LEVEL 3 CERTIFICATE

MATHEMATICS FOR ENGINEERING

H860/02

Paper 2

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet (sent with general stationery)
- Insert (inserted)
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Friday 14 January 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- The insert will be found in the centre of this document.
- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a scientific or graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **40**.
- This document consists of **4** pages. Any blank pages are indicated.

1 In this question use the formula for electrical power given in the pre-release document,

$$P_E = \tfrac{1}{2}C_P\rho A v_1^3.$$

Assume that the density of air is $1.2\,\text{kg}\,\text{m}^{-3}$.

(a) **(i)** A wind turbine is designed to have an optimal size for a wind speed of $8\,\text{m}\,\text{s}^{-1}$ and a maximum power coefficient of 0.35. The turbine is required to produce 10 kW of electrical power.

Calculate the area swept by the turbine rotor and the length of a rotor blade. **[3]**

(ii) For the same turbine, operating with a constant wind speed of $8\,\text{m}\,\text{s}^{-1}$, the power coefficient C_P increases uniformly from 0.25 to 0.35 during a thirty-minute interval. During the next hour the power coefficient remains constant at 0.35 and for a further thirty minutes the power coefficient decreases uniformly from 0.35 to 0.25. Fig. 1 shows a graph of the power coefficient for the two-hour period.

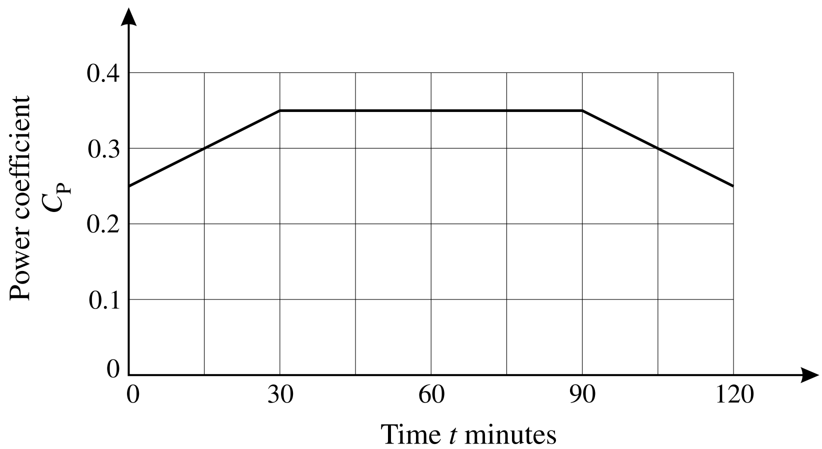

Fig. 1

Calculate the average power produced during this two-hour period. **[2]**

(b) For wind speeds of $2\,\text{m}\,\text{s}^{-1}$ or less, the rotor of a particular wind turbine remains stationary. The wind speed increases from $2\,\text{m}\,\text{s}^{-1}$ at a constant rate of $0.2\,\text{m}\,\text{s}^{-2}$ for one minute. During this period the tip speed ratio increases linearly from 0 to 4. Time, t seconds, is measured from the start of the one-minute interval.

(i) Express the wind speed during this one-minute interval in terms of t. **[2]**

(ii) Show that the rotor tip speed is given by

$$\tfrac{2}{15}t + \tfrac{1}{75}t^2.$$ **[2]**

(iii) The length of the rotor blade is 15 m.

Determine the angular acceleration of the rotor as a function of t during the one-minute interval. **[3]**

OCR
RECOGNISING ACHIEVEMENT

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- You must **not** bring your previous copy of the pre-release material into the examination.

Wind Power

In July 2009 the British Government published its Renewable Energy Strategy, which plans to reduce carbon emissions by 34% over the following decade. Among the proposals for implementation is the rapid deployment of wind turbines, which is likely to include 4000 new onshore installations and 3000 new installations at sea. It is clear that wind power will play a significant role in the reduction of carbon emissions in the future; however, wind turbines capture less than 50% of the power contained in the wind in an area swept by the turbine blades.

Large wind turbines typically have three rotor blades of over 20 m in length mounted on a central shaft. The blades are designed in such a way that the wind causes them to turn, rotating the shaft and driving an attached electrical generator, normally via a gear box. In optimum conditions large wind turbines can generate several megawatts of power. Conventional wind turbines, such as the one shown in Fig. 1, are mounted on a tower and continually adjust their position to face directly into the wind.

The power, P_W watts, of the wind passing through a given area, A m^2, is

$$P_W = \tfrac{1}{2}\rho A v_1^3$$

where ρ kg m^{-3} is the density of air and v_1 m s^{-1} is the speed of the wind.

The density of air reduces with increasing altitude and changes with variations in temperature. At sea level and 20 °C, air has a density of approximately 1.2 kg m^{-3}.

