

**Mathematics**

Advanced GCE

Unit **4725**: Further Pure Mathematics 1

**Mark Scheme for January 2011**

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1 (i)	$(7 \ 9)$	B1B1 2	Each element correct SC (7,9) scores B1
<hr style="border-top: 1px dashed black;"/>			
(ii)	$(18)$	B1* depB1 2	Obtain correct value Clearly given as a matrix
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(iii)	$\begin{pmatrix} 12 & -4 \\ 6 & -2 \end{pmatrix}$	M1  A1 A1 3 $\boxed{7}$	Obtain $2 \times 2$ matrix  Obtain 2 correct elements Obtain other 2 correct elements

2. (i)	$-12 + 13i$	B1B1 2	Real and imaginary parts correct
<hr style="border-top: 1px dashed black;"/>			
(ii)	$\frac{27}{37} - \frac{14}{37}i$	B1 M1  A1  A1 4 $\boxed{6}$	$z^*$ seen Multiply by $w^*$  Obtain correct real part or numerator  Obtain correct imaginary part or denom. Sufficient working must be shown

3		B1* M1*  A1* depA1 4 $\boxed{4}$	Establish result true for $n = 1$ or $2$ Use given result in recurrence relation in a relevant way Obtain $2^n + 1$ correctly Specific statement of induction conclusion
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4	<i>Either</i>  $\frac{a}{4}n^2(n+1)^2 + \frac{bn}{2}(n+1)$  $a = 4 \quad b = -4$ <i>Or</i>  $a + b = 0 \quad 4a + b = 12$  $a = 4 \quad b = -4$	B1 M1 A1  M1 A1 A1 6  M1 A1 A1  M1 A1 A1  $\boxed{6}$	Correct value for $\sum r$ stated or used Express as sum of two series Obtain correct unsimplified answer  Compare coefficients or substitute values for $n$ Obtain correct answers  Use 2 values for $n$ Obtain correct equations  Solve simultaneous equations Obtain correct answers
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5	$A^2$	B1 M1 A1cao 3 $\boxed{3}$	$(A^{-1})^{-1} = A$ seen or implied Use product inverse correctly Obtain correct answer
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6 (i) (a)	B1*	Vertical line
(b)	depB1 2	Clearly through ( 4, 0 )
	B1	Sloping line with +ve slope
	B1	Through ( 0, -2 )
	B1ft 3	Half line starting on y-axis 45° shown convincingly

(ii)	B1ft	Shaded to left of their (i) (a)
	B1ft	Shaded below their (i) (b) must be +ve slope
	B1ft 3	Shaded above horizontal through their (0, -2 )

**NB** These 3 marks are independent, but 3/3 only for fully correct answer.

**8**

7 (i) $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$	B1 B1 2	Each column correct
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(ii)	B1*	Enlargement or stretch in <i>x</i> <b>and</b> <i>y</i> axes
	depB1 2	Scale factor $\sqrt{3}$

(iii) (a)	B1	(2,0), (6,2) indicated
	B1	(8, 2) seen
	B1 3	Accurate diagram, including unit square

(b) $\det C = 4$	B1	Correct value found
	B1 2	Scale factor for area

**9**

8 (i) <i>Either</i>		
$\alpha + \beta = \frac{1}{2}, \alpha\beta = \frac{3}{2}$	B1	State or use both correct results in (i) or (ii)
$\alpha + \beta + \frac{\alpha + \beta}{\alpha\beta}$ or $\alpha + \beta + \frac{2}{3}(\alpha + \beta)$	M1	Express sum of new roots in terms of $\alpha + \beta$ and $\alpha\beta$
	M1	Substitute their values into their expression
$p = \frac{5}{6}$	A1 4	Obtain <b>given</b> answer correctly
<i>Or</i>		
$3u^2 - u + 2(= 0)$	B1	Substitute $x = \frac{1}{u}$ and obtain correct quadratic (equation)
	M1	Use sum of roots of new equation
	M1	Substitute their values into their expression
$p = \frac{5}{6}$	A1	Obtain <b>given</b> answer correctly

(ii)	$\alpha' \beta' = \alpha\beta + \frac{1}{\alpha\beta} + \frac{\beta}{\alpha} + \frac{\alpha}{\beta}$	B1	Correct expansion
	$\frac{\beta}{\alpha} + \frac{\alpha}{\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$	M1	Show how to deal with $\alpha^2 + \beta^2$
		A1	Obtain correct expression
	$q = \frac{1}{3}$	M1	Substitute their values into $\alpha'\beta'$
		A1	Obtain correct answer a.e.f.
<b>9</b>			
(i)		M1	Show correct expansion process for 3 x 3
	$\det \mathbf{M} = a^2 - 7a + 6$	M1	Correct evaluation of any 2 x 2
		A1	correct answer
(ii)		M1	Solve $\det \mathbf{M} = 0$
	$a = 1$ or $6$	A1A1	Obtain correct answer, ft their (i)
(iii)		M1	Attempt to eliminate one variable
		A1	Obtain 2 correct equations in 2 unknowns
		A1	Justify infinite number of solutions
			SC 3/3 if unique solution conclusion consistent with their (i) or (ii)
<b>10</b>			
(i)		M1	Use correct denominator
		A1	Obtain <b>given</b> answer correctly
(ii)		M1	Express terms as differences using (i)
		M1	Do this for at least 3 terms
		A1	First 3 terms all correct
		A1	Last 2 terms all correct
	$\frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2}$	M1	Show relevant cancelling
		A1	Obtain correct answer a.e.f.
(iii)	$\frac{1}{2}$	B1ft	$S_{\infty}$ stated or start at $n + 1$ as in (ii)
	$\frac{1}{n+1} - \frac{1}{n+2}$	M1	$S_{\infty}$ - their (ii) or show correct cancelling
	$\frac{1}{(n+1)(n+2)}$	A1	Obtain <b>given</b> answer correctly

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