

Principal Learning

Engineering

OCR Level 3 Principal Learning

Unit **F563**: Mathematical techniques and applications for engineers

Mark Scheme for January 2012

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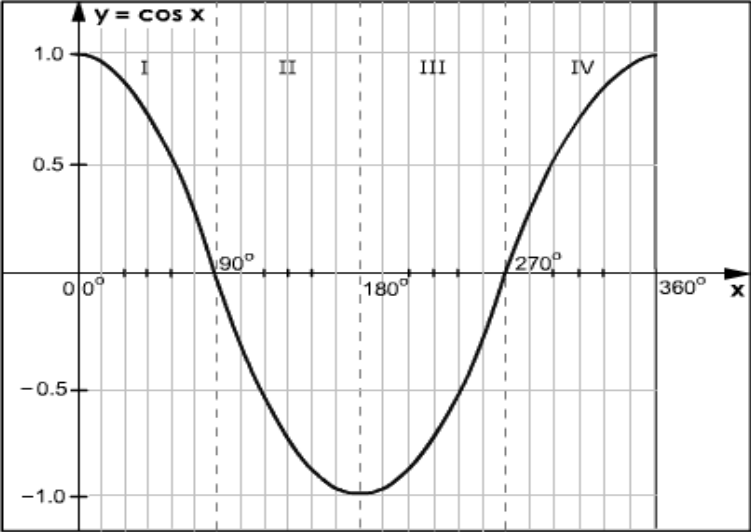
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Annotations

Annotation	Meaning
/	Correct
X	incorrect
BOD	Benefit of Doubt
ECF	Error Carried Forward

Section A

Question		Answer	Marks	Guidance
1		$-8(2x - 4) = -16x + 32$ $-16x$ $+32$	1 1	
2		$x^2 - 9 = (x + 3)(x - 3)$ $(x + 3)$ $(x - 3)$	1 1	
3		$(x + 4)/5 - (x + 3)/2 = (2x + 8 - 5x - 15)/10 = (-3x - 7)/10$ $(2x + 8 - 5x - 15)/10$ $(-3x - 7)/10$	1 1	
4		$5(x + 4) = 6x + 3$ $5x + 20 = 6x + 3$ $6x - 5x = 20 - 3$ $x = 17$	1 1	
5		Circumference of wheel = πd $= 800\pi$ Distance travelled = $250 \times 800\pi$ $= 628319$ mm correct to the nearest mm	1 1	
6		$\sin A = 80/125 = 0.64$ Angle A = $\sin^{-1} 0.64$ $= 40^\circ$ correct to the nearest degree	1 1	

Question	Answer	Marks	Guidance
7	<p>Graph of $y = \cos x$ Marks:</p> 	<p>1 1</p>	<p>Award one mark for general shape Award one mark for correct values</p>
8	<p>$\sin 50^\circ / \cos 60^\circ = 0.766/0.5$ $= 1.532$ $= 1.53$ correct to 2dp</p>	<p>1 1</p>	
9	<p>$y = 4x^3 + 5x^4$ $dy/dx = 12x^2 + 20x^3$ $12x^2$ $20x^3$</p>	<p>1 1</p>	
10	<p>$y = \sin x + \ln(4x)$ $dy/dx = \cos x + 1/x$ $\cos x$ $+ 1/x$</p>	<p>1 1</p>	
11	<p>$\int \sin x \cdot dx = -\cos x + C$ $-\cos x$ $+C$</p>	<p>1 1</p>	

Question	Answer	Marks	Guidance
12	$\int_4^5 3x^2 dx = \left[x^3 \right]_4^5$ $= 125 - 64 = 61$	1 1	
13	Mode = 7 Median = 8	1 1	
14	Height (mm) 141 – 150 / 151 – 160 / 161 – 170 / 171 – 180 Frequency 6 18 22 8 Cumulative Frequency 6 24 46 54 cf of 24 cf of 46 and 54	1 1	
15	Probability = 15/24 = 5/8 or 0.625	1 1	
	Total	30	

