Oxford Cambridge and RSA

## GCE

# Further Mathematics B (MEI) 

Y416/01: Statistics B

Advanced Subsidiary GCE

Mark Scheme for June 2019

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

## Annotations and abbreviations

| Annotation in scoris | Meaning |
| :--- | :--- |
| $\checkmark$ and $\boldsymbol{x}$ |  |
| BOD | Benefit of doubt |
| FT | Follow through |
| ISW | Ignore subsequent working |
| M0, M1 | Method mark awarded 0, 1 |
| A0, A1 | Accuracy mark awarded 0, 1 |
| B0, B1 | Independent mark awarded 0, 1 |
| SC | Special case |
| $\wedge$ | Omission sign |
| MR | Misread |
| Highlighting |  |
|  | Meaning |
| Other abbreviations <br> in mark scheme |  |
| E1 | Mark for explaining a result or establishing a given result |
| dep* | Correct answer only |
| cao | Or equivalent |
| oe | Rounded or truncated |
| rot | Seen or implied |
| soi | Without wrong working |
| www | Answer given |
| AG | Anything which rounds to |
| awrt | By Calculator |
| BC | This indicates that the instruction In this question you must show detailed reasoning appears in the question. |
| DR |  |

## Subject-specific Marking Instructions for AS Level Further Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The $A, M$ and $B$ annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.
b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.
c The following types of marks are available.

## M

A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

## E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.
d When a part of a question has two or more 'method' steps, the $M$ marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only - differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km , when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Accept any value within a probability model that agrees with the correct value to 4 d.p. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.
h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
j If in any case the scheme operates with considerable unfairness consult your Team Leader.

| Question |  | Answer | Marks | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | (a) | Distribution is $\mathrm{N}(4.96,0.15 / 50)$ $\mathrm{P}(\text { Mean }>5.00)=0.233(0.23260 \ldots)$ | $\begin{gathered} \hline \text { B1 } \\ \text { M1 } \\ \text { A1 } \\ {[3]} \\ \hline \end{gathered}$ | For $\mathrm{N}(4.96, \ldots$ For $\ldots 0.15 / 50)$ BC | $\begin{aligned} & \text { Alternative is to calculate } \\ & \text { total }>250 \text { with } \mathrm{N}(50 \times 4.96, \\ & 0.15 \times 50) \text { or } \mathrm{N}(248,7.5) \end{aligned}$ |
|  | (b) | Because by the central limit theorem, for large values of $n$, the distribution of the sample mean is approximately Normal | $\begin{aligned} & \text { B1 } \\ & \text { B1 } \\ & \text { B1 } \\ & {[3]} \end{aligned}$ | For CLT <br> For large values of $n$ For full answer. | Must mention SAMPLE mean |


| 2 | (a) | Because there is one head and the score is $3 \times$ number of heads. | $\begin{aligned} & \hline \text { E1 } \\ & {[1]} \\ & \hline \end{aligned}$ |  | Watch for candidates who work out Leila's score |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | (b) | Estimate of $\mathrm{P}(C>6)=\frac{8}{20}=0.4$ <br> Estimate of $\mathrm{P}(L>6)=\frac{11}{20}=0.55$ | B1 B1 [2] |  |  |
|  | (c) | $\begin{aligned} & \mathrm{P}(\text { Leila loses })=\frac{7}{20} \\ & =0.35 \end{aligned}$ | $\begin{gathered} \text { M1 } \\ \text { A1 } \end{gathered}$ [2] |  |  |
|  | (d) | Because the number of simulations is small. | $\begin{aligned} & \text { E1 } \\ & {[1]} \end{aligned}$ |  |  |
|  | (e) | Caleb did not win $50 \%$ of games as there were some draws, so may not be true. <br> A much larger sample would be needed to be reasonably sure of the conclusion. <br> Although both can get a max score of 12, Leila's minimum is 2 as compared to Caleb's 0 , so may not be true. <br> Leila's expected score is 7 but Caleb's is 6 , so may not be true. <br> Comparing the maximum values is not sensible. | E1 <br> E1 <br> [2] | For any two valid comments | Do not allow comments about simulation not being able to account for bias unless mention 'due to small sample' oe |


