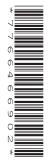


Monday 13 May 2019 – Afternoon AS Level Further Mathematics B (MEI)

Y410/01 Core Pure

Time allowed: 1 hour 15 minutes



You must have:

- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:

• a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Write your answer to each question in the space provided in the Printed Answer Booklet. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets [].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of 12 pages. The Question Paper consists of 4 pages.

Answer all the questions

1 In this question you must show detailed reasoning.

Find
$$\sum_{r=1}^{100} \left(\frac{1}{r} - \frac{1}{r+2}\right)$$
, giving your answer correct to 4 decimal places. [3]

2 The roots of the equation $3x^2 - x + 2 = 0$ are α and β . Find a quadratic equation with integer coefficients whose roots are $2\alpha - 3$ and $2\beta - 3$. [3]

3 In this question you must show detailed reasoning.

A and **B** are matrices such that $\mathbf{B}^{-1}\mathbf{A}^{-1} = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$.

(a) Find AB.

(b) Given that
$$\mathbf{A} = \begin{pmatrix} \frac{1}{3} & 1 \\ 0 & 1 \end{pmatrix}$$
, find **B**. [3]

4 (a) Find
$$\mathbf{M}^{-1}$$
, where $\mathbf{M} = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ -2 & 1 & 2 \end{pmatrix}$. [1]

(b) Hence find, in terms of the constant k, the point of intersection of the planes

$$x + 2y + 3z = 19,$$

-x+ y+2z = 4,
-2x+ y+2z = k. [3]

(c) In this question you must show detailed reasoning.

Find the acute angle between the planes x + 2y + 3z = 19 and -x + y + 2z = 4. [4]

5 Prove by induction that, for all positive integers n, $\sum_{r=1}^{n} \frac{1}{3^r} = \frac{1}{2} \left(1 - \frac{1}{3^n} \right)$. [6]

- 6 A linear transformation T of the *x*-*y* plane has an associated matrix **M**, where $\mathbf{M} = \begin{pmatrix} \lambda & k \\ 1 & \lambda k \end{pmatrix}$, and λ and *k* are real constants.
 - (a) You are given that det M > 0 for all values of λ .
 - (i) Find the range of possible values of *k*. [3]
 - (ii) What is the significance of the condition $\det \mathbf{M} > 0$ for the transformation T? [1]

For the remainder of this question, take k = -2.

- (b) Determine whether there are any lines through the origin that are invariant lines for the transformation T. [4]
- (c) The transformation T is applied to a triangle with area 3 units². The area of the resulting image triangle is 15 units².
 Find the possible values of λ. [3]
- 7 (a) Sketch on a single Argand diagram
 - (i) the set of points for which |z-1-3i| = 3, [3]
 - (ii) the set of points for which $\arg(z+4) = \frac{1}{4}\pi$. [3]
 - (b) Find, in exact form, the two values of z for which |z-1-3i| = 3 and $\arg(z+4) = \frac{1}{4}\pi$. [6]

8 In this question you must show detailed reasoning.

You are given that i is a root of the equation $z^4 - 2z^3 + 3z^2 + az + b = 0$, where a and b are real constants.

(a) Show that a = -2 and b = 2. [4]

[7]

(b) Find the other roots of this equation.

END OF QUESTION PAPER



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