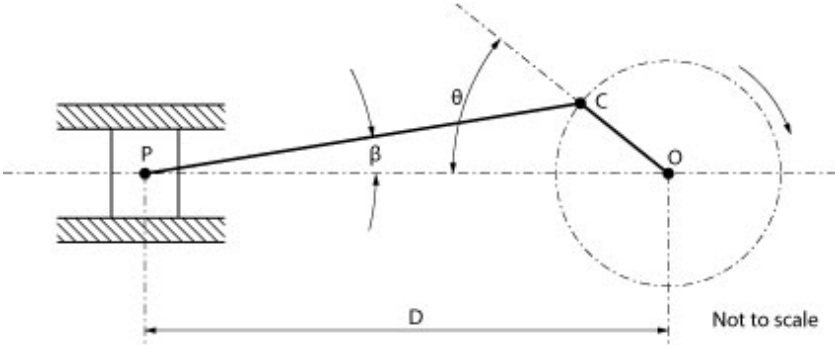
Section B

Question		Answer	Mark	Guidance
1	(a)	Given $V = \pi r^2 h$ Then $V = \pi \times 10^2 \times 80$ $V = 25132.74 \text{ mm}^3$ correct to 2 dp	1 1	
	(b) (i)	Transpose the formula $V = \pi r^2 h$ to make r the subject. Given $V = \pi r^2 h$ Then $r^2 = V/\pi h$ square root both sides then $r = \sqrt{V/\pi h}$	1 1	
	(ii)	Given $r = \sqrt{V/\pi h}$ Then $r = \sqrt{10000/100\pi}$ $r = \sqrt{100/\pi}$ $r = 5.64 \text{ mm}$ correct to 2 dp	1 1 1	Consider ECF from part (b)
	(c)	Given $X = \sqrt{Z^2 - R^2}$ Square both sides then $X^2 = Z^2 - R^2$ Add R^2 to both sides then $X^2 + R^2 = Z^2 - R^2 + R^2$ square root both sides then $Z = \sqrt{X^2 + R^2}$	1 1 1	

Question		Answer	Mark	Guidance
2	(a)	<p>Given equation $2x^2 - 3x - 4 = 0$</p> <p>Solution of quadratic equation by formula</p> $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ <p>where $a = 2$, $b = -3$ and $c = -4$</p> <p>so $x = \frac{3 \pm \sqrt{9 - 4 \times 2 \times -4}}{2 \times 2}$</p> $x = \frac{3 \pm \sqrt{9 + 32}}{4}$ $x = \frac{3 \pm \sqrt{41}}{4}$ $x = \frac{3 \pm 6.4}{4}$ $x = \frac{3 + 6.4}{4} = 9.4/4 = 2.35 \text{ s correct to 2 dp.}$ <p>Or $x = \frac{3 - 6.4}{4} = -3.4/4 = -0.85 \text{ s correct to 2 dp.}$</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p>	<p>Accept any other correct method of solution.</p>

Question	Answer	Mark	Guidance
2 (b)	<p>Given $s = ut + \frac{1}{2}at^2$</p> <p>Where $s = 42$, $u = 11$ and $a = 6$</p> <p>Then $42 = 11t + (\frac{1}{2} \times 6t^2)$</p> <p>So $3t^2 + 11t - 42 = 0$</p> <p>Solution of quadratic equation by formula</p> $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ <p>where $a = 3$, $b = 11$ and $c = -42$</p> <p>so $t = \frac{-11 \pm \sqrt{11^2 - 4 \times 3 \times -42}}{2 \times 3}$</p> $t = \frac{-11 \pm \sqrt{121 + 504}}{6}$ $t = \frac{-11 \pm \sqrt{625}}{6}$ $t = \frac{-11 \pm 25}{6}$ <p>Use $t = \frac{-11 + 25}{6} = \frac{14}{6} = 2.33$ s correct to 2 dp.</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>	

