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RECOGNISING ACHIEVEMENT

Level 3 Certificate

Mathematics for Engineering

OCR Level 3 Certificate

H860/02 Paper 2

Mark Scheme for June 2011

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1 a i	Force = 0.01 Mg cos(0) = 0.01 × 1500 × 9.8 × 1 = 147N	 1 **1**	
1 a ii	Initial speed = 0, Final speed = 20, Time = 12 Acceleration a = 20/12 Force = Ma = 1500 × 5/3 = 2500 Total force = 2500 + 147 = 2647 N	 1 1 **2**	Allow FT from a i Accept 20/12 and 2500 for 2 marks
1 a iii	For slope of 1 in 20, $\theta = \tan^{-1}(1/20) = 0.049958$ radians (2.8624°) F_r = 0.01 × 1500 × 9.8 × cos (0.049958) = 146.816594 F_i = 1500 × 9.8 × sin (0.049958) = 734.082969 Total force = 880.899563 N	1 1 1 **3**	Accept reasonable rounding Accept ECF from slope Accept ECF from slope 1 mark for each force or 2 marks for total
1 b	u = 20 a = 1.5 $v^2 = 20^2 + 2as = 400 + 2 \times 1.5 \times 300 = 1300$ $v = \sqrt{1300} = 36.06$ m s^{-1}	 1 1 **2**	 Allow 1 mark for $v^2 = 2 \times 1.5 \times 300 = 900$ and v = 30
1 c	$F_r + F_i$ = 880.899563 Total force = 2000 Remaining force = 2000 – 880.899563 = 1119.100437 $F = Ma$; $a = F/M$ = 1119.100437/1500 = 0.7460669 m s^{-2}	 1 1 **2**	Allow FT from a iii Accept reasonable rounding Remaining force = 2000 – 881.51 =1118.49 $F = Ma$; $a = F/M$ = 1118.49/1500 = 0.7456 m s^{-2}
2 a i	$F_d = 0.5 \times 1.2 \times 0.4 \times 2.5 \times v^2 = 0.6 \times 24^2 = 345.6$ N F_r = 1500 × 9.8 × 0.01 = 147 Total force = 345.6 + 147 = 492.6 Torque at wheel = 492.6 × 0.3 = 147.78 Torque at motor = 147.78/4.5 = 32.84 Nm	1 1 1 1 **4**	 Allow FT from 1 a i Allow 1 mark for $t_w = F \times 0.3$ seen Allow 1 mark for $t_m = t_w/4.5$ seen

2 a ii	Speed = 24 Wheel radius = 0.3 Rotational speed of wheel = 24/0.3 = 80 rad s^{-1} Rotational speed of motor = 80 × 4.5 = 360 rad s^{-1}	 1 1 **2**	 Accept 24/(2π × 0.3) ≈ 12.73 revs s^{-1} Accept 12.73 × 4.5 ≈ 57.29 revs s^{-1}
2 a iii	Current i = (rotational speed × torque)/(120η) i = (360 × 32.84)/(120 × 0.9) = 109.4667 A ≈ 110 A	 1 **1**	 Allow FT in both cases
2 b i	$F = 0.6\,v^2 + 147 = 250$ N $v = \sqrt{((250 - 147)/0.6)} = 13.1022$ m s^{-1}	1 1 **2**	Allow FT in both cases
2 b ii	Terminal velocity reached when $a \rightarrow 0$ ($F_a = 0$) Total force = $0.6v^2 + 147 \cos(5°) + 1500 \times 9.8 \times \sin(5°) = 1600$ $= 0.6v^2 + 146.4406 + 1281.1894$ $v = \sqrt{((1600 - 146.4406 - 1281.1894)/0.6)} = 16.9494$ m s^{-1}	 1 1 1 **3**	
3	Total force , $F = v^2 + 150$ Power = $(v^2+150)v$ Using i = power/V $i = (v^2 + 150)v/120$ Alternatively: Torque at wheel = $(v^2 + 150) \times l$ (l = wheel radius) Torque at the motor = $(v^2 + 150) \times l/r$ (r = gear ratio) Rotational speed of wheel = v/l (rads/s) Rotational speed of motor = $v \times r/l$ Current required i = (rotational motor speed × motor torque)/voltage $= (v \times r/l) \times ((v^2 + 150) \times l/r) / 120 = v \times (v^2 + 150)/120$	 1 1	 Allow 1 mark for use of Power = $F \times v$

	Battery time available = $10000/i = (10000 \times 120)/(v \times (v^2 + 150))$ min Journey time = $(100 \times 10^3)/(60 \times v)$ min For journey completion: $(100 \times 10^3)/(60 \times v) \leq (10000 \times 120)/(v \times (v^2 + 150))$ $v^2 + 150 \leq (10000 \times 120 \times 60)/(100 \times 10^3)$ $v^2 \leq 570$ $v \leq 23.87$ Maximum speed = 23.87 m s^{-1}	1 1 1 1 **6**	
4 a	Total force opposing motion = $F_d + F_r$ $F_d = ½ \times 1.2 \times 5/9 \times 3 \times v^2 = v^2$ $F_r = 9.8 \times 1200 \times 0.012755 = 150$ Driving force = 550 Total force = $550 - v^2 - 150 = 400 - v^2$ Force = Mass × acceleration = $1200\frac{dv}{dt} = 400 - v^2$ $1200\frac{dv}{dt} + v^2 = 400$	1 1 1 1 **4**	$\rho = 1.2$ was not given explicitly in this question so accept any reasonable attempt showing $0.8333\rho v^2$

4 b	$1200\frac{dv}{dt}+v^2=400$ $1200\frac{dv}{dt}+v^2-400=0$ $1200\frac{dv}{dt}+(v^2-400)=0$ $\frac{1200}{(v^2-400)}\frac{dv}{dt}=-1$ $\int\frac{1200}{(v^2-400)}dv=-\int dt$ $\int\frac{1200}{(v^2-400)}dv=-t+c=c-t$	 1 1 1 **3**	
4c	$\int\frac{1}{v^2-a^2}=\frac{1}{2a}\ln(\frac{v-a}{v+a})$ $=30(\ln((v-20)/(v+20))=C-t$ $\ln(\frac{v-20}{v+20})=K-t/30$ $\frac{v-20}{v+20}=Ae^{-t/30}$ $v=0$ when $t=0\rightarrow A=-1$ $\frac{v-20}{v+20}=-e^{-t/30}$ $v-20=(v+20)(-e^{-t/30})$ $v(1+e^{-t/30})=20(1-e^{-t/30})$ $v=\frac{20(1-e^{-t/30})}{(1+e^{-t/30})}$	 1 1 1 1 1 **5**	Accept alternative solutions using $\int\frac{1200}{(v-20)(v+20)}dv=\int\frac{30}{v-20}-\frac{30}{v+20}dv=c-t$ $=30(\ln(v-20)-\ln(v+2))=c-t$ $\ln\left(\frac{v-20}{v+20}\right)=K-t/30$ or $\int\frac{1}{a^2-v^2}=\frac{1}{a}\tanh^{-1}(\frac{x}{a})$ Accept 20 tanh (t/60) OE

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