

Mathematics (MEI)

Advanced GCE A2 7895-8

Advanced Subsidiary GCE AS 3895-8

OCR Report to Centres

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Reports should be read in conjunction with the published question papers and mark schemes for the examination.

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4751 Introduction to Advanced Mathematics

General Comments

Most candidates were able to answer most of the questions in Section A competently, with the exception of question 9 where relatively few scored both marks in either part. In section B, the questions proved to be accessible with many candidates attempting every part; however, question 12(iv) caused difficulties for most candidates as did the last part of question 10(iii). Examiners felt that those who did not attempt question 12(iv) did so because they did not know how to proceed rather than because they had run out of time.

Candidates' arithmetic with negative numbers and fractions was often poor, affecting in particular their work in questions 10(i) and 8 respectively.

A number of candidates used additional pages, with the most common questions for 'overflow' or second attempts being questions 10(ii) and 11(i).

Comments on Individual Questions

Section A

- 1 Most candidates knew how to find the equation of the perpendicular line. The main errors were using the same gradient of 5 or using the incorrect perpendicular gradient $\frac{1}{5}$. A few failed to write the equation in the requested form.
- 2 This was done well by most candidates. A minority did not deal correctly with the negative fractional index in part (i). In part (ii) numbers other than the correct 32 were sometimes seen, as was x^7 instead of x^{10} .
- 3 Most candidates preferred multiplying out to using the binomial expansion. The special case in the mark scheme for an answer of $3n^2 + 6n + 4$ had significant usage, from candidates treating an expression as if it was an equation. Some forgot after their cubing to subtract the n^3 as the question required. A few candidates thought that $(n + 2)^3 = n^3 + 8$.
- 4 Many candidates gained two of the three marks in part (i), but incorrectly dealing with $3\sqrt{2} \times -2\sqrt{2}$ was the usual error in the expansion.

In part (ii) many got as far as $\sqrt{54} = 3\sqrt{6}$ but were unable to relate the second term to this. A common error was to multiply everything by $\sqrt{6}$.
- 5 Solving the inequality was done well by many candidates. Some lost the final mark, usually those whose previous line was $-3x < 14$ rather than collecting the x terms on the right and obtaining $-14 < 3x$. The follow-through provisions in the mark scheme were much used and enabled almost all candidates to obtain some marks, as mistakes in handling the fractions and expanding brackets were common.
- 6 This rearrangement was done well by many candidates. Almost all attempted to collect terms, some making a sign error in the process. A minority did not then factorise the h terms. A few obtained the correct answer and then attempted to simplify it further and lost the final mark.

- 7 The translations were almost always correct. A few went down instead of up, or right instead of left, and received partial credit.
- 8 This question demonstrated again that most candidates have difficulties with completing the square. Most obtained a mark for 5 and many a mark for $\frac{3}{2}$, but only a minority had $+0.75$ or $+\frac{3}{4}$. Arithmetic errors were common, as were lack of brackets resulting in their $\left(\frac{3}{2}\right)^2$ not being multiplied by 5; sometimes brackets were used correctly and a method mark obtained but then the candidate omitted to multiply this term by 5. The final mark was sometimes obtained as a follow through mark, but many candidates gave the coordinates of the minimum point instead of the minimum value of y asked for, and others omitted this part of the question.
- 9 Few candidates obtained full marks on this question. Some omitted crucial albeit simple step(s) in an argument and a common fault in part (i) was to write, for example, 'If n is an even integer $n^3 + 1$ is an odd number' without explaining why this was so. Those who solved the inequality correctly in part (ii) then had only to select the correct symbol to obtain both marks but only a minority achieved this. In both parts, counterexamples were used effectively by some candidates; however, many thought it was sufficient to use one or two numerical examples when it was not.

Section B

- 10 (i) Finding the equation of the line through A and B was completed successfully by the majority of candidates. The main error seen was a sign error, either in working out the gradient (negative divided by negative given as negative instead of positive), or in expanding brackets and collecting terms.
- 10 (ii) Most tried to use gradients to show ABC was a right-angle. Many just stated 'grad BC = $-\frac{1}{3}$ ' without showing the calculation. The $m_1m_2 = -1$ rule was well used on the whole, although not always explicitly stated, with some just saying that 3 and $-\frac{1}{3}$ were perpendicular gradients. Those using Pythagoras to show angle ABC = 90° were often successful, but some lost a mark due to incorrect notation and / or lack of convincing steps, with $\sqrt{40} + \sqrt{10} = \sqrt{50}$ being seen on a number of scripts. A small number of candidates successfully found the equation of a line perpendicular to AB that went through C and then confirmed that B lay on this line. Some candidates worked very hard for their two marks, unnecessarily finding equations as they had not spotted the more direct methods. These more long-winded approaches were variable in terms of accuracy.

For the area, many correctly found the lengths needed but failed to simplify the surds to obtain 10. The alternative method of a rectangle minus three triangles was seen very occasionally.

- 10 (iii) Most found the mid-point of AC correctly but failed to score the final mark, with some omitting an explanation. Most successful explanations involved showing ABC was in a semi-circle, but many of these attempts did not mention diameter or semi-circle and were not sufficiently clear to score the mark. Some successfully showed that the right-angled triangle formed half of a rectangle with D as the centre and hence the same distance from A, B and C. The most common explanation was stating that 'D was the midpoint of the hypotenuse of a right angled triangle' (or words to that effect), which did not score. Weak attempts included the assumption that ABC was isosceles or that BD was the perpendicular bisector of AC or that A, B and C were three corners of a square.

- 11 (i)** Factorising the cubic was generally done very well. For the first demand, some did not use $f(-3)$ but divided successfully, although such candidates did not always conclude that finding the factor $(x + 3)$ meant that $x = -3$ was a root. The division, whether by long division or inspection, was generally done well. Candidates seem well-practised in this technique. When the quadratic had been arrived at correctly, the majority of candidates successfully found the two linear factors, although some using the formula failed to express the factors, hence losing the final two marks. The factor theorem was occasionally used to find the remaining factors, but generally did not lead to all factors being found. Some candidates confused the ideas of roots and factors.
- 11 (ii)** Sketching the cubic was well done by most candidates. A few forgot to show the y -intercept but most knew the correct shape and used their roots to show intersections.
- 11 (iii)** Most knew they had to equate the linear and cubic expressions for y and usually simplified to the correct cubic equation. Many were then unsure how to solve this. Many lost the $x = 0$ root by dividing by x , although these candidates often found the other two roots successfully. Some candidates started with the factorised form of $f(x)$ and divided both sides by $x + 3$; many of these lost the $x = -3$ root. Some tried to use the quadratic formula on the cubic equation.
- 12 (i)** The majority of candidates were able to write down the centre and radius successfully. Some radii were given as 20 instead of $\sqrt{20}$, and some centres as $(-2, 0)$ or $(0, 2)$ instead of $(2, 0)$.
- 12 (ii)** A significant minority of candidates forgot to find the negative square root when solving $y^2 = 16$ and so only found one intersection, but on the whole this was well done. Some also found where the circle cut the x -axis. Most sketched correctly, showing the y -intercepts found and their centre was correctly placed. However, a significant number of candidates took little care over their sketches, with many "circles" drawn poorly.
- 12 (iii)** Candidates who substituted $y = 2x + k$ into the equation of the circle were generally very successful, with only a few minor slips. However, candidates who decided to work backwards from the given result usually struggled.
- 12 (iv)** Many candidates did not know where to start, and the full four marks were rarely awarded. About a quarter of the candidates did not attempt the question and those that did make an attempt often substituted $x = 2$ or $x = 0$ at the start. Some successfully used $b^2 - 4ac = 0$ and found the correct values of k but many made errors, particularly taking c as -16 instead of $k^2 - 16$.

Some candidates found the equation of the normal, although few made further progress with this approach. A few candidates offered solutions using the gradient of the normal and finding the intersections with the circle by using a vector approach from the centre – a neat approach which usually scored full marks.

4752 Concepts for Advanced Mathematics

General Comments

Solutions were often concise and clearly set out, and by and large excellent use was made of electronic calculators. Nevertheless, some candidates lost easy marks by showing insufficient detail of their working, particularly when responding to a “show that” request, or by misquoting standard formulae.

Comments on Individual Questions

- 1 The overwhelming majority of candidates scored full marks on this question. A small minority either began summing from 1, or went as far as 8, and did not score. A few simply listed the terms and did not score, and a tiny fraction made errors with the arithmetic. A very small number of candidates tried to use formulae for arithmetic progressions or for geometric progressions.
- 2 This question was done very well, with many candidates obtaining full marks. Some lost an easy mark because they failed to simplify $10 \div 2.5$. Careless mistakes included the omission of “+ c” and $10 \div 2.5 = 25$. A more surprising error was arriving at $4^{2.5}/2.5$. A small number of candidates differentiated instead of integrating.
- 3 A significant minority of candidates factorised the expression and concluded that $0 < x < 7$, thus failing to score. The most common approach was to differentiate and most went on to obtain 3.5. The majority gave the correct answer, but some spoiled earlier work by giving answers such as $-3.5 < x < 3.5$ or $0 < x < 3.5$. A few candidates made sign errors and lost the last mark.
- 4 (i) Most candidates gave the correct answer. The most common error was $\log_a 1 = 1$.
- 4 (ii) Not quite so many were successful with this part. $\log_a a = 1$ and $1^{18} = 1$ was a surprisingly common error, 9 and 729 were even more common.
- 4 (iii) Most obtained the correct answer. 1 or 0 were the most common incorrect responses.
- 5 (i) This was done very well. Of those who were unsuccessful, nearly all realised that “2” was relevant, giving the answer as $\sin 2x$ or $\frac{1}{2}\sin x$.
- 5 (ii) Again, this was very well done. A few unsuccessful candidates gave the answer as $\sin 2x$ or $\sin \frac{1}{4}x$. Occasionally $\frac{1}{2}\sin x$ or $2 \sin x$ were seen.
- 6 Many candidates made the correct initial move and went on to correctly find the answer to the required precision. Only a few lost the accuracy mark due to inappropriate rounding (usually 2 d.p.) or poor calculator work. A surprisingly large number of candidates started with $\log 235 \times \log 5^x$, and didn’t score any marks. A few made the double error $\log 235 \times \log 5^x = \log 987$ so $\log 5^x = \log 987 - \log 235$, and went on to fortuitously obtain 0.892 for no marks. Other mistakes included $1180^x = 987$ and $\log 235 + 5^x = \log 987$.