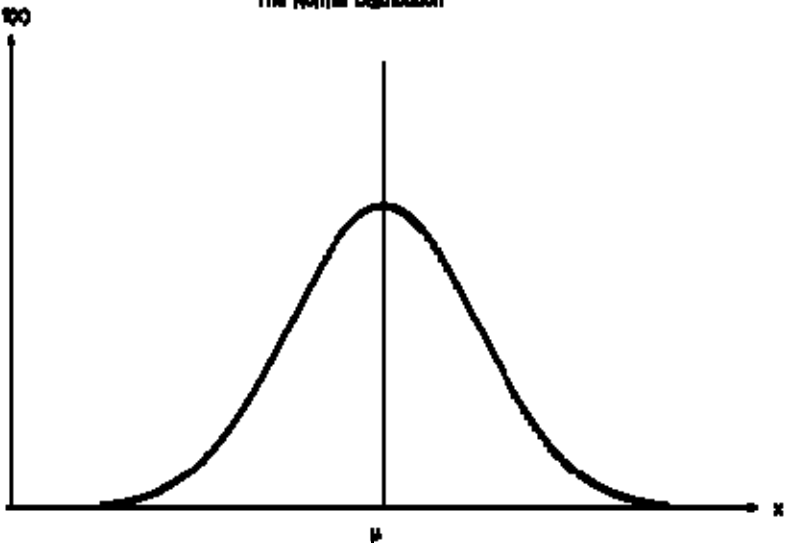
Question		Answer	Mark	Guidance
3	(a)	Length of arc = $(\pi r^{\circ})/180$		
		= $(\pi \times 72 \times 30)/180$	1	
		= 37.70 mm correct to 2dp	1	
	(b)	(i) peak value of the current		
		By observation: peak value = 25 A	1	
		(ii) frequency of supply		
		By observation: $2\pi ft = 400t$	1	
		$f = 400/2\pi$		
		= 63.66 Hz correct to 2 dp	1	
		(iii) periodic time		
		Periodic time $T = 1/f$	1	
		$T = 1/63.7$		
		= 0.02 s correct to 2dp	1	
		(iv) the current after 15 ms		
		Current i after 15 ms = $25 \sin 400 t$		
		= $25 \sin (400 \times 0.015)$	1	
		= $25 \sin 6$ (radians)	1	
		= -6.99 A correct to 2dp	1	

Question	Answer	Mark	Guidance
4 (a)	 <p>Sine rule: $PC/\sin \theta = OC/\sin \beta$ so $\sin \beta = OC \sin \theta / PC$ $\sin \beta = (100 \times \sin 55^\circ) / 400$ Angle $\beta = \sin^{-1}[(100 \times \sin 55^\circ) / 400]$ $= 11.82^\circ$ correct to 2 dp</p>	<p>1 1 1 1 1</p>	
(b)	<p>Angle OCP = $180 - 55 - 11.82 = 113.18^\circ$ Sine rule: $PC/\sin \theta = D/\sin OCP$ $D = PC \sin OCP / \sin \theta$ $D = (400 \sin 113.18^\circ) / \sin 55^\circ$ $D = 448.89$ mm correct to 2 dp OR $D = 100^2 + 400^2 - (2 \times 100 \times 400 \cos 113.18) = 448.88$ mm (1) (1) (1) (1) (1) OR $D = 400 \cos \beta + 100 \cos \theta = 448.88$ mm (2) (2) (1)</p>	<p>1 1 1 1 1 5 5</p>	

Question		Answer	Mark	Guidance
5	(a)	Given $y = 5e^{-3x}$ $dy/dx = -3(5e^{-3x})$ $dy/dx = -15e^{-3x}$	1	
	(b) (i)	initial velocity of the vehicle Given equation: $s = t^3 - 2t^2 + 3t$ Velocity = ds/dt $ds/dt = 3t^2 - 4t + 3$ The time $t = 0$ Initial velocity = $3(0^2) - 4(0) + 3$ $= 3 \text{ ms}^{-1}$	1 1 1	Do not deduct marks if units are not given in the answer
	(ii)	When $t = 5 \text{ s}$ Velocity = $ds/dt = 3t^2 - 4t + 3$ $= 3(5^2) - 4(5) + 3$ $= 58 \text{ ms}^{-1}$	1 1	
	(iii)	Acceleration = $dv/dt = d^2s/dt^2$ $ds/dt = 3t^2 - 4t + 3$ $d^2s/dt^2 = 6t - 4$ When $t = 5 \text{ s}$ $d^2s/dt^2 = 6(5) - 4$ $= 26 \text{ ms}^{-2}$	1 1 1	