Not all the power of the wind can be converted into electrical power by a wind turbine. The electrical power, P_E watts, captured by the turbine is

$$P_E = C_P P_W = \tfrac{1}{2} C_P \rho A v_1^3$$

where C_P is called the power coefficient and is the proportion of the wind power that can be converted into electrical power, and A is the area swept by the rotor.

A large turbine with a sweep diameter of 44 m (blades of length 22 m), an approaching wind speed of 7 m s^{-1}, a power coefficient of 0.4 and density of air of 1.2 kg m^{-3} gives a value of about 125 kW for P_E.

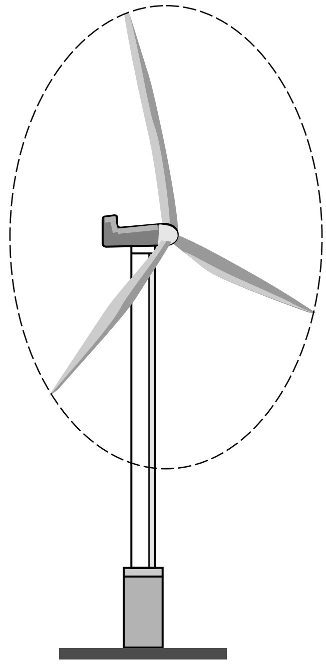

Fig. 1

The value of the power coefficient is a result of several factors. Albert Betz developed a theory which determines the maximum power that can be extracted by a wind turbine. The maximum value of the power coefficient according to this theory is called the *Betz limit*.

Wind speed is reduced as it passes through the swept area of the rotor. If the wind speed is v_1 as it approaches the swept area and v_2 as it leaves the swept area, then the power, P_T, taken from the wind is given by

$$P_T = \tfrac{1}{4}\rho A (v_1 + v_2)(v_1^2 - v_2^2).$$

The electrical power P_E produced by the turbine will be less than this as a result of other factors such as rotor blade drag, mechanical friction, copper losses and rectifier losses. Depending upon the design of the turbine and wind speeds, power coefficients of between 0.2 and 0.4 are typical.

Turbines are designed to provide optimum electrical power for particular ranges of wind speeds. Manufacturers will often state the total power output of turbines at an optimum wind speed. However, wind speeds change dramatically with time and it is useful to determine the distribution of wind speeds at particular sites. This is done by taking detailed measurements and tabulating the proportion of time that each range of speeds occurs. Experimentation shows that a good approximation to variation in wind speeds is given by the probability density function $\mathrm{f}(v)$, given by

$$\mathrm{f}(v) = \frac{\pi}{2}\frac{v}{\bar{v}^2}\mathrm{e}^{-\frac{\pi}{4}\frac{v^2}{\bar{v}^2}}$$

where v is the speed and $\bar{v}$ is the mean speed. Fig. 2 shows this probability density function for two different values of $\bar{v}$.

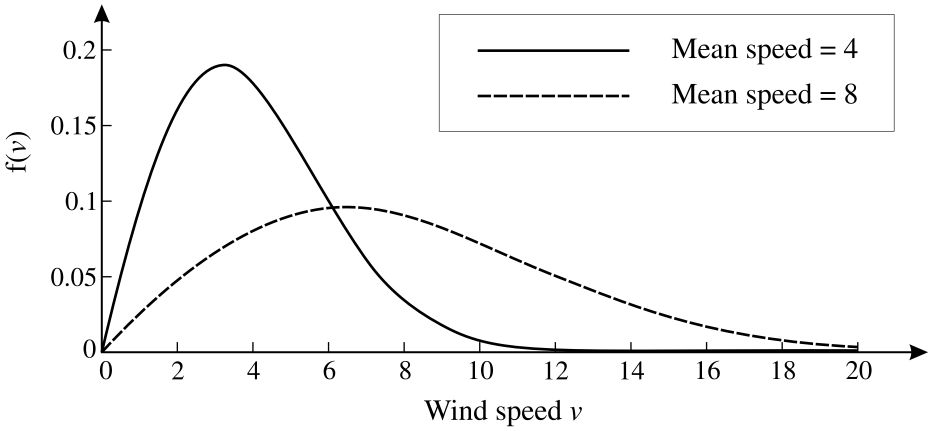

Fig. 2

This probability density function can then be used to determine the probability of finding wind speeds in a particular interval. For obvious reasons, wind turbines should be sited where average wind speeds are fairly high. However, very high wind speeds will take the wind turbine past its safe working limit and lead to damage of both mechanical and electrical components. In these situations a brake is applied to limit the angular speed of the rotor.

Although wind speed is a significant factor in the electrical power that a turbine can produce, it is the *rotor tip speed ratio* that provides a more realistic measure to determine the power coefficient. The rotor tip speed ratio is the ratio of the speed of the tip of the rotor to the speed of the wind; typical values range from 4 to 10. The graph that relates the dependence of power coefficient to tip speed ratio is usually plotted from measurements taken at particular sites and has the general shape shown in Fig. 3. The formula for this relationship can be approximated by placing a curve through a number of known measurements.