- 7** The vast majority of candidates made the wrong initial move and obtained $\log y = \log a + \log x^b$. Of those who did earn the first method mark for $x^b = y - a$, a disappointing proportion failed to progress, writing either $\log x^b = \log y - a$ or $\log y - \log a$. Most earned a SC1 for $\log x$ appearing at some point.
- 8** The vast majority of candidates were comfortable with substituting $\sin^2\theta = 1 - \cos^2\theta$ and successfully derived the required result. Many went on to obtain all three roots correctly, although weaker candidates struggled to solve the quadratic equation; and $\sin\theta = 1$ and $\sin\theta = \frac{3}{4}$ were occasionally seen. 270° was the most frequently missed root, and occasionally $90 + 48.6$ instead of $180 - 48.6$ was presented. Very few candidates found extra values in the range; even fewer worked in radians.
- 9** This was done very well indeed, with many candidates obtaining full marks. A few candidates rounded prematurely and obtained $r = 0.63$ and $b = 20.2$, and an even smaller proportion inverted r to obtain 1.6. A few candidates found r and neglected to find b . Nearly all candidates used the formula for the sum of the first 15 terms correctly. Occasionally $1 - r^{15}$ was used in the numerator, along with $r - 1$ in the denominator, and sometimes 2 was substituted instead of 15. Very few candidates resorted to summing all fifteen terms directly.
- 10** Most candidates identified two correct equations and went on to solve them simultaneously – and were generally successful. $a + 10d = 11$ was quite a common error, as was $3030 = 20(2a + 19d)$. Those who were only able to identify one equation correctly occasionally resorted to trial and improvement, and were usually unsuccessful.
- 11 (i)** Most candidates were awarded both marks, but a surprising number were unable to convince the examiners that what they were drawing was a parabola, and some drew curves which were clearly cubic. Some marked the correct intercepts, and then tried to make their curve fit, often with disastrous results. Too many candidates failed to indicate the x and y intercepts, thus losing an easy mark.
- 11 (ii)** This was very well done indeed, with most candidates scoring full marks. Occasionally $2x - 4 = 0$ so gradient = 2 was seen, and there were occasional errors in finding the value of y .
- 11 (iii)** Most knew what to do here and made it clear that they were working with the negative reciprocal of the gradient of the tangent, and showed sufficient detail of the working in obtaining the correct equation. Many went on to obtain the correct quadratic equation, although occasionally sign errors led to an incorrect term (usually $-25x$ instead of $-23x$). Most were able to solve their quadratic successfully – although a surprising number resorted to using the formula (and sometimes slipped up) instead of using the fact that one of the roots was already known and factorising. A small minority of candidates made extra work by finding and solving a quadratic in y and then substituting back for x . Not all were successful.
- 12 (i)** Many candidates did not adopt the expected approach. Rather they substituted $x = 0$ and then tried a variety of other values. This rarely earned both marks, as -3 was usually missed. A good proportion did factorise, but were then unable to complete the answer successfully, making errors such as $x^2 = -9$ or $x = \pm 9$.

- 12 (ii)** Nearly all candidates found the first and second derivatives successfully: the next move was often to substitute $x = 0$ into the second derivative to confirm the nature of the turning point. Often candidates ran out of steam at this point. Of those who set dy/dx equal to 0, many either missed the negative root, or missed the root $x = 0$ or missed both.
- 12 (iii)** On the whole this was done very well. A few candidates used the function from question 11, and some integrated from -3 to 3 instead of from 0 to 3 . A tiny number differentiated instead of integrating, or integrated their first derivative from part (ii).
- 13 (i)** The Cosine Rule was by far the most popular approach, and most candidates were successful in deriving the required result. Most were successful in finding the correct area of the sector, although a few used a radius of 5 or 6 instead of 11. The majority correctly applied $\frac{1}{2}ab\sin C$ to find the area of the triangle. Common errors were $\frac{1}{2} \times 5 \times 5$ or $\frac{1}{2} \times 7 \times 5$. Many who found the correct perpendicular height were successful in finding the area, but some made slips such as finding the area of either half or double the required triangle.
- 13 (ii)(A)** There were many excellent responses to this question. A few candidates rounded up to 44 instead of truncating to 43. A significant minority simply used 7.4×1.55 for the arc length, but generally went on to earn the method mark for dividing by 0.8. A few candidates worked with areas, or used $r\theta$ with θ in degrees, and didn't score.
- 13 (ii)(B)** The majority of those who were successful in part (A) went on to be successful in this part.

4753 Methods for Advanced Mathematics (Written Examination)

General Comments

The paper proved to be a good, fair test of candidates' attainment. All but the very weakest candidates managed to accumulate over 20 marks, and over 70% of candidates gained over half the marks. Getting over 65 marks was rare, however, and there were a number of quite demanding tests for the more able candidates. Virtually all candidates attempted all the questions and part questions. The usual variability of presentation, algebraic fluency (use of brackets, etc.) and accurate use of notation was evident.

It might be helpful to advise candidates that the answer booklets are designed to provide ample space for answers, and they should not worry if they fail to fill the space available. They should also be made aware that, in the case of offering more than one attempt at a solution, it is the last complete attempt which is marked, not the best. Sometimes this cost candidates marks – it is worth their while to indicate which attempt they wish to be marked.

One aspect of the syllabus which might be worth drawing specific attention to is transformations and their specification. Students should be encouraged to use the words *translation* (not 'move', 'shift', etc., or vector only), *one-way stretch* (not 'squash', 'squeeze', etc.), and *reflection* (not 'flip'). Descriptions which refer to coordinates (e.g. *y*-coordinates are doubled) score no marks. In fact, many of these descriptions were actually condoned in this paper, but in general will not be allowed.

Comments on Individual Questions

Section A

- 1 The derivative of $\tan x$ was usually familiar, but those candidates who started with $\sin 2x/\cos 2x$ usually got lost in algebraic complexity. A surprising number lost marks through giving the derivative of $\tan 2x$ as $\sec^2 x$, or omitting the '2' in $2 \sec^2 2x$. However, better candidates just wrote the result down.
- 2 This question was often well done. Marks were lost through omitting essential brackets, and stating that $1 + \ln x^2 = 1 + 2 \ln x$. Very occasionally, *fg* and *gf* were the wrong way round.
- 3 There was a mixed response to the question, with plenty of faultless answers, but others with errors in $v = 2 \sin \frac{1}{2} x$, e.g. $v = \sin \frac{1}{2} x$ or $-2 \sin \frac{1}{2} x$ or $\frac{1}{2} \sin \frac{1}{2} x$. Occasionally there was insufficient working to show that the given result had been established: candidates are well advised to include ample working.
- 4 This simple two-mark question was well answered, with the majority of candidates correctly identifying the counter-example $8^3 = 512$. Some candidates, however, did not understand what was meant by 'units digit'.

- 5 Candidates achieved mixed success here, with part (ii) answered a little better than part (i). Unlike in recent papers, we condoned inaccurately specified transformations, as the spirit of the question was to deduce the formula for the transformed function. In part (i), quite a few used the x -stretch after translating one to the right (instead of before). One-way stretching in the x -direction seemed to be more popular than in the y -direction. The form of the final function was often incorrect.

In part (ii), successful candidates were equally split between using a reflection in Ox (sometimes described as a one way stretch in the y -direction with scale factor -1) and a translation of π in the x -direction. The final function was a little more successfully done.

- 6 In part (i), the first two marks for finding the radius when $t = 2$ were readily achieved. Not so the next two, with some generally rather poor attempts to differentiate $20(1 - e^{-0.2t})$. Quite a few candidates substituted $t = 2$ into $e^{-0.2t}$ to get $e^{-0.4}$, then differentiated this as $-0.4e^{-0.4}$. Some simply divided their value of r by 2.

Part (ii) offered some accessible marks for stating the chain rule, and for $dA/dr = 2\pi r$. The final mark depended on getting $dr/dt = 2.68$ from part (i).

- 7 Part (i) was very well done – it is pleasing to see how well implicit differentiation is understood, and the algebra to derive the given result was generally done well.

In part (ii), many fully correct answers notwithstanding, some failed to get beyond the first M1 for $y = x^2$; others who substituted for y in the implicit function sometimes erred with $(x^2)^3 = x^5$.

- 8 Part (i) was an easy two marks for nearly all candidates. However, sometimes it was difficult to tell whether it was made clear that the point $(3, 3)$ lies on the line $y = x$.

In part (ii), both the product and quotient rules were seen – perhaps the product rule is slightly easier to sort out in this case. Although most gained the initial M1A1 for this, the algebra required to derive the given answer, either by using a common denominator or factoring out $(x - 2)^{-1/2}$, was poorly done. Most candidates should have been able to recover to get the derivative at $x = 3$, and $4/7$ was a common mark for the part. The final mark, using this result to examine the symmetry of the function, was the preserve of more able candidates. Many thought that the P had to be a turning point for the graph to be symmetrical about $y = x$.

Part (iii) achieved mixed success. It was pleasing to see that most gained the B1 for $du = dx$; most got the second B1 for $(u + 2)\sqrt{u}$; thereafter, the 'M' for splitting the fraction was often lost – some used integration by parts here with some success (a sledgehammer to crack a nut?). Those who got beyond this hurdle often gained all 6 marks. The final 3 marks were often omitted, but the best candidates got all 9 marks; the most common error here was to use the triangle with vertices $(0, 0)$, $(11, 0)$ and $(11, 11)$ rather than the trapezium formed by removing the triangle with vertices $(0, 0)$, $(3, 0)$ and $(3, 3)$.

- 9 Part (i) offered two straightforward marks. Many approximated for $\ln(4/3)$, but we ignored this in subsequent working.

In part (ii), the hint proved valuable and was taken by nearly all candidates. However, many found the derivative of $\ln(2x)$ as $1/(2x)$ and lost two marks. Those who avoided this error usually scored all 4 marks.

Inverting the function in part (iii) was less successful than usual. This might have been caused by candidates using the 'hint' from the previous part to write $x = \ln 2y - \ln(1 + y)$, and then getting stuck. The gradient in the last part as the reciprocal of that in part (ii) was better answered than in previous papers.

Finally, part (iv) was the least well answered question. The new ' u ' limits of 1 and $2/3$ were usually present, but many lost the minus sign from $du = -e^{-x}dx$, and few gave fully convincing 'shows'. The last result was rarely done, though it was not possible to gather whether this was due to difficulty or lack of time.

4754 Applications of Advanced Mathematics (C4)

General Comments

This paper consists of two parts. Paper A was of a similar standard to that of recent years but Paper B proved to be more difficult than recent comprehensions.

In Paper A a good standard of work was seen. There were sufficient questions to challenge the more able candidates and some excellent scripts were seen. At the same time, there were sufficient accessible questions for all candidates. The most disappointing loss of marks arose from the omission of the constants of integration in 9(ii) and 9(iv). Often, when these were neither included nor found, five marks were lost.

The comprehension was well answered by able candidates and several scored full marks but the weaker and average candidates found this more difficult and seemed often not to understand what was required. Particularly disappointing was 5(iii), a relatively straightforward question, which was not often answered correctly.

- Candidates would be advised to read questions carefully and to give answers to the required degree of accuracy.
- In general, when answers are given in the question, candidates should take care to give all stages of working when establishing these given results.
- In the comprehension in particular, candidates should consider their answers carefully before writing in the answer boxes. Often crossed out work leaves little space for its replacement.
- Constants of integration should be included.

Comments on Individual Questions

Paper A

- 1 Good marks were obtained by all candidates who started with the correct partial fractions. Some candidates failed to split the fraction into three parts whilst others incorrectly started from $\frac{A}{2x-1} + \frac{B}{x^2}$ or $\frac{Ax+B}{2x-1} + \frac{C}{x^2}$. Another common error was to misread $2x-1$ as $2x+1$.
- 2 Candidates used both the approach from $\text{invtan } 2\theta$ and from the double angle formula, the latter method being more common. Most candidates scored marks here. Common errors included only giving one solution (particularly in the invtan method) and failing to give the required accuracy.
- 3 Most candidates scored the first four marks with the 'R' method generally being well understood. The second part was also often fully correct. Common errors included failing to find both coordinates, using an incorrect method for finding the angle such as equating the correct expression to 1 instead of to R , using degrees instead of radians or failing to give the required accuracy.
- 4 (i) The evaluation using the trapezium rule was usually correct. There were occasional errors including the use of the wrong formula – usually only multiplying the first two terms by $\pi/16$.