| Question |  | Answer | Marks | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | (a) | $\mathrm{P}(<23)=0.748$ (0.74750...) | $\begin{aligned} & \hline \text { B1 } \\ & {[1]} \end{aligned}$ | BC |  |
|  | (b) | ```A to C distribution \(X: \quad \mathrm{N}(50,25)+1\) minute stopped \(\mathrm{P}(\) Total Journey \(<50)=\mathrm{P}(X<49)\) \(=0.421\) ( \(0.42074 \ldots\)...)``` | $\begin{gathered} \text { B1 } \\ \text { B1 } \\ \text { M1 } \\ \text { A1 } \\ {[4]} \end{gathered}$ | For Normal and mean <br> For variance oe, using $\mathrm{N}(51,25)$ and $\mathrm{P}(<50)$ BC |  |
|  | (c) | $\begin{aligned} & \text { Total time : } \mathrm{N}(145,80) \\ & \mathrm{P}(>150)=0.288 \quad(0.28807 \ldots) \end{aligned}$ | $\begin{aligned} & \hline \mathbf{B 1} \\ & \text { B1 } \\ & \text { B1 } \\ & {[3]} \\ & \hline \end{aligned}$ | For mean For variance BC | $\begin{aligned} & \text { Or mean }=5 \times 29 \\ & \text { variance }=5 \times 4^{2} \end{aligned}$ |
|  | (d) | It seems likely that the assumption is valid, since it is unlikely that a delay on one day would affect another day. | $\begin{aligned} & \hline \text { B1 } \\ & {[1]} \end{aligned}$ | Allow sensible alternative such as 'There may be bad weather for a particular week which might make journeys slower and mean that the assumption is not valid.' <br> 'There may be roadworks for a period of some days which would affect journey times and mean that the assumption is not valid.' | Or eg 'Not independent because if buses run late on one day, it may affect the service on the next day' |


| Question |  | Answer | Marks | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | (a) | $\begin{aligned} & {\left[\mathrm{F}(2)=1 \text { so } k\left(12 \times 2-2^{2}\right)=1\right.} \\ & \text { So } 20 k=1 \text { or } k=1 / 20 \text { so } k=0.05 \end{aligned}$ | $\begin{aligned} & \hline \text { M1 } \\ & \text { A1 } \\ & {[2]} \\ & \hline \end{aligned}$ |  |  |
|  | (b) | $\begin{aligned} & \mathrm{P}(1 \leq X \leq 1.5)=\mathrm{F}(1.5)-\mathrm{F}(1) \\ & =0.05\left(12 \times 1.5-1.5^{2}\right)-0.05\left(12 \times 1-1^{2}\right) \\ & =0.2375 \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \\ & {[2]} \end{aligned}$ |  |  |
|  | (c) | $\begin{aligned} & 0.05\left(12 m-m^{2}\right)=0.5 \\ & 0.05 m^{2}-0.6 m+0.5=0 \\ & m=0.901 \text { (reject } m=11.099 \text { ) } \end{aligned}$ | $\begin{gathered} \text { M1 } \\ \text { M1 } \\ \text { A1 } \\ {[3]} \end{gathered}$ | BC | Accept $6-\sqrt{ } 26$ (0.9009) |
|  | (d) | $\begin{aligned} \mathrm{f}(x) & =0.05(12-2 x) \\ \mathrm{E}(X) & =\int_{0}^{2} 0.05 x(12-2 x) \mathrm{d} x \text { or } \int_{0}^{2} x\left(\frac{3}{5}-\frac{1}{10} x\right) \mathrm{d} x \\ & =0.933 \end{aligned}$ <br> Maximum value (in range of $x$-values) $=0$ so mode $=0$ <br> The mean is the largest. | M1 <br> M1 <br> A1 <br> M1 <br> A1 <br> [5] | For differentiation <br> BC <br> For using range <br> Dep on all 3 values correctly obtained | Max M1M1A1M0A0 if the mode is not found <br> Must justify the value of the mode |


