OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4754A

Applications of Advanced Mathematics (C4) Paper A

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Tuesday 13 January 2009
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

Section A (36 marks)

1 Express $\dfrac{3x+2}{x(x^2+1)}$ in partial fractions. **[6]**

2 Show that $(1+2x)^{\frac{1}{3}} = 1 + \frac{2}{3}x - \frac{4}{9}x^2 + \dots$, and find the next term in the expansion.

State the set of values of x for which the expansion is valid. **[6]**

3 Vectors **a** and **b** are given by $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

Find constants λ and μ such that $\lambda\mathbf{a} + \mu\mathbf{b} = 4\mathbf{j} - 3\mathbf{k}$. **[5]**

4 Prove that $\cot\beta - \cot\alpha = \dfrac{\sin(\alpha-\beta)}{\sin\alpha\sin\beta}$. **[3]**

5 **(i)** Write down normal vectors to the planes $2x - y + z = 2$ and $x - z = 1$.

Hence find the acute angle between the planes. **[4]**

(ii) Write down a vector equation of the line through $(2, 0, 1)$ perpendicular to the plane $2x - y + z = 2$. Find the point of intersection of this line with the plane. **[4]**

6 **(i)** Express $\cos\theta + \sqrt{3}\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is acute, expressing α in terms of π. **[4]**

(ii) Write down the derivative of $\tan\theta$.

Hence show that $\displaystyle\int_0^{\frac{1}{3}\pi} \frac{1}{(\cos\theta + \sqrt{3}\sin\theta)^2}\,d\theta = \frac{\sqrt{3}}{4}$. **[4]**

Section B (36 marks)

7 Scientists can estimate the time elapsed since an animal died by measuring its body temperature.

(i) Assuming the temperature goes down at a constant rate of 1.5 degrees Fahrenheit per hour, estimate how long it will take for the temperature to drop

(A) from 98 °F to 89 °F,

(B) from 98 °F to 80 °F. **[2]**

In practice, rate of temperature loss is not likely to be constant. A better model is provided by Newton's law of cooling, which states that the temperature θ in degrees Fahrenheit t hours after death is given by the differential equation

$$\frac{d\theta}{dt} = -k(\theta - \theta_0),$$

where θ_0 °F is the air temperature and k is a constant.

(ii) Show by integration that the solution of this equation is $\theta = \theta_0 + Ae^{-kt}$, where A is a constant. **[5]**

The value of θ_0 is 50, and the initial value of θ is 98. The initial rate of temperature loss is 1.5 °F per hour.

(iii) Find A, and show that $k = 0.031\,25$. **[4]**

(iv) Use this model to calculate how long it will take for the temperature to drop

(A) from 98 °F to 89 °F,

(B) from 98 °F to 80 °F. **[5]**

(v) Comment on the results obtained in parts **(i)** and **(iv)**. **[1]**

[Question 8 is printed overleaf.]

8 Fig. 8 illustrates a hot air balloon on its side. The balloon is modelled by the volume of revolution about the x-axis of the curve with parametric equations

$$x = 2 + 2\sin\theta, \quad y = 2\cos\theta + \sin 2\theta, \quad (0 \leqslant \theta \leqslant 2\pi).$$

The curve crosses the x-axis at the point A (4, 0). B and C are maximum and minimum points on the curve. Units on the axes are metres.

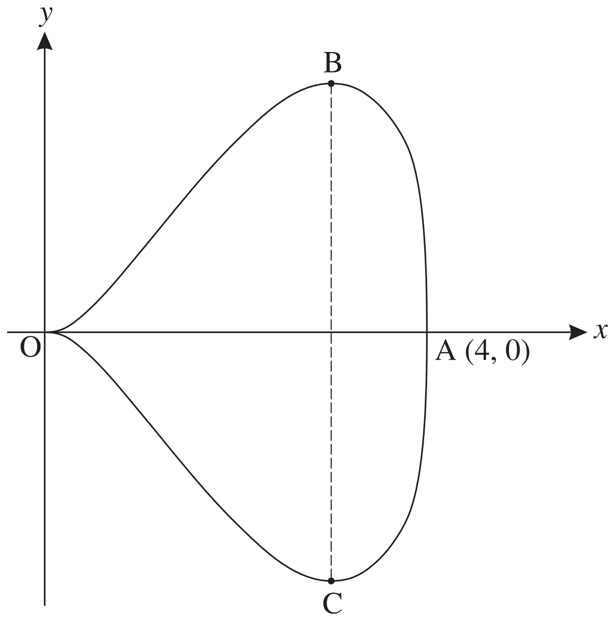

Fig. 8

(i) Find $\frac{dy}{dx}$ in terms of θ. **[4]**

(ii) Verify that $\frac{dy}{dx} = 0$ when $\theta = \frac{1}{6}\pi$, and find the exact coordinates of B.

Hence find the maximum width BC of the balloon. **[5]**

(iii) (*A*) Show that $y = x\cos\theta$.

(*B*) Find $\sin\theta$ in terms of x and show that $\cos^2\theta = x - \frac{1}{4}x^2$.

(*C*) Hence show that the cartesian equation of the curve is $y^2 = x^3 - \frac{1}{4}x^4$. **[7]**

(iv) Find the volume of the balloon. **[3]**