- 4 (ii)** In the second part it was pleasing to see that the explanations were so much better than they often have been in the past. Many scored this mark having given a complete explanation.
- 5** There were many fully correct solutions here. Some candidates still only found the scalar product of $2\mathbf{i}-\mathbf{j}+4\mathbf{k}$ with one vector in the plane and felt that this established perpendicularity. In order to obtain full marks candidates should read the question and answer it as stated i.e....'Verify'... and then...'Hence'. Some solutions seemed to involve many mixed responses including finding the scalar product of $2\mathbf{i}-\mathbf{j}+4\mathbf{k}$ with the position vectors of points.
- 6** This question used a more unusual approach to the binomial expansion. There were many excellent solutions showing good algebra skills. Most candidates scored the first two marks for $pq=-1$ and $p(p-1)q^2/2=2$ although some candidates abandoned the question at that stage. Others failed to eliminate one of the variables or made algebraic errors. Those who found p and q correctly usually gave the correct range.
- 7** The solution was started well in a very large majority of cases with most candidates equating components. Nearly all of the problems arose as a result of missing verification/checking. In many cases, either the third pair of equations was not used, or one parameter (usually λ) was not used at all. Equating $2 = 4+\mu$ and substituting for μ in the second equation was not enough.
- 8 (i)** Many candidates correctly obtained $dy/dx = 1/t$ although not all gave a satisfactory explanation of why this was equal to $\tan \theta$. Often those who did used a diagram to help them.
- 8 (ii)** This was one part of the paper that discriminated particularly well between the good and the average candidates. The gradient of QP was usually found correctly. Many gave up at this point. There were some good solutions from the few who then proceeded to use the double angle formula to show that $2\theta = \varphi$. Although candidates seemed to have a good idea of why angle $TPQ = \theta$, the explanations and lack of geometrical statements were disappointing. Again, often those with the best explanations supported them with diagrams. Some failed to attempt this final part.
- 8 (iii)** This proved to be relatively straightforward and a full seven marks was usually scored provided that candidates were able to find the correct limits for their integral.
- 9 (i)** The differentiation was usually correct and the use of the chain rule usually lead to full marks for those that started correctly.
- 9 (ii)** The integration here was not difficult. Most candidates scored either three marks or five marks depending upon whether they included a constant of integration. It was very disappointing to see how common this error was.
- 9 (iii)** Those who started correctly with $dV/dt = -kx$ usually obtained full marks. Some candidates had given up by this point.
- 9 (iv)** The separation of variables and integration were again generally well answered by those who attempted them. As before, the constant was rarely included or found and as it was non-zero in this case, some confused attempts at the final part were seen. As a result, three marks were usually lost here. Those who did include the constant were usually successful in scoring all six marks.

Paper B

- 1** There were a lot of completely wrong answers. Some focused on the width not being constant or it becoming a circle and did not refer to the vertices being opposite vertices in polygons with an even number of sides.
- 2** Whilst this was probably the best answered question on the comprehension, there were still disappointing responses. In some cases the labelling in (i) was unclear. Sometimes areas were shaded leading one to wonder if they realised what an arc was. Some only showed one arc. Part (ii) was usually correct.
- 3** Many candidates failed to explain and give reasons sufficiently carefully despite the instruction ...*'justifying your answer carefully'*. In particular, 90° was used without stating why and without any reference to a tangent. Some referred to pentagons or assumed, without explanation, that the shape was a regular hexagon.
- 4 (i)** Those who explained from a starting point of sectors of area $\frac{1}{3}\pi r^2$ were usually successful. Many did not mention $\frac{1}{3}\pi$ or 60° .
- 4 (ii)** In part (ii) many did not realise what was required and failed to attempt to substitute for R in terms of l in the given expression. Some fortuitously found $0.765R^2$ or $0.765r^2$. As a special case, some marks were obtained by those who, instead of showing $0.765l^2$ lay in the range indicated in lines 28 and 29 as required, showed that $11.28r^2 < 12.24r^2 < 12.56r^2$ or that $2.82R^2 < 3.06097R^2 < 3.14R^2$ instead.
- 5 (i)** There were some good solutions here. Since the answer was given, the working needed to be complete in order to achieve full marks. Some candidates had squashed solutions following a lot of crossing out.
- 5 (ii)** Good solutions were seen here from some candidates – usually using $l - 2CE$ and following correct work in (i). There were also many confused attempts at showing that the side length was the given answer but using incorrect methods. Others completely omitted this part.
- 5 (iii)** This should have been an easy mark to obtain but it was probably more often wrong than right. A variety of incorrect answers were commonly seen including 21 (from $50x(\sqrt{2}-1)$).

4755 Further Concepts for Advanced Mathematics (FP1)

General Comments

The paper appeared to be largely accessible, with many good scores obtained from high quality responses. There did appear to be a surprising number of misreads this session, especially in the early questions. Notation was not always conventional and this could be unhelpful to the candidate. In particular the careful use of brackets is recommended. Candidates need to ensure that all diagrams are clear and in pencil, with minimal alterations; where such are needed they should be thoroughly erased. It seemed that there were more occasions this time where responses were put in the wrong space in the answer book. This is understandable for a candidate in full flow, but extremely unhelpful for the examiner.

Comments on Individual Questions

- 1 (i) This was usually well done, except for not uncommon misreadings of the figures, and sometimes of q , in **A** or **B**.
- 1 (ii) The majority of candidates chose to multiply out **BA** in full. Some candidates failed to produce a 3×3 matrix. The chief error was to claim that **BA** did not exist. Not many chose the economical route of considering the resulting order of a (3 by 2) matrix multiplying a (2 by 3) matrix, which saved a lot of work.
- 2 This was usually well done, with most candidates scoring full marks. Errors in finding D (as 57, or -51 , sometimes 51) were the most common.
- 3 This question was also done well. Substitution to find p was the favourite starting point, but many candidates chose to find the linear and quadratic factors first, either by inspection, long division, or matching coefficients. There were some mistakes in finding the roots of the quadratic equation, often caused by careless notation.
- 4 This was well done by many. However, some candidates tried to treat $\sum r^2(r-1)$ as $(\sum r^2)(\sum r-1)$, scoring no marks. Those that did not begin by taking out the factors n and $(n+1)$ often failed satisfactorily to complete the factorisation of their quartic in n . Another error which occurred was to write down the summation initially as $\sum r^2 - \sum r$, for which it was possible to score a maximum of 3 marks.
- 5 Most candidates chose to use the relationships $\sum \alpha$, $\sum \alpha\beta$ and $\alpha\beta\gamma$, then the sums and products of the new roots. This could lead to mistakes in the resulting expansions and substitutions. The more successful used the substitution $w = 2(z - 1)$ and achieved the required result more quickly, especially if they were conversant with the cubic expansion. This method could also lead to the error of using $w = 2z - 1$.
- 6 This type of question always differentiates between candidates. Some know the words but not always their logical sequence nor the meaning of some of their phrases. This particular question challenged the algebraic manipulation of many and not a few fudged their work to the result that they knew was wanted. It was apparent that many problems here would have been alleviated by a rigorous approach to using brackets. It was not that uncommon to see 4×3^k turn into 12^k .

- 7 (i)** Many did not write conventional co-ordinates, and are fortunate that the scheme allows $x = 0$, $y = 1/3$ etc.
- 7 (ii)** The examiners want to see three distinct equations here. Most candidates found $y = 2$, and only a few gave $x = 3$ and $x = -3$.
- 7 (iii)** For this question the mark scheme is necessarily sketchy, as many substitutions could be used. It is important that evaluations of the numerical expressions are given to demonstrate the conclusions about the approaches to the asymptotes. The conclusions are wanted in words. An algebraic argument was not a popular choice, and, where attempted, was insufficiently thorough.
- 7 (iv)** Many clear, well presented graphs were seen. Some were carelessly drawn or had incomplete annotation. Some were wrong, particularly in the left hand branch. Alterations can be difficult to decipher and need careful erasing.
- 7 (v)** This was not well answered on the whole. The best solved an equation to find the value of x where $y = 2$. As an inequality this should only be solved by multiplying by $(x^2 - 3)$ if there is an argument to explain that this expression is positive. Some candidates forgot the part of the graph in $-\sqrt{3} < x < \sqrt{3}$.
- 8 (i)** Well answered by most candidates, with circles placed in more or less the right place and with sufficient annotation on the diagram to see what was intended. The most common error was to see the centre at -4 on the real axis.
- 8 (ii)** Many candidates correctly placed A and B on the tangents from O. A small minority got them muddled up. A frequently seen mistake was to place A and B at the top and bottom of the circle. Some candidates put A and B on the real axis, forgetting that in these positions both had arguments of zero.
- 8 (iii)** Shading began to obscure some of the earlier notation. This part was often not well answered. Most often that part of the locus to the right of the circle was forgotten. Some candidates shaded inside the circle, contravening the condition $|z - 4| \geq 3$.
- 8 (iv)** Not all those who managed to place A and B correctly were able to complete this section successfully. The geometry of the diagram was not appreciated, leading to the wrong trigonometric function being used. Many candidates guessed that $\pi/4$ was the answer. Those who had A and B wrongly positioned could not score. A high proportion of candidates did not attempt a solution.
- 9 (i)** Quite a number of candidates ignored the wording of this question and simply showed by evaluation that \mathbf{R}^4 was \mathbf{I} . A surprisingly high proportion of candidates thought that \mathbf{R} represented a reflection. Those who correctly identified both the rotation and the full turn did not always make explicit that this was equivalent to the identity transformation, as represented by \mathbf{I} . There were several instances of confusion between matrices and the images of objects, making the explanation less than coherent.
- 9 (ii)** The matrix was usually correct, apart from those who believed the determinant of \mathbf{R} was -1 . The transformation was often correct, but the description frequently left out the centre of the rotation. Those who had the direction wrong in (i), where this was condoned, were penalised here. It was surprising that some candidates gave the correct transformation in this section having described \mathbf{R} as representing a reflection in part (i).

- 9 (iii)** Overall this was quite well done. Some candidates simply substituted the number 60 for θ in the given formula which scored no marks. It was interesting that quite a few thought of 60 as $\frac{2}{3}$ of 90, whence **S** became $(\frac{2}{3})\mathbf{R}$.
- 9 (iv)** Those that attempted this question mostly answered it very well, with excellent explanation. Reflections, however, could not score.
- 9 (v)** With credit available for the correct use of an incorrect **S** many candidates earned the first two marks. A small minority evaluated **SR** instead of **RS**. The final explanation was for correct transformations only, and a few candidates were perceptive enough to realise that all rotations were commutative. Claiming this for “transformations”, however, was not sufficient, nor correct.

4756 Further Methods for Advanced Mathematics (FP2)

General Comments

The overall standard of work was most impressive, with over 30% of candidates scoring 60 marks or more, and fewer than 5% scoring 20 marks or fewer. Question 1 (calculus) and Question 3 (matrices) were the best done questions, with Question 2 (complex numbers) close behind: indeed, there was a marked improvement in work on complex numbers this series. Question 4 (hyperbolic functions) was found the most difficult by some margin, while very few candidates attempted the alternative Question 5 (investigations of curves).

