Oxford Cambridge and RSA

## Thursday 6 June 2019 - Afternoon

## AS Level Further Mathematics B (MEI)

## Y416/01 Statistics b

Time allowed: 1 hour 15 minutes

You must have:

- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:

- a scientific or graphical calculator


## INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer all the questions.
- Write your answer to each question in the space provided in the Printed Answer Booklet. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.


## INFORMATION

- The total number of marks for this paper is $\mathbf{6 0}$.
- The marks for each question are shown in brackets [ ].
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of 12 pages. The Question Paper consists of 8 pages.


## Answer all the questions.

1 It is known that the red blood cell count of adults in a particular country, measured in suitable units, has mean 4.96 and variance 0.15 .
(a) Find the probability that the mean red blood cell count of a random sample of 50 adults from this country is at least 5.00.
(b) Explain how you can find the probability in part (a) despite the fact that you do not know the distribution of red blood cell counts.

2 Leila and Caleb are playing a game, using fair six-sided dice and unbiased coins.

- Leila rolls two dice, and her score $L$ is the total of the scores on the two dice.
- Caleb spins 4 coins and his score $C$ is three times the number of heads obtained.

The winner of a game is the player with the higher score. If the two scores are equal, the result of the game is a draw. The spreadsheet in Fig. 2 shows a simulation of 20 plays of the game.

|  | A | B | C | D | E | F | G | H |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | First <br> dice | Second <br> dice | Total (Leila's <br> score) $\boldsymbol{L}$ | Coin <br> $\mathbf{1}$ | Coin <br> $\mathbf{2}$ | Coin <br> $\mathbf{3}$ | Coin <br> $\mathbf{4}$ | Caleb's <br> score $\boldsymbol{C}$ |
| 2 | 1 | 2 | 3 | H | T | T | T | 3 |
| 3 | 6 | 1 | 7 | T | H | T | T | 3 |
| 4 | 2 | 6 | 8 | H | H | T | T | 6 |
| 5 | 2 | 5 | 7 | T | H | H | H | 9 |
| 6 | 1 | 5 | 6 | T | H | T | T | 3 |
| 7 | 5 | 2 | 7 | H | H | H | H | 12 |
| 8 | 1 | 1 | 2 | H | T | H | T | 6 |
| 9 | 2 | 6 | 8 | T | H | T | H | 6 |
| 10 | 6 | 2 | 8 | H | T | H | T | 6 |
| 11 | 1 | 3 | 4 | T | H | H | H | 9 |
| 12 | 6 | 1 | 7 | T | H | T | T | 3 |
| 13 | 3 | 1 | 4 | T | T | T | T | 0 |
| 14 | 3 | 6 | 9 | H | T | H | H | 9 |
| 15 | 2 | 3 | 5 | T | H | H | H | 9 |
| 16 | 2 | 5 | 7 | H | H | H | H | 12 |
| 17 | 1 | 5 | 6 | H | H | T | H | 9 |
| 18 | 5 | 6 | 11 | T | H | H | H | 9 |
| 19 | 4 | 2 | 6 | T | H | H | T | 6 |
| 20 | 6 | 5 | 11 | T | T | H | H | 6 |
| 21 | 1 | 1 | 2 | T | T | T | T | 0 |

Fig. 2
(a) Explain why the value of $C$ in row 2 is 3 .
(b) Use the spreadsheet to estimate $\mathrm{P}(C>6)$ and $\mathrm{P}(L>6)$.
(c) Use the spreadsheet to estimate the probability that Leila loses a randomly chosen game.
(d) Explain why your answers to parts (b) and (c) may not be very close to the true values.
(e) Leila claims that the game is fair (that Leila and Caleb each have an equal chance of winning) because both she and Caleb can get a maximum score of 12 and also in the simulation she won exactly $50 \%$ of the games.
Make 2 comments about Leila's claim.