| Question |  | Answer | Marks | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | (a) | $\begin{aligned} & \text { Sample mean }=262.975 \\ & \text { Sample standard deviation }=1.213(1.2127 \ldots) \end{aligned}$ | $\begin{aligned} & \hline \text { B1 } \\ & \text { B1 } \\ & {[2]} \end{aligned}$ | $\begin{aligned} & \hline \mathbf{B C} \\ & \mathbf{B C} \end{aligned}$ |  |
|  | (b) | Normal probability plot is roughly straight Very high $p$-value <br> Both suggest that the data may be Normally distributed | $\begin{aligned} & \text { E1 } \\ & \text { E1 } \\ & \text { E1 } \\ & \text { [3] } \end{aligned}$ | Dep on at least one previous E mark | Do not allow ‘strong correlation' |
|  | (c) | $\mathrm{H}_{0}: \mu=264 \quad \mathrm{H}_{1}: \mu<264$ <br> Where $\mu$ is the population mean melting temperature <br> Test statistic is $\frac{262.975-264}{1.213 / \sqrt{8}}$ $=-2.391$ <br> Refer to $t_{7}$ <br> Critical value (1-tailed) at $5 \%$ level is 1.895 <br> $-2.391<-1.895$ so significant (reject $\mathrm{H}_{0}$ ) <br> Sufficient evidence to suggest that the batch might be contaminated with another type of nylon | $\begin{gathered} \text { B1 } \\ \text { B1 } \\ \text { M1 } \\ \text { A1 } \\ \text { M1 } \\ \text { A1 } \\ \text { M1 } \\ \text { A1 } \\ \hline 8] \end{gathered}$ | Hypotheses in words only must include "population". <br> For definition in context. <br> FT their mean and/or sd <br> Must be $t_{7}$ <br> Or $2.391>1.895$ | No marks for Wilcoxon test unless in part (b) candidate says 'not Normal' <br> If two-tailed test used can get all marks except first B1 with critical value $=2.365$ <br> If critical value $=2.365$ but one-tailed test allow Max M1A0M1E0 for last 4 marks |
|  | (d) | A test based on the Normal distribution | $\begin{aligned} & \text { E1 } \\ & \text { [1] } \end{aligned}$ |  |  |


| Question |  | Answer | Marks | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 6 | (a) | Confidence interval does not suggest that the mean weight is different from 250 g because the upper bound is 250.1255 so the interval contains 250 | $\begin{aligned} & \hline \text { B1 } \\ & \text { B1 } \\ & {[2]} \end{aligned}$ |  | Max B1B0 if no element of doubt. Max B1B0 if upper bound not stated. |
|  | (b) | Confidence interval is given by $248.92 \pm 1.645 \times 0.61506$ <br> $247.91<\mu<249.93$ or $248.92 \pm 1.01$ so the manager is correct | B1 <br> M1 <br> A1 <br> [3] | For 1.645 |  |
|  | (c) | No, it is not appropriate as the size of a confidence interval should be decided before the interval is calculated because otherwise the level can be adjusted to provide the conclusion that it desired. | $\begin{aligned} & \text { E1 } \\ & \text { E1 } \\ & \text { [2] } \end{aligned}$ | Must have some justification | Allow E1E0 BOD if 'No' and there is a rather unclear reason or if a larger sample is suggested |
|  | (d) | Width of interval $=2 \times 1.96 \times \sqrt{\frac{s^{2}}{40}}=1.9$ Variance $s^{2}=9.397$ | M1 <br> A1 <br> [2] |  | Must use 1.96 <br> NB Sample sd $=3.065$ which gets M1A0 |
|  | (e) | In repeated sampling, $95 \%$ of confidence intervals constructed in this way will contain the true population mean. | $\begin{aligned} & \text { E1 } \\ & \text { E1 } \\ & {[2]} \end{aligned}$ |  |  |

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