For future series candidates would be well advised to look for simpler methods: the quadratic formula is not the most appropriate way to solve $\lambda^2 - 7 = 0$. Also, enough detail should be given in working to convince the examiner that the candidate has validly obtained a given answer: full marks (or, indeed, very many marks at all) will not be given if it appears “as if by magic”.

This was the first series in which the paper was marked on-line, and a printed answer book used. This caused no problems. A small number of candidates used the blank spaces for Question 5 to work on other questions; it is better if they use additional sheets.

Comments on Individual Questions

1 (Calculus: polar curves, Maclaurin series, standard integral)
The mean mark on this question was about 14 out of 18.

1 (a) The cardioid in (i) was usually correct, although some could have been bigger! Errors included a sharp point at the right-hand extremity and missing the pole altogether.

In (ii), the area of the curve was very well done, and there were many concise and efficient solutions. Most knew how to deal with the integral of $\cos^2\theta$ although a few gave $\cos^3\theta/3$, and the integral of $\cos\theta$ was given as $-\sin\theta$ fairly frequently.

1 (b) Although many correct answers were seen, this was the least well done part of question 1. Many candidates wasted time by deriving the Maclaurin series for $\sin x$ and $\cos x$, although the instruction in the question was “write down”. Then most realised that they had to divide $\sin x$ by $\cos x$, although a few attempted to divide $\cos x$ by $\sin x$. However, having obtained $\frac{x - \frac{1}{6}x^3}{1 - \frac{1}{2}x^2}$, many then began to differentiate $\tan x$ repeatedly: this was often managed correctly and the required result obtained. Comparatively few reached the approximation for $\tan x$ by writing the quotient as $(x - \frac{1}{6}x^3)(1 - \frac{1}{2}x^2)^{-1}$ and using the binomial expansion.

1 (c) All but a very few realised that this was an arcsin integral: the most common error was to obtain $\frac{1}{2}\arcsin\frac{x}{2}$ rather than $2\arcsin\frac{x}{2}$ as the result. A few candidates obtained $\arcsin 2x$ and then appeared surprised that they could not evaluate $\arcsin 2$.

2 (Complex numbers: infinite series, fourth roots)
The mean mark on this question was about 13 out of 18. Work on this topic appeared significantly better than in previous series.

2 (a) Most candidates could recognise $C + jS$ as a geometric series, and sum it to infinity, although a substantial number produced a formula for the sum to n terms. It was particularly pleasing to see many candidates explicitly checking that the sum to infinity existed.

Many stopped at this point, but there was a pleasing improvement in the number of candidates who were able to realise the denominator. Those who left their expressions in terms of exponential functions as late as possible generally had less trouble with the manipulation than those who introduced trigonometric functions earlier. A very common error was to give the 1 from the numerator as part of S , which is the imaginary part, and/or get the sign wrong.

2 (b) The response to this part-question on fourth roots of a complex number was most encouraging, with well over half the candidates scoring 9 or 10 out of 10. Efficient methods were used. Common errors included: omitting z from the Argand diagram; giving the argument of z as $\pm\pi/3$ or $5\pi/6$; and quoting the modulus of the product of the fourth roots as $4 \times \sqrt[4]{2}$. A few found the sum of the roots instead.

3 (Matrices: characteristic equation, eigenvalues, eigenvectors and the Cayley-Hamilton theorem)
The mean mark on this question was about 14 out of 18.

3 (i) Well over 80% of candidates scored full marks in this part. A variety of methods were seen, including Sarrus' method and even the elegant use of elementary operations to produce a zero in an appropriate place before finding the determinant.

3 (ii) Virtually all candidates obtained the quadratic factor $\lambda^2 - 7$. Solving $\lambda^2 - 7 = 0$ caused more of a problem! The quadratic formula was often used and a few candidates gave only the positive root, or quoted the roots as ± 7 .

3 (iii) This part caused the most problems. Most knew the method to obtain an eigenvector,

although having obtained $y = -2x$ many gave the eigenvector as $\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$ or similar, as

has happened in previous series. Relatively few candidates took account of the instruction "find an eigenvector...of unit length". The intention in the last section was that candidates would observe that $\mathbf{M}^2\mathbf{v} = 25\mathbf{v}$ etc. but relatively few candidates did this: indeed, many calculated \mathbf{M}^2 and \mathbf{M}^{-1} and worked from there, even performing trigonometrical feats to produce directions in terms of angles which were inappropriate in three dimensions anyway.

3 (iv) The Cayley-Hamilton theorem was well known and this part was generally very well done, although arithmetic and sign errors were quite common. Just a few calculated \mathbf{M}^2 as a 3×3 matrix and tried to work from there.

4 (Hyperbolic functions: tanh and artanh)
This question caused the most difficulty: the mean mark was about 10 out of 18. Fewer than 10% of candidates scored full marks. Many candidates dropped many or all of their h s, writing tanh as tan etc. This was condoned!

- 4 (i)** The vast majority knew the correct exponential form of $\tanh t$ and could draw the graph, although not all included the important information that it is bounded by $y = \pm 1$.
- 4 (ii)** Many carried this out very efficiently but some new “laws” of logarithms were invented. A substantial number used the quadratic formula to find e^y from $e^{2y}(1-x) - (1+x) = 0$, which led to manipulation and sign issues. Some forgot to give the range of validity but for other candidates this gained them their only mark in this part. Quite a few candidates started with $\operatorname{artanh} x = \frac{\operatorname{arsinh} x}{\operatorname{arcosh} x}$ and their logarithmic equivalents, which was often followed by copious quantities of manipulation leading magically to the “correct” answer.
- 4 (iii)** This part was not well done. Many carried out the instruction to differentiate $\tanh y = x$ but could not relate $\operatorname{sech}^2 y$ to $1 - x^2$. Some got as far as $\cosh^2(\operatorname{artanh} x)$ which they then tried to express in exponential form. This usually succeeded only in filling the answer space, although some completely correct solutions by this route were seen.
- The differentiation of the logarithmic form given in part (ii) was rarely carried through correctly or efficiently. The easiest way is to use a (correct) log law to split the logarithm but many tried to differentiate $\frac{1+x}{1-x}$ using the quotient rule and got into a tangle, making various manipulation and sign errors.
- 4 (iv)** The vast majority used integration by parts appropriately but integrating $\frac{x}{1-x^2}$ caused considerable difficulty: indeed, many stopped at this point. Those who went further often did use the result in part (ii) to introduce logarithms, but often needed to show much more detail when producing the given answer.
- 5** (Investigations of curves)
- Few candidates attempted this question, but some good answers were seen, especially to part (i).

4758 Differential Equations (Written Examination)

General Comments

Candidates showed a good level of understanding of the methods of solving differential equations being examined in this paper. As always, the vast majority were able to solve second order linear differential equations and do so with a pleasing degree of accuracy. Unusually, this year candidates seemed to find it more difficult to decide on which three questions to attempt and many answered at least some parts of all four questions, presumably leaving it to the examiner to determine which three were their best. The first two parts of Question 1 and the middle parts of Question 2 proved to be stumbling-blocks for a significant number of candidates. These covered topics which are less routine and depended on an ability to apply syllabus and subject knowledge in less familiar scenarios.

Comments on Individual Questions

- 1** The majority of candidates seemed less than confident with the aspects of this topic that were being tested in the first three parts of the question. There was evidence that they had some relevant knowledge, but not always an ability to apply it appropriately to answer the given requests. The routine request in part (iv) was, as always, a reliable source of marks.
- 1 (i)** There were a pleasing number of well-presented solutions to this first request, although many candidates gave little, if any, justification for the signs of the terms.
- 1 (ii)** Responses to this part of the question were variable in quality. Candidates appeared to have some knowledge of what was required, but there was a lot of confusion when matching an appropriate set of values of k with the different damping situations. The sketches were rarely fully correct, with only a minority of candidates using the initial condition as the starting-point for their graph.
- 1 (iii)** Very few correct sketches were seen. The common error was not to show the initial conditions.
- 1 (iv)** This routine application of the method for the solution of a second order linear differential equation was familiar territory for the candidates and the majority earned most of the marks. The coefficients of the trigonometric terms in the particular integral, though not simple fractions, were often found correctly, displaying a pleasing accuracy in the algebra involved.
- 2** Attempts at this question were variable in quality, with a significant number of candidates successfully negotiating part (i), only to come to a grinding halt in part (ii) and omit the remaining parts.
- 2 (i)** The majority of candidates who attempted this part offered a concise and accurate solution.
- 2 (ii)** Only a minority of candidates realised that, having separated the variables, the use of partial fractions was necessary in order to integrate the integral involving P . Those who did follow this approach invariably earned most of the marks available, although often losing the final mark by not expressing t in terms of P . Those who did not use partial fractions offered a wide variety of incorrect integration methods, usually involving incorrect algebra.

- 2 (iii)** This was omitted by the significant minority of candidates who had abandoned their attempts at part (ii)
- 2 (iv)** Only a small number of candidates realised the need to differentiate the given expression for the derivative of P , in order to find the maximum *rate* of growth. The need to differentiate was highlighted by the use of italics in the question, but this strong hint was not taken.
- 2 (v)** Candidates showed themselves to be very competent at using Euler's method. Many, however, did not round their answer to a whole number, as required by the request for a population size.
- 3** This question was a popular choice, with most candidates having adequate knowledge and understanding of most of the topics covered to score some marks in each part.
- 3 (i)** Most candidates did not attempt to sketch the requested isoclines. The direction indicators were usually correct, although sometimes lacking in sufficient quantity, and often only in the right hand quadrants. A more comprehensive set of indicators would have helped in drawing the solution curves requested in part (iii).
- 3 (ii)** This did not seem to be well-understood.
- 3 (iii)** For those who had drawn adequate tangent fields in part (i), the task of sketching the two curves was a simple proposition and some very neat curves were seen. For others, who had worked only in the first and fourth quadrants in part (i), their solution curves were only partially correct.
- 3 (iv)** The integrating factor method was used well by most candidates. It was particularly pleasing to see that the need for integration by parts was not a stumbling-block, the only common errors being with signs.
- 3 (v)** Candidates had no trouble in finding the complementary function and made only numerical errors, if any, when finding a particular integral.
- 3 (vi)** Many candidates obtained the correct solution after applying the given condition. To sketch the solution curve, candidates needed to realise that for large positive values of x the exponential term was dominant, and for large negative values of x the trigonometric terms were dominant. Some recognised one or other of these, but few offered sketches that displayed both.
- 4** This question was attempted by all candidates and many scored high marks.
- 4 (i)** Candidates are very familiar with this type of question and they work through the various stages methodically. Unusually, the second order differential equation for x in terms of t was not given in the question (as a check), but it was pleasing to see that the majority worked accurately and found the correct general solution for x . A small number of candidates used the wrong form of particular integral.
- 4 (ii)** Candidates knew what to do here, and the majority correctly used the product rule in differentiating their general solution for x .
- 4 (iii)** The initial conditions were always applied, the main loss of marks being due to earlier slips in signs.
- 4 (iv)** A degree of rigour was expected in an acceptable solution to this final part. Very few candidates were able to offer convincing arguments to either of the requests.

4761 Mechanics 1

General Comments

This paper was well answered. There were few very low scores and most candidates were clearly well prepared for it. Many of them used the conventions for writing mathematics well, and so were able to communicate their intentions effectively. There were, however, some who experienced difficulty with the questions involving modelling.