Question		Answer	Mark	Guidance
6	(a)	Given equation: $F = 6s^2 + 4s$ $\text{Work done} = \int_{4.5}^6 (6s^2 + 4s) ds.$ $= [2s^3 + 2s^2]_{4.5}^6$ $= [(2 \cdot 6^3 + 2 \cdot 6^2) - (2 \cdot 4.5^3 + 2 \cdot 4.5^2)]$ $= 504 - 222.75$ $= 281.25 \text{ J}$	 1 1 1	
	(b) (i)	$\text{Area} = \int_0^{\pi/2} \cos x \cdot dx$ $= [\sin x \cdot dx]_0^{\pi/2}$ $= \sin \pi/2 - \sin 0$ $= 1$	 1 1 1 1	

Question		Answer	Mark	Guidance
	(ii)	$\text{Area} = \int_{\pi/2}^{\pi} \cos x \cdot dx$ $\text{Area} = [\sin x \cdot dx]_{\pi/2}^{\pi}$ $= \sin \pi - \sin \pi/2$ $= -1$	1 1 1	Accept +1
7	(a) (i)	The probability of something happening is the likelihood or chance of it happening	1	
	(ii)	An independent event is one in which the probability of an event happening does not affect the probability of another event happening.	1 1	
	(b) (i)	<p>The probability of selecting at random a relay, p, is given by the ratio: Number of relays/Total number of components</p> $p = \text{Number of relays/Total number of components}$ $= 36/(36 + 39)$ $= 36/75$	1 1	
	(ii)	<p>The probability of selecting at random a choke, q, is given by the ratio: Number of chokes/Total number of components</p> $q = \text{Number of chokes/Total number of components}$ $= 39/(36 + 39) \text{ or } (1 - 36/75)$ $= 39/75$	1	

Question	Answer	Mark	Guidance
(c)	<p>Total number of components = $36 + 39 = 75$</p> <p>The probability of randomly selecting a relay on the first draw is $36/75$</p> <p>There are now 35 relays in a batch of 74.</p> <p>The probability of randomly selecting a relay on the second draw is $35/74$</p> <p>The probability of selecting a relay on the first and second draw is $(36/75) \times (35/74) = 0.23$</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p>	
8	<p>(a)</p> <p style="text-align: center;">The Normal Distribution</p> 	2	One mark for shape and one mark for symmetry

Question		Answer					Mark	Guidance	
(b)	(i)	Raw (x)	Freq(f)	fx	x – mean	(x – mean)²	f(x – mean)²	Award one mark for each correct answer 60 , 416 , 227.87	
		Score							
		2.0	6	12	-4.93	24.31	145.86		1
		6.0	8	48	-0.93	0.87	6.96		1
		7.0	24	168	0.07	0.0049	0.07		1
		8.0	16	128	1.07	1.15	18.40		1
10.0	6	60	3.07	9.43	56.58				
		$\Sigma f = 60$ $\Sigma fx = 416$		$\Sigma f(x - \text{mean})^2 = 227.87$				Be aware that the mean calculated in part (b)(ii) is used In part (b)(i) Consider ECF from part (b)(ii)	
(b)	(ii)	$\Sigma f = 60$						Consider ECF from part (b)(i)	
		$\Sigma fx = 416$							
		Mean = $416/60 = 6.93$ (correct to 2 dp)					1		
		$\Sigma f(x - \text{mean})^2 = 227.87$					1		
		Variance = $[\Sigma f(x - \text{mean})^2] / \Sigma f$					1		
		= $227.87/60 = 3.80$ (correct to 2 dp)					1		
Standard Deviation = $\sqrt{\text{variance}} = \sqrt{3.80} = 1.95$ (correct to 2 dp)					1				

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