3 A bus runs from point $A$ on the outskirts of a city, stops at point $B$ outside the rail station, and continues to point C in the city centre.
The journey times for the sections A to B and B to C vary according to traffic conditions, and are modelled by independent Normal distributions with means and standard deviations as shown in the table.

|  | Journey time (minutes) |  |
| :---: | :---: | :---: |
|  | Mean | Standard deviation |
| A to B | 21 | 3 |
| B to C | 29 | 4 |

(a) Find the probability that a randomly chosen journey from A to B takes less than the scheduled time of 23 minutes.

For every journey, the bus stops for 1 minute when it reaches B to drop off and pick up passengers.
(b) Find the probability that a randomly chosen journey from A to C takes less than the scheduled time of 50 minutes.

Mary travels on the bus from the station at B to her workplace at C every working day. You should assume that times for her bus journeys on different days are independent.
(c) Find the probability that the total time taken for her five journeys on the bus in a randomly chosen week is at least $2 \frac{1}{2}$ hours.
(d) Comment on the assumption that times on different days are independent.

4 The cumulative distribution function of the continuous random variable $X$ is given by

$$
\mathrm{F}(x)= \begin{cases}0 & x<0 \\ k\left(12 x-x^{2}\right) & 0 \leqslant x \leqslant 2 \\ 1 & x>2\end{cases}
$$

where $k$ is a constant.
(a) Show that $k=0.05$.
(b) Find $\mathrm{P}(1 \leqslant X \leqslant 1.5)$.
(c) Find the median of $X$, correct to 3 significant figures.
(d) Find which of the median, mean and mode of $X$ is the largest of the three measures of central tendency.

5 A technician is investigating whether a batch of nylon 66 (a particular type of nylon) is contaminated by another type of nylon.
The average melting point of nylon 66 is $264^{\circ} \mathrm{C}$. However, if the batch is contaminated by the other type of nylon the melting point will be lower. The melting points, in ${ }^{\circ} \mathrm{C}$, of a random sample of 8 pieces of nylon from the batch are as follows.
$\begin{array}{llllllll}262.7 & 265.0 & 264.1 & 261.7 & 262.9 & 263.5 & 261.3 & 262.6\end{array}$
(a) Find

- the sample mean,
- the sample standard deviation.

The technician produces a Normal probability plot and carries out a Kolmogorov-Smirnov test for these data as shown in Fig. 5.


Fig. 5
(b) Comment on what the Normal probability plot and the $p$-value of the test suggest about the data.
(c) In this question you must show detailed reasoning.

Carry out a suitable test at the $5 \%$ significance level to investigate whether the batch appears to be contaminated with another type of nylon.
(d) Name an alternative test that could have been carried out if the population standard deviation had been known.

6 The label on a pack of strawberries in a large batch states that it holds 250 g of strawberries. A random sample of 40 packs from the batch is selected and software is used to produce a $95 \%$ confidence interval for the mean weight of strawberries per pack. An extract from the software output is shown in Fig. 6.

| Sample Mean | 248.92 |
| :--- | :--- |
| Standard Error | 0.61506 |
| Sample Size | 40 |
| Confidence Level | 0.95 |
| Interval | $248.92 \pm 1.2055$ |

Fig. 6
(a) Explain whether the confidence interval suggests that the mean weight of strawberries per pack in the batch is different from 250 g .
(b) A manager looking at the data says that the conclusion would have been different if a $90 \%$ confidence interval had been used.
Determine whether the manager is correct.
(c) Explain briefly whether or not it is appropriate for the manager to vary the confidence level before coming to any conclusions.
(d) On another occasion, using the same sample size, a $95 \%$ confidence interval for the mean weight of strawberries per pack is [248.05, 249.95]. Find the sample variance in this case.
(e) Explain the meaning of a $95 \%$ confidence interval.

BLANK PAGE

## OCR

Oxford Cambridge and RSA

## Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series. If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.
For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.
OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