There was no evidence of candidates being under time pressure.

Comments on Individual Questions

1 Motion round a smooth pulley

There was a wide spread of marks on this question. While many candidates scored full marks, there were also plenty who did not, including a few who did not know how to start. Nearly all of those who did not score full marks failed to write down two correct equations of motion, with sign errors particularly common. Some correctly used the overall equation, $5g - 3g = 8a$, to obtain $a = 2.45$ but were then unable to go on to find the tension in the string.

2 Equilibrium of an object under three forces

This was the least well answered question on the paper.

The question started with a simple geometrical request and almost all candidates were able to provide a satisfactory answer.

In the next part they were asked to draw a triangle of forces. This was not done well. Many candidates did not seem to know the meaning of the term ‘triangle of forces’ and drew an ordinary force diagram instead (which was given some credit). Those who attempted to draw a triangle of forces were often unsuccessful with incorrect arrows and labels particularly common. Another common mistake was to think that the tensions in both strings were equal.

In the third and final part, candidates were asked to calculate the tensions in the two strings. Those who had drawn a correct triangle of forces in part (ii) almost invariably went on to obtain correct answers. Most candidates, however, worked from horizontal and vertical equilibrium equations and many of them were successful although algebraic and arithmetical errors were not uncommon.

3 Motion with variable acceleration

This question was about two runners. One travelled at constant speed while the other had a two-stage motion, accelerating to maximum speed and thereafter travelling at constant speed. While this presented no difficulty to many candidates, there were others who were unable to deal with the two stages and consequently lost several marks.

In the last part, candidates were asked to show that one girl had caught up with the other at a given time. Some candidates did not seem to realise that a few words would be expected in their answers to such a question.

4 Describing motion

In part (i) of this question candidates were asked to “read” a vector equation and extract information from it. Nearly all did this well but a few did not see the point in part (i)(B) and gave an answer of 9.8 instead of 10 for g .

In part (ii) candidates were asked to use the given equation to find a displacement and most obtained full marks. The most common mistake was not to appreciate that displacement is a vector quantity.

In part (iii), candidates were asked to deduce the equation of a trajectory from the given equation, and this was very well answered.

5 Vectors

This question was well answered.

In part (i) candidates were required to show that two vectors were of the same magnitude and a large majority did so correctly.

In part (ii) they were asked to show that two vectors were parallel and most knew how to do this. However, a few made the mistake of trying to divide one vector by another.

In part (iii) candidates were asked to show two vectors on a grid and to find the angle between them. Most were able to do this but many lost a mark by not putting arrows on their vectors.

6 The stopping distance of a car

Almost all candidates got started on this question and many worked successfully through to the end and obtained full marks.

Parts (i) and (ii) required the use of *suvat* equations and $F = ma$ and a large majority of candidates obtained full marks.

In part (iii), candidates had to take a driver’s reaction time into account and many did not see how to do this. This was important for the rest of the question and a pleasing number were able to recover and score well in part (iv) and in part (v), where the car was being driven down a slope and so the stopping distance was greater. Most candidates were able to deal with motion on the slope.

The question ended with a calculation of the percentage increase in the stopping distance of the car because it is on a given slope; information which is useful for drivers.

7 Modelling the motion of a projectile

Most candidates scored quite well on this question but many dropped a few marks as they went through its various parts. It was pleasing to see that many candidates clearly understood the process of setting up a model, testing it and then refining it.

In part (i) candidates were asked to derive the standard results for the flight time and range of a projectile. This was well answered but it was also common to see marks lost because of unconvincing arguments about the time of flight. A number of candidates lost marks by missing out essential steps in the derivations; the results were given so a high standard was expected.

In part (ii), candidates used the formulae to obtain a number of values that they would need in the rest of the question. Nearly all candidates got this part right; the most common cause of losing marks was not reading the question carefully and so missing out some of the answers.

Part (iii) was the first of four places where candidates were required to make some comment; some did not appreciate that this was expected to be based on the information that had just been given in the question and made general statements instead.

In part (iv) the standard projectile model was refined by allowing a constant horizontal retardation. Candidates were required to derive a given equation for x but many omitted to do so. They then had to use the equation for a given angle of projection and comment on the result; many lost a mark by not commenting.

In part (v) candidates were required to use the model with a different angle of projection and comment on its accuracy and this led into the final part where they were asked to suggest how the model could be further improved. While a few candidates gave up before the end, most obtained some marks for these parts.

4762 Mechanics 2

General Comments

Candidates found this paper to be very accessible and many scored well on the majority of the questions. The presentation of the responses was, in general, of a pleasingly good standard. As always, it is important to stress the need for clear and labelled diagrams, particularly when dealing with forces. Armed with a good diagram, the evidence suggests that a candidate is more likely to achieve accuracy.

Comments on Individual Questions

- 1** This question tested an understanding of work, energy and power and it was pleasing to see that most candidates demonstrated a working knowledge of the principles and methods involved. In some cases, there was some confusion with the given units, but most handled successfully the necessary conversion to standard units.
- 1 (i)** The vast majority of candidates scored full marks on this question and it was pleasing to note how competently the mix of units was handled.
- 1 (ii)** Again, this was well-answered. A minority of candidates did not appreciate that a mass of 8 tonnes is equivalent to 8000 kg.
- 1 (iii)** A common error in the application of the work-energy equation was the omission of the work done by the driving force on the bus, calculated in part (i).
- 1 (iv)** About two thirds of candidates scored full marks on this response, offering solutions which indicated a good understanding of the principles involved. For the remaining candidates, the most common error was the omission of g in the weight term when applying Newton's second law of motion. The successful solutions almost invariably used $P = Fv$ followed by Newton's second law of motion. Those candidates who attempted to combine the two stages in a single expression, often seemed to confuse themselves.
- 1 (v)** The majority of candidates realised that the total force was the sum of the weight component parallel to the slope and the new resistance to motion. No marks were earned until this sum was multiplied by the constant speed. A minority of candidates did not attempt this essential multiplication and offered the total force as the answer for the power.
- 2** The standard of the presentation of the solutions to this question on centres of mass was pleasingly high. Candidates have learned to set out their calculations in a way which enables them to work through methodically and, usually, accurately. This sound approach enabled them to be unfazed by the three-dimensional shape configured in part (iii).
- 2 (i)** The vast majority of candidates scored full marks on this question. In the work of the other candidates, there was almost always the loss of a single mark resulting from a miscalculation in one of the co-ordinates of the centre of mass of one of the component parts.
- 2 (ii)** Most candidates showed that they understood which angle was needed and many correctly calculated its value. The common error was a miscalculation of one of the lengths in the right angled triangle being used.

- 2 (iii)** As in part (i), solutions were usually very well-presented and accurate. Errors were minor slips in calculations within the application of a known method.
- 2 (iv)** Again, many candidates had a clear vision of the triangle needed to calculate the required angle and did so with brevity. Others seemed to struggle with identifying the angle, a minority drawing very complicated three-dimensional diagrams that did not seem to help them. A minority of candidates made little or no attempt at any calculation.
- 3** Most candidates scored well on this question, with many aided by clear diagrams with labelled forces. In part (iii), there was evidence of some candidates trying to merge the methods of resolution and of taking moments.
- 3 (i)** Almost all candidates used resolution and the principle of moments to secure full marks.
- 3 (ii)** Many candidates scored full marks. Other candidates did not realise that the reaction at the peg was unchanged from part (i).
- 3 (iii)** There were some excellent concise solutions to this part of the question. Some candidates, however, seem to believe that there are two moments equations, one horizontal and one vertical, about a single point. Forces were resolved and then each component used in a moments equation, in what appeared to be a merging of resolution and moments ideas. Other candidates were not able to make good use of the instruction in the question that they should resolve in a *suitable* direction. Unnecessarily, the reaction at the peg was introduced into the mix.
- 4** This was the least well-answered question on the paper, with part (b) being a source of few marks for many candidates. Unsurprisingly, working in the general case, rather than with particular numerical values, proved more challenging for many.
- 4 (a) (i)** This was usually well-answered, although there was, at times, confusion between velocity and linear momentum.
- 4 (ii)** The response to this part of the question was mixed in quality. The preferred method was to use Newton's second law of motion to find the acceleration, and then use the *suvat* equations to find time and distance. For some, the confusion between velocity and linear momentum continued. A surprising number of candidates, having found the acceleration correctly, then assumed zero acceleration and used 'distance equals speed times time.'
- 4 (iii)** Candidates demonstrated their knowledge of Newton's experimental law and the principle of conservation of linear momentum, but often their efforts were hindered by algebraic or arithmetical inaccuracies.
- 4 (b) (i)** Most candidates were able to write down the components of the speed of the ball before and after the collision with the wall, but many did not assign the appropriate, or indeed any, direction to these components.
- 4 (ii)** This was the least well-answered part of the whole paper. The common response was to fill the page with spurious equations relating speeds and angles. The key was to note the deflection through 90° and use this to find an expression for $\tan \alpha$.
- 4 (iii)** Most candidates found this final mark on the paper difficult to gain. A coherent and convincing argument for the given result was required, and not often seen. A significant minority of candidates substituted particular numerical values and hoped that this was sufficient to prove the general case.

4763 Mechanics 3

General Comments

The work on this paper was generally of a very high standard, with candidates setting out their solutions clearly. Most candidates had a good working knowledge of the topics being examined, and it was pleasing to see very many confident answers. The question on circular motion was answered rather better than in the past, although some candidates would benefit from drawing a clear diagram. The topic which caused the most trouble was simple harmonic motion.

Comments on Individual Questions

1 (*Dimensional analysis*)

All parts of this question were generally answered well, with the exception of that involving conversion between systems of units.

- (i) Almost all candidates derived the dimensions of surface tension correctly.
- (ii) Only about one third of the candidates were able to carry out the conversion from one set of units to another. The most common error was to divide instead of multiply by 60^2 . Some included the conversion factor for length (100) in their calculations, even though the dimensions of surface tension do not include L.
- (iii) Most candidates could show that the given equation was dimensionally consistent. The dimensions of ρ and g were almost universally stated correctly, but some made errors in the manipulation of negative powers.
- (iv) Most found the radius accurately, but a surprising number failed to rearrange the formula correctly to give r . Many gave the reciprocal of the correct answer. Some did not convert the height 25 cm into SI units.
- (v) Almost all candidates obtained three equations by considering the powers of M, L and T, although the equation resulting from L, $-3\beta + \gamma = 1$, was quite often incorrect.
- (vi) Most candidates approached this by finding the value of k , with just a few using a ratio method. Slips were often made in the substitutions, but about half obtained the correct answer.

2 (*Circular motion*)

This question was answered somewhat better than similar questions in previous papers; in particular the work on the two strings problem in the last part was most impressive. However, many candidates did not include a clear diagram, and wrote down equations involving $\sin \theta$ and $\cos \theta$ without any indication of which angle was intended to be θ . When the final answer was incorrect, this sometimes meant that intermediate marks could not be awarded.

- (i) Most candidates considered the radial equation of motion, although there were several resolving and sign errors. Conservation of energy was also needed, to find the angle of the string, but not all candidates realised this. About 40% of the candidates obtained the correct answer.
- (ii) Almost all candidates answered this correctly, by resolving vertically to find the tension.

