 312 1003 440"> <tr> <td>First draw</td> <td>Second draw</td> <td>Third draw</td> </tr> <tr> <td>R</td> <td>R</td> <td>F</td> </tr> <tr> <td>R</td> <td>F</td> <td>R</td> </tr> <tr> <td>F</td> <td>R</td> <td>R</td> </tr> </table> <p>Two round head screws (R) and one pozidrive screw (P) can also be obtained in any of the following ways:</p> <table data-bbox="427 579 1003 707"> <tr> <td>First draw</td> <td>Second draw</td> <td>Third draw</td> </tr> <tr> <td>R</td> <td>R</td> <td>P</td> </tr> <tr> <td>R</td> <td>P</td> <td>R</td> </tr> <tr> <td>P</td> <td>R</td> <td>R</td> </tr> </table> <p>There are now six ways to achieve the combinations specified. The probability calculations are shown in the table.</p> <table data-bbox="427 855 1003 1126"> <tr> <td>First draw</td> <td>Second draw</td> <td>Third draw</td> </tr> <tr> <td>R</td> <td>R</td> <td>F</td> </tr> <tr> <td>R</td> <td>F</td> <td>R</td> </tr> <tr> <td>F</td> <td>R</td> <td>R</td> </tr> <tr> <td>R</td> <td>R</td> <td>P</td> </tr> <tr> <td>R</td> <td>P</td> <td>R</td> </tr> <tr> <td>P</td> <td>R</td> <td>R</td> </tr> <tr> <td>[1]</td> <td>[1]</td> <td>[1]</td> </tr> </table> <p>The probability of having the first combination or the second or the third is given by the addition of the probabilities.</p> <p>Probability = (3 x 0.06349) + (3 x 0.02506) = 0.266 correct to 3dp.</p>	First draw	Second draw	Third draw	R	R	F	R	F	R	F	R	R	First draw	Second draw	Third draw	R	R	P	R	P	R	P	R	R	First draw	Second draw	Third draw	R	R	F	R	F	R	F	R	R	R	R	P	R	P	R	P	R	R	[1]	[1]	[1]	<p>[1]</p>	<p>Probability</p> <p>$64/170 \times 63/169 \times 76/168 = 0.06349$</p> <p>$64/170 \times 76/169 \times 63/168 = 0.06349$</p> <p>$76/170 \times 64/169 \times 63/168 = 0.06349$</p> <p>$64/170 \times 63/169 \times 30/168 = 0.02506$</p> <p>$64/170 \times 30/169 \times 63/168 = 0.02506$</p> <p>$30/170 \times 64/169 \times 63/168 = 0.02506$</p> <p>[2]</p>
First draw	Second draw	Third draw																																																	
R	R	F																																																	
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[1]	[1]	[1]																																																	
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Question		Expected Answer			Mark	Rationale/Additional Guidance																					
8	(a)	Machine 1 produces ribbons that are closer to 900mm in length on average but the lengths are less consistent than machine 2. Machine one could produce ribbons less than 900 mm but this is less likely with machine 2. Remove machine 1			[2]																						
	(b)	(i)	<table border="1"> <thead> <tr> <th>Age of machine (years)</th> <th>Frequency</th> <th>Cumulative frequency</th> </tr> </thead> <tbody> <tr> <td>2</td> <td>4</td> <td>4</td> </tr> <tr> <td>2.5</td> <td>8</td> <td>12</td> </tr> <tr> <td>3</td> <td>14</td> <td>26</td> </tr> <tr> <td>3.5</td> <td>14</td> <td>40</td> </tr> <tr> <td>4</td> <td>5</td> <td>45</td> </tr> <tr> <td>4.5</td> <td>3</td> <td>48</td> </tr> </tbody> </table>	Age of machine (years)	Frequency	Cumulative frequency	2	4	4	2.5	8	12	3	14	26	3.5	14	40	4	5	45	4.5	3	48		[1]	
Age of machine (years)	Frequency	Cumulative frequency																									
2	4	4																									
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3	14	26																									
3.5	14	40																									
4	5	45																									
4.5	3	48																									
		(ii)			[3]																						
		(iii)	Median time = 2.95 years Accept a value between 2.85 and 3.05		[1]																						
		(iv)	Lower quartile = 2.5 years Accept a value between 2.4 and 2.6 Upper quartile = 3.4 years Accept a value between 3.3 and 3.5		[1] [1]																						
		(v)	Inter-quartile range = 3.4 – 2.5 = 0.9 years		[1]																						
			Total		[10]																						

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