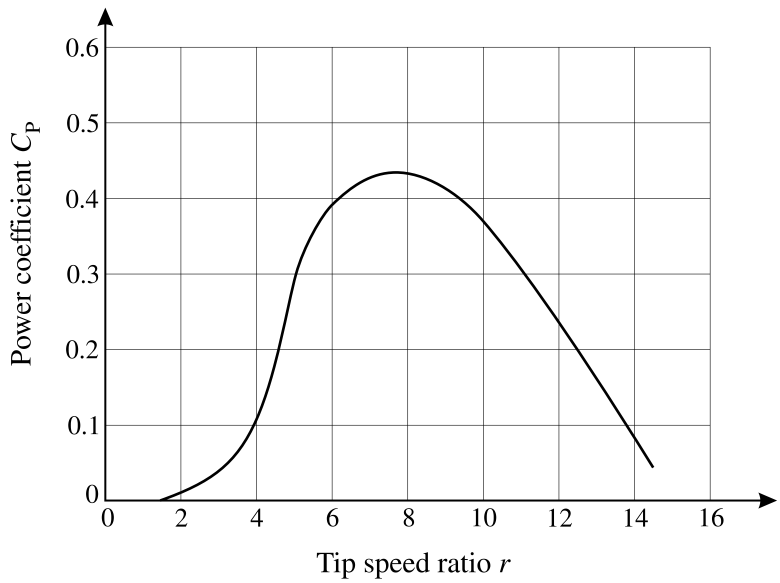

Fig. 3

Current developments taking place in the design of wind turbines and the careful siting of wind turbine farms seek to optimise the amount of electrical power that can be produced; however, the Betz limit is the main factor that places a maximum value of about 0.6 on the power coefficient of any wind turbine. It remains to be seen how effective wind power will become at providing a significant input to the National Grid.

2 **(a)** In an ideal case, the power coefficient $C_P = \frac{P_T}{P_W}$.

(i) Using the formulae for P_T and P_W given in the pre-release document, show that

$$C_P = \tfrac{1}{2}\left(1 + \frac{v_2}{v_1}\right)\left(1 - \frac{v_2^2}{v_1^2}\right).$$ **[2]**

(ii) Substitute $r = \frac{v_2}{v_1}$ into the formula given in part **(i)** and calculate the value of r which maximises C_P. **[4]**

(iii) Calculate the Betz limit, which is the greatest value of the power coefficient, C_P. **[1]**

(b) The manufacturer of a particular wind turbine uses the following model to relate the power coefficient, C_P, to the wind speed, $v\,\mathrm{m\,s^{-1}}$.

$$C_P = \begin{cases} 0, & v \leqslant 2 \\ \frac{1}{4}(v-2)e^{-\frac{1}{5}(v-2)}, & 2 < v < 20 \\ 0, & v \geqslant 20 \end{cases}$$

(Note that when $v \geqslant 20$ the turbine brake is applied to prevent instability.)

(i) Calculate the maximum value of the power coefficient C_P using this model. **[5]**

(ii) Sketch a graph of the power coefficient C_P against wind speed v for the interval $0 \leqslant v \leqslant 25$. **[2]**

3 Variations in wind speed at a particular proposed site have been measured and are summarised in Table 3.

Wind speed ($\mathrm{m\,s^{-1}}$)	Percentage of time
$0 \leqslant v < 2$	3.6
$2 \leqslant v < 4$	12.6
$4 \leqslant v < 6$	18.6
$6 \leqslant v < 8$	19.2
$8 \leqslant v < 10$	16.7
$10 \leqslant v < 12$	12.6
$12 \leqslant v < 14$	8.4
$14 \leqslant v < 16$	4.7
$16 \leqslant v < 18$	2.5
$18 \leqslant v < 20$	1.1

Table 3

(a) Show that the mean wind speed is approximately $8\,\mathrm{m\,s^{-1}}$. **[2]**

(b) Give the formula for the probability density function for wind speed based on this data in the form

$$f(v) = \frac{\pi}{2}\frac{v}{\bar{v}^2}e^{-\frac{\pi}{4}\frac{v^2}{\bar{v}^2}}$$

as stated in the pre-release document. **[1]**

(c) Use the probability density function given in part **(b)** to determine the probability that the wind speed is between $5\,\mathrm{m\,s^{-1}}$ and $9\,\mathrm{m\,s^{-1}}$. **[4]**

4 Measurements have been taken from a particular wind turbine to determine the relationship between rotor tip speed ratio and power coefficient expressed as a percentage. A summary of these measurements is shown in Table 4.

Tip speed ratio (r)	0	5	10	15
Power coefficient (C_P)	0	40	45	30

Table 4

Assume that the relationship between C_P and r is a third-order polynomial

$$C_P = ar^3 + br^2 + cr + d.$$

(a) Write down a set of linear equations from which the values of a, b, c and d could be determined. **[2]**

(b) Calculate the values of the constants a, b, c and d. **[5]**