- (iii) Most candidates found the period of the conical pendulum correctly, although there was sometimes confusion between the speed and the angular speed.
- (iv) Almost all candidates made good attempts to resolve vertically and form the horizontal equation of motion, and about three-quarters found the two tensions correctly.

3 (*Elasticity and simple harmonic motion*)

This was found to be the hardest question. The average mark was about 10 (out of 18), compared with about 14 for each of the other questions.

- (i) Most candidates used conservation of energy to find the natural length correctly. However, about a quarter assumed that the jumper was in equilibrium at the lowest point.
- (ii) Most candidates derived the result correctly. There were sometimes sign errors in the equation of motion, and the expression for the tension was occasionally wrong, for example $T = 300(x - 2.45)$ instead of $T = 300x$.
- (iii) Many obtained the correct value, $c = 2.45$ but $c = -2.45$ was also quite frequently seen. Several candidates did not state the maximum value of y , and common incorrect values given here were 14 and 2.45. About a quarter of the candidates scored no marks in this part.
- (iv) Although the formulae $A\omega$ and $A\omega^2$ for the maximum speed and acceleration were very often stated, many candidates did not realise that the amplitude was the maximum value of y found in part (iii).
- (v) This was found to be the most difficult item on the paper. It was sometimes omitted altogether, and about one third of the candidates scored no marks. Only about 10% of the candidates answered it correctly. There were two stages to be considered: constant acceleration for a distance equal to the natural length of the rope, then simple harmonic motion while the rope was stretched. Some candidates considered only one of these stages, and several calculated the free-fall time for an incorrect distance. The second stage required some careful thought (the neatest method was to solve $11.55 \cos 2t = -2.45$) and few candidates found this time correctly; by far the most common error was to assume that this was either one quarter or one half of the period.

4 (*Centres of mass*)

The methods for finding the centres of mass of a solid of revolution and a lamina were very well understood, and a great deal of confident and accurate work was seen in this question.

- (a) About three-quarters of the candidates found the centre of mass of the solid of revolution correctly. Any errors made were usually just careless slips, sometimes involving powers of a .
- (b) (i) Most candidates found the x -coordinate correctly. When finding the y -coordinate, mistakes were quite often seen in squaring the expression for y (such as forgetting to square the 16) and in the integration. Also, the factor $\frac{1}{2}$, usually present at the start, was often dropped at some later stage in the calculation.
- (b) (ii) This part was sometimes omitted, but most candidates understood how to work with the centres of mass of a composite body. The calculations were usually carried out accurately.

4766/G241: Statistics 1

General Comments

The level of difficulty of the paper appeared to be appropriate for the candidates and there was no evidence of candidates being unable to complete the paper in the allocated time. Most candidates were well prepared for the paper and lower-scoring candidates scored marks throughout the paper, rather than on just a few questions. In general candidates supported their numerical answers with appropriate explanations and working, although in the more discursive questions, such as question 7 parts (iv) and (v), many candidates found it difficult to produce succinct answers and instead produced a rather ‘rambling’ solution. Presentation was generally satisfactory. Most candidates had adequate space in the answer booklet without having to use additional sheets, and very few candidates attempted parts of questions in answer sections intended for a different question/part. Once again many candidates over-specified some of their answers, despite recent Examiner’s reports warning against this. This was particularly the case in Question 2, where for instance many candidates gave the new mean in part (ii) as £12.903, thus losing a mark. It is pleasing to report that there was much less over-specification of probabilities than in previous sessions. It is also pleasing to report that there was less use of point probabilities in the hypothesis testing in question 6. Please note that in future papers from January 2013 onwards, the definition of p will be expected to include ‘in the population’ – see question 6(ii)A.

Comments on Individual Questions

- 1 (i)** Many candidates scored full marks in this easy start to the paper. A number of candidates omitted a key and rather more did not align the numbers well enough – particularly the last line ‘1 1 4 4 6’.
- 1 (ii)** This was another well answered question. The most common error was an answer of 7.5, as a result of forgetting to add on the stem value of 20.
- 1 (iii)** The majority of candidates stated that median was preferable and mentioned outliers or extremes. However, as these data have no clear outliers, candidates were expected to comment on the skewness of the data to gain full credit. Some candidates suggested that the mean was a better measure for these data, but credit was only given for a very convincing reason for this.
- 2 (i)** Almost all candidates found the mean correctly and the majority also found the standard deviation, although this was sometimes over specified, thus losing a mark. The main difficulty found by candidates was in using the formula for S_{xx} . Very few found the root mean square deviation rather than the standard deviation.
- 2 (ii)** Although most candidates used the correct method by multiplying their values from part (i) by 1.02, a good number lost marks for over-specification, very often giving the value of the mean as 12.903. Some candidates multiplied by 1.2 rather than 1.02. A number of candidates ‘started again’ to work out the new mean and standard deviation from scratch.
- 2 (iii)** Most candidates scored both marks by adding 0.25 to the mean and saying that the standard deviation was unaltered (from part (i)). A few candidates incorrectly added 0.25 to their original standard deviation and also occasionally 25p became £25 to give 37.65 for the mean. A few candidates used their new values from (ii) instead of their original values found in part (i).

- 3 (i)** Many candidates produced a fully correct tree diagram although a significant number added additional branches in the fifth round. Some were confused by the results in rounds 1 and 2 and tried to incorporate those, leading in most cases to significant loss of marks in parts (ii) and (iii). The probabilities 0.4/0.6 were often reversed, but 0.7/0.3 were nearly always correct. Labelling was often correct but some candidates either gave no labels or labelled the first set of branches only.
- 3 (ii)** For those with the correct tree this probability was almost universally then found correctly. Credit was given for an attempt at follow through probabilities that matched the correct form of tree diagram.
- 3 (iii)** Candidates with a correct tree almost always gained full marks and many others gained a follow through mark.
- 4 (i)** Very few candidates realised that all they had to say was, “because $P(T|M) \neq P(T)$ ”. Many candidates attempted $P(T) \times P(M) \neq P(T \cap M)$ often with success, but there was much confusion among some candidates with many incorrect statements.
- 4 (ii)** There were many correct responses, but a significant number of candidates assumed independence (despite the question stating a lack of independence) and calculated $P(T) \times P(M) = 0.1815$.
- 4 (iii)** This was done well by most candidates although often as a follow through from an incorrect $P(T \cap M)$.
- 5 (i)** Marks could only be scored if candidates wrote down the four correct alternatives, *GB, BG, BBG, BBBG* in some form, and unfortunately the majority of candidates failed to do this. Those who did so almost universally scored full marks and almost all of the rest scored zero. A disappointing number of candidates showed that $11/16$ was $(1 - \text{the sum of the remaining probabilities})$, which of course gained no credit.
- 5 (ii)** This was very well answered, with many candidates scoring the full 5 marks. A few found $E(X^2)$ and stated that that was $\text{VAR}(X)$. It was pleasing to see that far fewer candidates than in previous years divided the mean and/or the variance by 5, or by other spurious factors.
- 6 (i) (A)** The majority of candidates used the binomial formula rather than tables, but most answers were correct by either method, except for occasional over specification.
- 6 (i) (B)** Many correct answers were seen from tables, but $P(X \leq 6) - P(X \leq 3)$ was a fairly common error, gaining just one mark and $P(X \leq 6) \times P(X \geq 3)$ was occasionally seen. Some candidates added individual probabilities often successfully.
- 6 (i) (C)** Generally very well answered.
- 6 (ii) (A)** The hypotheses were correctly stated in most cases although a few candidates gave a two tailed alternative hypothesis. More candidates than in the past are now giving an acceptable definition of p . However, please note that in future papers from January 2013 onwards, the definition of p will be expected to include ‘in the population’. For example, in this paper, a suitable definition of p would have been: ‘Let p = probability that a randomly selected student in the population is a smoker’.
- 6 (ii) (B)** (ii)(B) The reasons for H_1 being $p < 0.25$ were correct in most cases although some candidates simply stated the meaning of H_1 .

- 6 (iii)** This part was generally well done, and rather better done than in previous sessions. However, for those candidates not gaining full credit, a common error was the use of poor notation – the notation $P(X = 1)$ was often seen instead of $P(X \leq 1)$, despite candidates then writing down the correct $P(X \leq 1) = 0.0213$. The comparison with 5% was often not shown, losing the final 2 marks. It is pleasing to report that point probabilities are being used rather less than previously. A small number of candidates, having correctly found the probabilities and carried out the comparison, then stated the wrong critical region, thus demonstrating an insecure understanding.
- 6 (iv)** Only the best candidates seemed to attempt this part in the way expected by the structure of the question. Many candidates did not realise that they could use their answer to part (iii) and started again. Some who had part (iii) correct now used point probabilities, thus losing 2 marks. A small number of candidates failed to make a conclusion in context, thus losing the final mark. On this occasion, as there were only 2 marks available, a statement of the form ‘there is insufficient evidence to reject the null hypothesis’ was not insisted on, but instead a statement such as ‘accept the null hypothesis’ was condoned, provided that this was followed by a conclusion in context. However, in future sessions such statements may not be condoned.
- 7 (i)** This was very well answered. A few candidates lost marks due to either leaving the answer as 40 without giving the percentage, or working out that 80% had a birth weight of under 6kg rather than 20% over 6kg.
- 7 (ii)** Again this was very well answered, with only a small number of the weakest candidates giving the location as the median and quartiles.
- 7 (iii)** Most candidates found the upper and lower limits and correctly stated that there were a few outliers at the top end of the distribution. Some candidates used the median instead of the quartiles to work out the limits, whilst others multiplied the IQR by 2 instead of 1.5. A few candidates wrongly suggested that there were some outliers at the lower end of the distribution. Candidates often gave vague reasons for including or not including the outliers in the calculations, and few simply stated that there was nothing to suggest that these outliers were not genuine items of data.
- 7 (iv)** There were some very good answers to this question, which were precise and concise. However not all candidates quoted the figures of 3.6 and 0.8, which were needed to gain full marks. Some candidates found it hard to refer to the ‘central tendency’ or ‘average’ and ‘variation’ and simply referred to the median (mean in some cases) and the range, thus losing 2 marks.
- 7 (v)** This question was poorly answered. Many candidates did not realise that the median and IQR would remain unchanged. Several candidates were awarded 1 mark for stating that the range would have been increased. There were many candidates who ‘waffled’ and gave no substantive comments.
- 7 (vi)** (vi) Many candidates struggled with this question. Some used probabilities of ‘less than 3.9’ rather than ‘greater than 3.9’. Other candidates found both probabilities but did not know what to do with them. Some gave the probability for crossbred as $\frac{170}{200}$ rather than $\frac{165}{200}$. However, roughly one third of candidates produced a fully correct solution.

4767 Statistics 2

General Comments

Once again, the overall level of ability shown by candidates taking this paper was very impressive. Most candidates demonstrated proficiency in the use of approximating distributions. A small number of candidates lost accuracy marks through providing final answers given correct to 5 or more significant figures. Candidates also showed a good understanding when carrying out hypothesis tests; it should be noted that the preferred form of conclusion is one which is not too assertive and which states clearly, in context, whether or not the evidence supports the alternative hypothesis. When stating hypotheses about a population mean, the explicit appearance of the word "population" has hitherto been insisted on. With effect from the June 2012 examination, it will be assumed that correct use of the correct notation μ will imply that this is a population mean. If any other notation is used, or if the hypotheses are stated verbally, use of the word "population" will continue to be insisted on.

