

ADVANCED GCE
MATHEMATICS
Further Pure Mathematics 3

4727

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:

None

Friday 29 January 2010
Morning

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

- 1 Determine whether the lines

$$\frac{x-1}{1} = \frac{y+2}{-1} = \frac{z+4}{2} \quad \text{and} \quad \frac{x+3}{2} = \frac{y-1}{3} = \frac{z-5}{4}$$

intersect or are skew.

[5]

- 2 H denotes the set of numbers of the form $a + b\sqrt{5}$, where a and b are rational. The numbers are combined under multiplication.

(i) Show that the product of any two members of H is a member of H .

[2]

It is now given that, for a and b not both zero, H forms a group under multiplication.

(ii) State the identity element of the group.

[1]

(iii) Find the inverse of $a + b\sqrt{5}$.

[2]

(iv) With reference to your answer to part (iii), state a property of the number 5 which ensures that every number in the group has an inverse.

[1]

- 3 Use the integrating factor method to find the solution of the differential equation

$$\frac{dy}{dx} + 2y = e^{-3x}$$

for which $y = 1$ when $x = 0$. Express your answer in the form $y = f(x)$.

[6]

- 4 (i) Write down, in cartesian form, the roots of the equation $z^4 = 16$.

[2]

(ii) Hence solve the equation $w^4 = 16(1-w)^4$, giving your answers in cartesian form.

[5]

- 5 A regular tetrahedron has vertices at the points

$$A(0, 0, \frac{2}{3}\sqrt{6}), \quad B(\frac{2}{3}\sqrt{3}, 0, 0), \quad C(-\frac{1}{3}\sqrt{3}, 1, 0), \quad D(-\frac{1}{3}\sqrt{3}, -1, 0).$$

(i) Obtain the equation of the face ABC in the form

$$x + \sqrt{3}y + (\frac{1}{2}\sqrt{2})z = \frac{2}{3}\sqrt{3}.$$

[5]

(Answers which only verify the given equation will not receive full credit.)

(ii) Give a geometrical reason why the equation of the face ABD can be expressed as

$$x - \sqrt{3}y + (\frac{1}{2}\sqrt{2})z = \frac{2}{3}\sqrt{3}.$$

[2]

(iii) Hence find the cosine of the angle between two faces of the tetrahedron.

[4]

6 The variables x and y satisfy the differential equation

$$\frac{d^2y}{dx^2} + 16y = 8 \cos 4x.$$

(i) Find the complementary function of the differential equation. [2]

(ii) Given that there is a particular integral of the form $y = px \sin 4x$, where p is a constant, find the general solution of the equation. [6]

(iii) Find the solution of the equation for which $y = 2$ and $\frac{dy}{dx} = 0$ when $x = 0$. [4]

7 (i) Solve the equation $\cos 6\theta = 0$, for $0 < \theta < \pi$. [3]

(ii) By using de Moivre's theorem, show that

$$\cos 6\theta \equiv (2 \cos^2 \theta - 1)(16 \cos^4 \theta - 16 \cos^2 \theta + 1). \quad [5]$$

(iii) Hence find the exact value of

$$\cos\left(\frac{1}{12}\pi\right) \cos\left(\frac{5}{12}\pi\right) \cos\left(\frac{7}{12}\pi\right) \cos\left(\frac{11}{12}\pi\right),$$

justifying your answer. [5]

8 The function f is defined by $f : x \mapsto \frac{1}{2-2x}$ for $x \in \mathbb{R}$, $x \neq 0$, $x \neq \frac{1}{2}$, $x \neq 1$. The function g is defined by $g(x) = ff(x)$.

(i) Show that $g(x) = \frac{1-x}{1-2x}$ and that $gg(x) = x$. [4]

It is given that f and g are elements of a group K under the operation of composition of functions. The element e is the identity, where $e : x \mapsto x$ for $x \in \mathbb{R}$, $x \neq 0$, $x \neq \frac{1}{2}$, $x \neq 1$.

(ii) State the orders of the elements f and g . [2]

(iii) The inverse of the element f is denoted by h . Find $h(x)$. [2]

(iv) Construct the operation table for the elements e , f , g , h of the group K . [4]

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