Comments on Individual Questions

- 1 (i) Well answered, with some issues concerning scaling of axes and lack of labels. Unusual scales sometimes led to inaccurate plotting of points.
- 1 (ii) Generally very well answered. Errors in ranking and arithmetic (in squaring and adding up the d^2) were noted, as well as incorrect rounding of final answers. Very few candidates failed to rank their data.
- 1 (iii) Most candidates correctly stated their hypotheses in terms of "association" and "no association" and referred to the context of the question. However, most candidates lost a mark by not making it clear what population their hypotheses applied to. Many candidates referred to "correlation" in their hypotheses and often provided hypotheses written in terms of ρ . Critical value and comparisons were substantially correct, but some candidates lost the final mark either by being too assertive in their claim or by neglecting to refer to the alternative hypothesis.
- 1 (iv) Many candidates failed to score on this part of the question. The words 'bivariate', 'Normal' and 'distribution' were seen often but not always together. The need for the **underlying population** to be bivariate Normal was not made clear by candidates as many seemed not to appreciate the difference between 'data' and 'population'. Successful responses made clear that the shape of the points on the scatter diagram indicated that a test based on Spearman's rank correlation coefficient was more appropriate; a small number of candidates recognised that one of the variables was discrete, thus invalidating the p.m.c.c. test.
- 2 (i) Many candidates did not provide clear enough explanations to justify the use of a Poisson distribution as a suitable model; those referring to "events" rather than "errors" were penalised.
- 2 (ii) Parts (A) and (B) were both well answered with the majority of candidates gaining full marks. Marks lost were usually in part (B), and due either to rounding errors or to mistakes when attempting to apply $1 - P(X \leq 1)$.
- 2 (iii) Very well answered.

- 2 (iv)** Many candidates failed to score a mark here; of those that did score marks, not many gained all four available marks. For those who clearly knew what was needed, often one mark was lost through incomplete justification of the final answer. Some candidates appeared to be searching for k such that $P(X = k) < 0.01$ and produced $k = 16$ as their solution. As this value is close to the correct answer, and frequently given as the answer by those using the correct method, credit could only be given to well-explained work.
- 2 (v)** Well answered with many gaining full marks. Commonly, marks were lost through not using the appropriate continuity correction. Some errors regarding incorrect variance, or incorrect use of variance when standardising, led to loss of marks.
- 3 (i)** Very well answered with most gaining full marks. Marks were lost typically through inaccurate use of Normal tables or confusion caused by one of the z values being zero.
- 3 (ii)** The expected Normal approximation caused problems for many; most correctly identified the mean but many struggled to obtain the correct variance. Again, many candidates struggled with the continuity correction. Some candidates correctly identified a Binomial distribution but then failed to use a Normal approximation. This led to a variety of wrong responses the most common of which was to try to work out $P(- = 40)$ using their Binomial distribution.
- 3 (iii)** There were many incorrect responses giving 1204 hours instead of 996. This resulted from candidates using 2.326 rather than -2.326 in their calculation. Commonly, marks were lost when answers were given to 5 or more significant figures.
- 3 (iv)** Many candidates scored at least 6 of the 8 available marks for this part of the question. Typically, for these candidates, marks were lost through failure to correctly define μ as the population mean or through being too assertive in the final conclusion. Once again a variety of approaches was seen, with that outlined in the mark scheme being the most popular. Some candidates attempted a one-tailed test despite the clear instruction in the question. Those failing to realise that the test statistic was based on a sample mean were heavily penalised.
- 4 (i)** Very well answered.
- 4 (ii)** Very well answered.
- 4 (iii)** This question was answered well by many. Common mistakes included reversal of hypotheses, use of “correlation” in hypotheses, incorrect addition of the contributions provided, incorrect critical value and over-assertive conclusions.
- 4 (iv)** Some good answers were seen but many candidates chose to ignore the instructions given within the question and answered it in their own way. Thus, instead of focussing on “each place” the type of bird was chosen as the key factor; such responses could still earn full marks. Despite being asked to “use the table of contributions” many failed to refer to it in their comments. In such questions, candidates are required to recognise that large contributions support the alternative hypothesis and small contributions support the null hypothesis. Comments such as “the large contribution of 60.7489 shows that there were many more thrushes observed in the garden than would be expected if there was no association” were seen, but many candidates simply reeled off figures from the tables of observed and expected frequencies without any reference to contributions or attempt at interpretation.

4768 Statistics 3

General Comments

There were 288 candidates (compared with 274 in January 2011) for this sitting of the paper. There were many very competent scripts and yet all too often candidates (including good ones) were seen to lose marks for carelessness, especially at the ends of questions. The topic “Sampling methods” continues to be one on which even good candidates do poorly. The numerical work was accurate except that candidates tended to quote confidence intervals to an excessive level of precision.

Invariably all four questions were attempted. Marks for Questions 1, 2 and 3 were found to be higher on average than Question 4.

Comments on Individual Questions

- 1 (a)** As mentioned above, candidates struggle on this topic. It was disappointing to see just how many did not appear to know the correct definition of a simple random sample. The usual (incorrect) answer given was “all members of the population are equally likely”. There were also many who said “all samples are equally likely”, omitting “of the required size”. The most common “difficulty” suggested was that the sample would not be representative. This is more a consequence of taking a simple random sample rather than a difficulty with the process of obtaining one.
- 1 (b) (i)** Answers to this part of the question were usually fine. There were many completely correct solutions, and most marks lost were as a result of careless errors and/or omissions.
- 1 (b)(ii)** Many completely correct answers were seen. Bizarrely there were candidates who, having scored full or nearly full marks for the test in the previous part, then switched away from the distribution t_{11} and so were penalised heavily. Many candidates insisted on quoting their final answer to a level of accuracy that was inappropriate.
- 2 (i)** This part, intended to be a straight-forward opener to the question, was answered correctly by the vast majority of candidates.
- 2 (ii)** Many candidates answered this correctly too, but at least as many interpreted the requirement “differ by 25” incorrectly, finding $P(S_1 - S_2 < 25)$ when they were meant to find $P(-25 < S_1 - S_2 < 25)$. A noticeable few considered the sum of two small packets rather than the difference.
- 2 (iii)** Once again there were many correct answers to this part. It continues to be the case that candidates are not rigorous about the notation they use. For example, they insist on writing “2S” when they mean “ $S_1 + S_2$ ”, thus appearing not to know the difference, and in some cases it became obvious that they really did not.
- 2 (iv)** This was another part that was generally well-answered. Sometimes candidates got confused and addressed the wrong tail of the distribution.
- 2 (v)** This was mostly answered correctly. There were problems with excessive accuracy in the final answer and, occasionally, over the choice of the percentage point to use.

- 3 (a)** For the chi-squared test, the hypotheses were broadly acceptable in most cases. Candidates usually realised to combine the last two classes in the table and so ended up with the correct value for the test statistic. However a very large proportion of them then failed to take account of the estimated parameter in the model and so used the wrong number of degrees of freedom to find the wrong critical value.
- 3 (b)(i)** A correct answer to this part was very rare indeed. A common answer went along the following lines, “There are differences between men and women and pairing eliminates these differences.”
- 3 (b)(ii)** The Wilcoxon test was almost always done correctly but with just a bit more care needed over the wording of the final conclusion.
- 4 (i)** Answers to this part were disappointing. Most candidates seemed unable to interpret symbolically the phrase “is proportional to”. Many assumed, but without justification, that the required probability was the ratio of two areas.
- 4 (ii)** This was correct most of the time. Candidates who did not obtain the correct result had usually treated a^2 as a variable, ending up with a complicated expression based on the quotient rule. Sometimes there was no indication of the variable with respect to which they would attempt to differentiate.
- 4 (iii)** For candidates who obtained the correct probability distribution function in part (ii), this part followed on quite easily, except that many were less than diligent about making sure that the final step towards the printed answer was shown carefully and convincingly. For the other candidates, if they knew what to do and carried it out carefully then most of the marks were still available.
- 4 (iv)** Many correct answers were seen. However candidates often forgot to divide the variance by the sample size or they divided it by 10. The other flaw often seen was that candidates wrote “ $R \sim \dots$ ” when it should have been “ $\bar{R} \sim \dots$ ”
- 4 (v)** While a good number of well thought out answers were seen for this part, there were very many that lacked clarity and seemed not to have taken account of the preceding part.

4771 Decision Mathematics 1

General Comments

Candidates generally did well on this paper. Section A was found to be very straightforward, as it should have been. In question 3 many candidates did lose marks for failing to answer the question. They hoped, or assumed, that the examiner would take the last deductive step for them – from working to solution. The examiner will not do this.

In section B there were some more challenging part marks, particularly the “Explain” in Q4(v) and the “Describe” in Q5(iv). Many candidates had difficulty with explaining and/or describing. Question 5 was the most difficult, with many candidates imagining that they had somehow to simulate numbers of gifts, rather than who received each gift.

Comments on Individual Questions

1 Almost all candidates succeeded with part (i). In part (ii) most answers were correct, with a smattering of “56”s for the number of edges, and some that were one or two off in their obviously non-structured counting.

2 A surprising number of candidates, though still only a small proportion, could not follow the instruction to reverse the order of the digits of a three-digit number. These were not slips. Such candidates would systematically, in all parts of the question, do some other transformation of the three digits.

Many candidates did not score both marks in part (iii) as a consequence of submitting an incomplete answer.

3 Most candidates could draw the graph, though inaccuracies and misreads abounded. As always, it was acceptable for candidates to use a sketch to drive a solution involving the solution of simultaneous linear equations. Other candidates needed an accurate graph to read off the points.

The required solution required the optimal point and optimal value, and many, many candidates, having done all of the work, failed to provide these. As remarked above, the examiner will not do the candidate’s work on his/her behalf!

4 In D1 papers there is always a sizeable proportion of candidates who do not succeed in applying Dijkstra, and this paper was no exception. Few candidates who did not apply the algorithm successfully subsequently recovered shortest path marks, because the shortest paths were not clear to see ... examiners were alert for “ACF...” and “ACFB...” Candidates were often successful with the minimum connector in part (iv), but few could manage the mark for part (v). Most answers represented, logically, no more than a rephrasing of the question. The few high quality answers seen noted that vertex D could be connected into the network by using DF, an arc of length 2, but that FD is not in the shortest path from A to D. Students might usefully learn that the mathematical way to disprove is to provide a specific counterexample.

- 5** As remarked above, a substantial minority of candidates lost many marks on this question by attempting to simulate numbers of gifts, rather than by simulating, repeatedly, which person receives the next gift.

Some candidates attempted to answer part (iii) theoretically. Theoretic answers are provided in the mark scheme for comparison and interest, but candidates received no marks for such attempts. They were required to use their simulations to estimate the probabilities.

In part (iv) most candidates, but not all, noted the essence – that some random numbers would have to be rejected.

- 6** The CPA question was answered well. There were very few, but some, candidates who forfeited most of the marks by attempting activity-on-node.

Some candidates seemed to think that a dummy activity could have two directions. There were several candidates who simply omitted activity D.

Others allowed a plethora of dummy activities. This is not of itself wrong, but many subsequently lost forward and backward pass accuracy marks because of the extra complexity introduced by their superfluous dummies.

In part (iv) some candidates seemed to think that they could show how two people could do the job as quickly as possible without showing who does what and when.

4776 Numerical Methods (Written Examination)

General Comments

The standard of work exhibited by the candidates was rather better than the average over recent years. It was pleasing that, on the whole, candidates were thinking numerically; that is, they showed an appreciation of both the power and the limitations of numerical methods.

Comments on Individual Questions

1 Solution of an equation (bisection)

Virtually everyone was able to locate the root using change of sign in part (i). The bisection method in part (ii) was usually done successfully, though some were not careful enough about the maximum possible error (mpe). The mpe initially is 0.1, but some candidates took it to be 0.5. A minority of candidates made no reference to mpe at all, and just iterated until they were happy with their solution. Such attempts did not receive full marks.

2 Newton's forward difference interpolation formula

Apart from some errors in signs, the difference table in part (i) was done well. Most were able to find an expression for $f(x)$, though there were the usual confusions between values of x and values of $f(x)$ from a few. The substitution in part (iii) was usually, but not invariably, done correctly.

3 Errors and accuracy

This was a pretty routine question about working to limited precision. Those who lost marks generally did so because they did not follow the instructions carefully enough. A small number of candidates lost marks through not rounding correctly to the required number of significant figures. In part (iii) some reference to subtracting nearly equal quantities was expected.

4 Errors and accuracy

Though the idea of rounded percentages not summing to 100 can hardly be new to candidates, quite a few of them found this question difficult. In part (i) it was not enough to say that rounding has occurred: that, in itself, would not necessarily give the wrong total. In this case it must be that the number rounded up exceeds the number rounded down. In part (ii), The maximum and minimum figures are 102 and 98, but candidates produced arguments for figures as high as 104 and as low as 96. It was quite common to see an asymmetrical and curious pair of figures such as 103 and 98.5 – the latter not even being an integer!

5 Numerical integration

The routine calculations were generally done well. The final answer, however, was often based on the agreement between the two Simpson's rule values rather than on a consideration of the change. The two values are 0.534609 and 0.534593; the difference is 16 points and so the next difference is likely to be 1, giving 0.53459 as secure to 5 significant figures.

6 Numerical differentiation

The numerical values in parts (i) and (ii) were generally obtained correctly. It was expected that candidates would say that the ratios of differences indicate that the forward difference method has first order convergence and the central difference method has second order convergence. The algebra in part (iii) was disappointing; the main problem appeared to be that candidates could not express the given information algebraically. Part (iv) required the formula in part (iii) to be applied to the answers from part (ii), these being for the second order method. A majority wrongly applied the formula to some or all of the answers from part (i).

7 Solution of an equation (fixed point iteration)

Part (i), locating the roots, was very easy. Part (ii) required candidates to determine, by differentiation, that the graph has only one turning point and hence (using the information from part (i)) that the equation has exactly two roots. This reasoning defeated those who were convinced that sextics must have six roots and/or five turning points. The iteration in part (iii) was generally well done, but in part (iv) the reasoning was often sloppy. Some argument to the effect that the derivative is greater than 1 *throughout* an interval containing the root was required. The majority were able to find a convergent iteration in part (v); most chose the inverse of the iteration in part (iii) which is, of course, guaranteed to converge.

Coursework

General Administration

Centres are reminded that the deadline date for the submission of marks to the board is December 10. This is to ensure that a sample request can be generated which will give centres the chance to receive it and despatch the sample to the moderator before breaking up for the Christmas break. Most centres complied most helpfully but a number did not, resulting in the receipt of coursework by the Moderator well into January. A few centres sent all the work in good time to the Moderator but failed to submit their marks to the board. Without knowledge of the sample determined by the board, Moderators are unable to proceed which causes the same problem as above. It should be noted that the same comment was made last January and our experience is that there has been no change in this aspect of the moderation process. However, the despatch of the centre Authentication form (CCS160) was much improved with only a few Centres having to be chased by OCR for them. Centres are reminded that the marks will not be validated (and therefore added in to the unit totals) without sight of this form signed by all the assessors.

Assessors are also requested to fill in the cover sheets fully. The loaded marks made available to Moderators are only identified by candidate number. Consequently, when a set of coursework tasks are received by the Moderator which are identifiable only by candidate name then there are difficulties with matching the work with the correct name and number.

It is helpful to Moderators if comments are made on cover sheets to indicate where marks are being withheld and why. It is also helpful if an annotation is made on the script where the work has been checked. It is disturbing to note that a number of assessors ticked work that was incorrect.

Centres are reminded that it is a requirement to supply a brief report on the Oral Communication.

The following comments are made to assist assessors in their task of interpreting the criteria. Most centres were fully conversant with these and it is clear that the task of assessment was carried out with great diligence and professionalism. However, there are a number of centres where the assessor was less well informed and (usually) awarded a mark that was more generous than was justified by the work seen.

4753/02 – Methods for Advanced Mathematics, C3

The following points should be subject to a penalty of half a mark. When there are four or more such errors in the assessment then the mark awarded goes outside the tolerance and an adjustment of marks is made.

Change of sign

- Most candidates do a decimal search. The root should be stated (rather than a range being given) and it should be correct to at least 3 decimal places. A number of candidates took, for instance, the range [1.11, 1.12] and asserted that the root was 1.115 correct to 3 decimal places.
- A graph of the function does not constitute an illustration.
- The following equations should not be used to demonstrate failure: trivial equations, equations with a root that is found in the table, equations with no roots. In this latter case candidates sometimes choose a very poor scale on the y-axis (perhaps going up in tens or worse) so that what is happening with a graph near to the axis cannot be seen clearly. Candidates then assert that the graph just touches when in fact it does not. A change of scale will indicate whether it cuts in two places or does not cut the axis at all.

Newton-Raphson method

- The roots should be found to at least 5 significant figures. We expect to see the working for at least one root which demonstrates an understanding of the method. This means seeing the formula developed from the general Newton Raphson formula for the particular equation (including sight of the derived function). Screenshots of “Autograph” may be used for subsequent roots but does not in itself demonstrate an understanding of the method.
- If an equation is used which has only one root, the second mark should not be awarded.
- As with the previous method, a graph of the function does not constitute an illustration. We expect to see two clear tangents which match the iterates.
- Error bounds need to be established, typically by change of sign, rather than simply stated.
- This method can be shown to fail if an initial value close to one root actually converges to another. Typically, an initial value that is “close to the root” may be an integer either side of the root. Taking an initial value that is not close enough to “demonstrate” failure to converge to a stated root is not acceptable. Likewise, we do not expect a “contrived” initial value just because it happens to be a turning point.

Rearrangement method

- Although there is no stipulation for error bounds in this criterion, nor is there any demand for a specific accuracy, it is expected that candidates will give a specific value for the root, and be aware of the accuracy of their root. It seems reasonable in a numerical process to expect values to be given to at least 3 decimal places.
- A graphical illustration will show either a staircase or cobweb diagram. This diagram should match the iterates found. The magnitude of $g'(x)$ can be discussed in two ways. The gradient function, $g'(x)$ can be found and calculated for a value of x that is close to the root and referred to the criterion for convergence. (The initial value of x is not usually close enough.) Alternatively, the gradient of the curve $y = g(x)$ can be discussed in general terms in relation to the way in which the curve cuts the line $y = x$ (which has gradient 1).
- The same equation should be used to demonstrate failure. The same rearrangement may be used to attempt to find another root, or a different rearrangement may be used to find the same or another root.
- As with the success, a clear diagram should be drawn to demonstrate divergence using the iterates found and the value of $g'(x)$ discussed.

Comparison

- When making a comparison of the fixed point methods, the same initial value should be used to find the same root to the same degree of accuracy.
- Without this, the discussion of speed of convergence is not valid. It is expected that the number of iterates required in each method to find the root should form part of this discussion.
- Candidates should refer to the hardware and software available to them in working this task. Different candidates will have different resources and will come to different conclusions.

Terminology

- Many assessors give the full mark here regardless of the terminology used. Typical errors which should be penalised are: failure to write equations (referring, for instance to $y = f(x)$ as an equation), incorrect language (for instance “I am going to find the root of the graph”) and candidates who word process their reports but are unable to write subscripts and powers properly.

4758/02 – Differential Equations

Only a small number of centres entered candidates for this particular piece of coursework, which is usual for the winter series. Occasionally there is a problem, at this stage of the year, of not having knowledge of the work necessary to complete certain tasks fully. For example, calculating the runway length in ‘Aeroplane Landing’ by an analytical rather than a numerical method can be a little problematic. However, the former is still expected and the calculation should be marked accordingly.

Despite previous reports, some candidates are still rejecting their initial model for the ‘Aeroplane Landing’ on the basis of the first nine seconds. The second phase of the motion, which should be investigated fully before proceeding, is ignored. It is not valid to make a judgement on the suitability or otherwise of a model without testing the model for the whole motion

Where possible it is expected that, when comparing the predicted and observed data, both table and graphical forms should be used.

When undertaking a modelling / experimental task (marked under Scheme B), care must be taken to avoid circular arguments. This occurs if only one set of data is produced. A model is created, and the data is then used to calculate parameters which are then used to predict the same data for comparison. The preferred method, for example in ‘Paper Cups’, is to use a set of observations for say one cup to predict the outcomes for say 5 cups or so. However, this comment does not apply to modelling investigations (marked under Scheme A) as usually there is only one set of data available.

Finally, although it does not affect the marking of the script, for modelling exercises especially, it helps the narrative if the data is presented and discussed at the beginning of the exercise.

4776/02 – Numerical Methods

The vast majority of candidates submitted tasks on numerical integration. Most candidates selected appropriate problems, but a few did not express them well, or did not explain why they were appropriate. Assessors did not always penalise accordingly.

Most used a sensible strategy, but often the justification for the selection of the algorithm was sketchy or non-existent. Many resorted to regurgitating bookwork and were nearly always given undue credit.

Nearly all knew they had to successively divide at least as far as 64 strips: many went beyond this. A small minority only went as far as 16 strips and were not penalised until the external moderator saw the work. A few candidates inadvertently made a systematic error in computing the function values, and therefore evaluated a different integral to the one stated. It is expected that assessors will note this and penalise accordingly.

Nearly all used a spreadsheet well, but many missed the point of the second mark, and were given credit for either describing which software they used or just printing out the formulae. They are expected to explain how the algorithm was implemented – usually by annotating the spreadsheet cell formulae. Many did the right thing here, but left the reader to work out what had been done by scrutinising the spreadsheet. Some detail is expected in the commentary!

A few mistakenly extrapolated from early values of Simpson’s Rule and achieved a less precise answer than their previous best estimate. A few used extrapolated values of M and T to obtain S – this is not valid and a penalty should be applied. Only a few used external sources – such as a value for π to inappropriately find relative error. This should not score – but it was sometimes given full credit.

In the final domain some candidates did not give a definitive statement of what they considered to be their best answer, so they should not get the first mark. Six significant figures is not the aim of the coursework – it is a guideline – so candidates should give the best answer they can from their working, and justify the precision quoted from their error analysis. Some candidates tend to quote all the figures from their extrapolated value as “the answer”. Some other candidates tend to be very conservative and say they are confident with the 6 significant figures they quote (and often as many as 10 significant figures are quoted). Many only gave limitations of the spreadsheet – which is not usually relevant. Few commented on r or problems with estimating undefined values of the function such as 0^0 .

A few assessors gave credit in the first 6 domains for comments made at interview or in discussion; this is not appropriate.

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